

CHAPTGR : 1.0 (PROPGRTIGS OF FLUID)

Actually the word hydraulics has been derived from the Greek word Hydous which means water.

Hydraulics is that branch of science which deals with water at rest as well as in motion.

It is that branch of engineering science which is based on experimental observation of water flow.

Fluid mechanics may be defined as the branch of engineering science which deals with the behaviour of fluid under the conditions of rest as well as in motion.

FLUID:~ Fluid may be defined as a substance which is capable of flowing. It may be liquid or gas. It has no definite shape of its own but conforms to the shape of the containing vessel. It may be divided into three parts statics, kinematics and dynamics.

Statics:~ The study of incompressible fluids under statics conditions is called hydrostatics and that dealing with the compressible fluids is termed as aerostatics.

Kinematics:~ It deals with the velocities, acceleration and the patterns of flow only. Forces or energy causing velocity and acceleration are not dealt under this heading.

Dynamics: - The branch of science which deals with the pressure forces are considered for the fluids in motion. In other words forces, energy causing velocity and acceleration are considered under this heading.

Physical properties of fluid: - The following are the properties of fluid such as Density, ~~specific~~ specific volume, specific weight, specific gravity.

Density or Mass Density: - Density or mass density of a fluid is defined as the ratio of the mass of fluid to its volume. Mathematically

$$\rho = \frac{\text{Mass of the fluid}}{\text{Volume of the fluid}} = \frac{m}{V}$$

$$\boxed{\rho = \frac{m}{V} \text{ Kg/m}^3}$$

In S.I system $\rho = \text{kg/m}^3$. The value of density for water is 1 g/cm^3 or 1000 kg/m^3

Specific Volume: - Specific volume of a fluid is defined as the volume per unit mass. It is denoted by v . Mathematically

$$v = \frac{\text{Volume of the fluid}}{\text{mass of the fluid}} = \frac{V}{m}$$

$$v = \frac{1}{\text{mass of fluid}} = \frac{1}{\rho} \quad \therefore v = \frac{1}{\rho}$$

Therefore specific volume is the reciprocal of density or mass density $\nu = \frac{m^3}{kg}$. It is commonly applied to gases.

specific weight or weight Density:~ Specific weight or weight density of a fluid may be defined as the ratio of the weight of the fluid to its volume. Mathematically $(w) = \frac{\text{Weight of the Fluid}}{\text{Volume of the Fluid}}$.

But weight is nothing but force.

$$(w) = \frac{\text{mass of the fluid} \times \text{Acceleration due to gravity}}{\text{Volume of the fluid}}$$

$$(w) = \frac{m \times g}{V} = \frac{m}{V} \times g \quad \therefore w = fg$$

For the purposes of all calculations, relating to hydraulics and hydraulic machines, the value of specific weight or weight density (w) for water is $9.81 \times 1000 \text{ N/m}^3$ or, 9.81 KN/m^3 . In S.I system. In M.K.S system the unit of 'w' is 1000 Litres/m^3 .

specific gravity:~ Specific gravity is defined as the ratio of the weight density (Density) of a fluid to the weight density (Density) of a standard fluid. For liquids the standard fluid is taken as water as a standard substance at 4°C . For gases the standard fluid is taken as air. It is also called as relative density. It has no unit and is denoted by 's'.

Mathematically (s) for liquid = $\frac{\text{wt Density of liquid}}{\text{wt Density of water}}$

$$\text{or, } s = \frac{\text{Density of liquid}}{\text{Density of water.}}$$

$$(s) \text{ for gases} = \frac{\text{wt Density of gas}}{\text{wt Density of air}}$$

$$(s) \text{ for gases} = \frac{\text{Density of gas}}{\text{Density of air.}}$$

$$\text{wt Density of liquid} = (s) \times \text{wt Density of water.}$$

$$= s \times 1000 \times 9.81 \text{ N/m}^3$$

$$= s \times 9.81 \text{ KN/m}^3$$

$$\boxed{\text{wt Density of liquid} = s \times 1000 \times 9.81 \text{ N/m}^3}$$

$$= s \times 9.81 \text{ KN/m}^3$$

If the specific gravity of a fluid is known. Then the density of the fluid will be equal to sp. gravity of fluid multiplied by the density of water. For example, the specific gravity of mercury is 13.6, hence density of mercury = $13.6 \times 1000 = 13600 \text{ kg/m}^3$.

Problem: Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.

Solution: Data given.

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad [1 \text{ litre} = 1000 \text{ cm}^3]$$

from to mass $\frac{M}{N_A N_t} \times \text{mole specific weight} = \text{volume of fluid}$, we know (M) problem (iii)

$$(\text{ans}) \quad \frac{\rho_w}{N_A N_t} = 6989 \cdot m \cdot$$

$$\frac{18.6 \times 1000}{N_A N_t} = B_f = (\text{m}) \text{ specific weight} (\text{m}) \quad (\text{ii})$$

$$(\text{ans}) \quad \frac{\rho_w}{B_f} \cdot 1000 = S$$

$$\frac{\rho_w}{B_f} \cdot 1000 = 1000 \times 1.0 =$$

from to figure (i) $\times S = f$ related to figure (i), as
from to figure (i) $= (S)$ Avogadro's number specific weight mole (i)
from to figure (i) $= (S)$ Avogadro's number specific weight (i)

$$1.0 = \text{specific weight} = S$$

$$\frac{\rho_w}{B_f} \cdot 1000 = \frac{\rho_w}{1000} = \text{specific weight} = \text{specific weight}$$

Solution \sim Data given

$$1.0 = \text{specific weight}$$

→ need to get f for every one to problem and
problem \sim calculate the density, specific weight

$$(\text{ans}) \quad S = 0.7135 \quad \text{or, } S = 0.7135$$

$$\frac{1000}{1.0} = \frac{\text{mass of water}}{\text{mass of air}} = (S) \text{ specific gravity} \quad (\text{iii})$$

(ans)

$$\frac{\rho_w}{\rho_a} \cdot 5.41t = f \quad \left[\frac{\rho_w}{B_f} \cdot S \cdot 1.0 = \frac{18.6}{1000 \cdot 1.0} = \frac{B_f}{m} = f \right]$$

$$\frac{B_f}{m} = f, \text{ as } B_f = (m) \text{ useful mass, we know } (f) \text{ figure (i)} \quad (\text{ii})$$

$$(\text{ans}) \quad \frac{\rho_w}{N_A N_t} \cdot 1000 = m$$

$$\frac{\rho_w}{N_A N_t} \cdot 1000 = 1000 \times 1.0 = \frac{\rho_w}{N_A N_t} =$$

$$\frac{\text{mass of air}}{\text{mass of water}} = (m) \text{ volume of water} \quad (\text{ii})$$

$$N_t = \text{volume}$$

$$\text{or, } w = \frac{W}{\text{Volume}} \quad \text{or, } W = w \times \text{volume of fluid}$$

$$\text{or, } W = 6867 \times 0.001 = 6.867 \text{ N}$$

$$\text{or, } W = 6.867 \text{ N} \quad (\text{Ans})$$

Problem: Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of 6 m^3 and weight of 44 KN .

Solution: Data given.

$$\text{Volume of liquid} = 6 \text{ m}^3$$

$$\text{Weight of liquid} = 44 \text{ KN}$$

$$(i) \text{ Specific weight } (w) = \frac{\text{Weight of liquid}}{\text{Volume of liquid}}$$

$$\text{or, } w = \frac{44}{6} = 7.333 \text{ KN/m}^3$$

(ii) Specific mass or mass density (ρ)

$$\text{we know that } w = \rho g \therefore \rho = \frac{w}{g} = \frac{7.333 \times 1000}{9.81}$$

$$\text{or, } \rho = 747.5 \text{ kg/m}^3$$

$$(iii) \text{ Specific volume } (v) = \frac{1}{\rho} = \frac{1}{747.5} = 0.00133 \frac{\text{m}^3}{\text{kg}}$$

$$\therefore v = 0.00133 \frac{\text{m}^3}{\text{kg}}$$

$$(iv) \text{ Specific gravity } (s) = \frac{\text{Weight Density of liquid}}{\text{Weight Density of water}}$$

$$\text{or, } s = \frac{7.333}{9.81} = 0.747 \quad \text{Ans}$$

Viscosity: Viscosity of a liquid is its property which controls its rate of flow.

Viscosity may be defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layers of the fluid.

Let us consider two layers of a fluid.

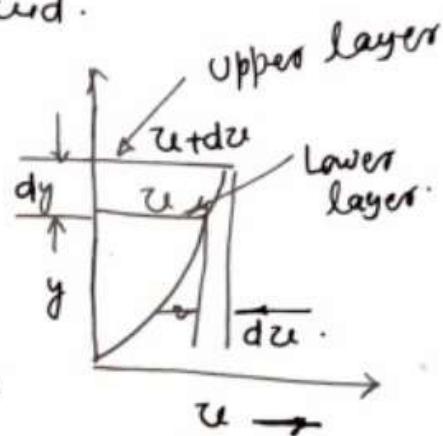
dy = The distance between two layers apart move one over the other at different velocities.

u = Velocity of lower layer

$u+du$ = Velocity of upper layer.

The viscosity together with relative velocity cause a shear stress acting between the fluid layers.

In the figure the top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is directly proportional to the rate of change of velocity with respect to y . Symbolically it is denoted by τ .



Mathematically $\tau \propto \frac{du}{dy}$ or $\boxed{\tau = \mu \frac{du}{dy}}$

$$\text{or, } \mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = \frac{\text{shear stress}}{\left(\frac{\text{change of velocity}}{\text{change of distance}}\right)}$$

$$\text{or, } \mu = \frac{\text{Force/Area}}{\left(\frac{\text{Length/Time}}{\text{Length}}\right)} = \frac{\text{Force} \times (\text{Length})^2}{\text{Length} \times \frac{\text{Time}}{\text{Length}}}$$

$$\text{or, } \mu = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2} = \frac{\text{Kg f - Sec}}{\text{m}^2} \text{ in M.K.S system.}$$

$$\text{or, } \mu = \frac{\text{Dyne - Sec}}{\text{cm}^2} \text{ in C.G.S system.}$$

$$\text{or } \mu = \frac{\text{Newton - sec}}{\text{m}^2} \text{ in S.I system.}$$

The unit of viscosity in C.G.S system is called as poise, which is equal to $\frac{\text{dyne - sec}}{\text{cm}^2}$

$$\mu = \frac{\text{Dyne - Sec}}{\text{cm}^2} = \text{poise} = \frac{1}{10} \frac{\text{N-S}}{\text{m}^2}$$

$$\text{Because } 1 \text{ Dyne} = 10^{-5} \text{ N.}$$

$$\mu = \frac{10^{-5} \text{ N-Sec}}{10^{-4} \text{ m}^2} = \frac{1}{10} \frac{\text{N-Sec}}{\text{m}^2}$$

$$\therefore \mu = \frac{1}{10} \frac{\text{N-Sec}}{\text{m}^2}$$

Kinematic Viscosity: Kinematic viscosity is defined as the ratio between dynamic viscosity and density of the fluid. It is denoted by $\nu(\text{nu})$.

$$\text{Mathematically } \nu = \frac{\text{Dynamic Viscosity}}{\text{Density}} = \frac{\mu}{\rho}.$$

$$\text{or, } \nu = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{\text{Volume}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}}$$

$$\text{or, } \nu = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}} = \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}. \quad [\because F=ma]$$

$$\text{or, } \boxed{\nu = \frac{(\text{Length})^2}{\text{Time}}}$$

Unit: In m.k.s and S.I system the unit of kinematic viscosity is m^2/sec .

In C.G.S system kinematic viscosity is written as cm^2/sec and is known as Stoke.

$$\nu(\text{nu}) = \frac{\text{cm}^2}{\text{sec}} = (10)^{-2} \frac{\text{m}^2}{\text{sec}} = 10^{-4} \text{m}^2/\text{sec}.$$

Since $1 \text{cm}^2 = 10^{-4} \text{m}^2$

one centistoke means $= \frac{1}{100} \text{ Stoke}$.

$$1 \text{ litres} = 1000 \text{ cm}^3$$

$$1 \text{ litres} = \frac{1}{1000} \text{ m}^3$$

Problem: A plate 0.05mm distant from a fixed plate moves at 1.2 m/sec and requires a force of 2.2 N/m^2 to maintain this speed. Find the viscosity of the fluid between the plates.

Solution: Data given

u = velocity of the moving plates = 1.2 m/sec.

$$dy = \text{Distance between the plates} = 0.05 \text{ mm} \\ = \frac{0.05}{1000} = 0.00005 \text{ m.}$$

F = Force on the moving plate = 2.2 N/m^2

du = change of velocity = $u - 0 = 1.2 - 0 = 1.2 \frac{\text{m}}{\text{sec}}$.

Viscosity of the fluid.

$$\text{We know that } \tau = \mu \frac{du}{dy} \therefore \mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

$$\text{or, } \mu = \frac{2.2}{\left(\frac{1.2}{0.00005}\right)} = \frac{2.2}{24000} = 9.17 \times 10^{-5} \frac{\text{N-s}}{\text{m}^2}$$

$$\text{or, } \mu = 9.17 \times 10^{-5} \times 10 = 9.17 \times 10^{-4}$$

$$\text{or, } \mu = 9.17 \times 10^{-4} \text{ poise} \quad \left| \begin{array}{l} \text{since} \\ 1 \text{ poise} = \frac{1}{10} \frac{\text{N-s}}{\text{m}^2} \end{array} \right.$$

Problem: The space between two square flat parallel plates is filled with oil. Each side of the plate is 720mm. The thickness of the oil film is 15mm. The upper plate, which moves

at 3 m/sec required a force of 120 N to maintain the speed. Determine (i) The Dynamic viscosity of the oil (ii) The Kinematic viscosity of oil if the specific gravity of oil is 0.95

Solution: Data given.

$$\text{Each side of a square plate} = 720 \text{ mm} = \frac{720}{1000} \text{ m} \\ = 0.72 \text{ m}$$

$$\text{The thickness of the oil} = dy = 15 \text{ mm} = \frac{15}{1000} \text{ m} \\ = 0.015 \text{ m}$$

$$\text{Velocity of the upper plate} = 3 \text{ m/sec.}$$

$$du = \text{change of velocity between plates} = 3 - 0 \\ = 3 \text{ m/sec.}$$

$$F = \text{Force required on upper plate} = 120 \text{ N.}$$

$$\text{We know that } \tau = \text{shear force} = \frac{\text{Force}}{\text{Area}} = \frac{120}{(0.72)^2}$$

$$\tau = 231.5 \text{ N/m}^2$$

$$(i) \text{ Dynamic viscosity } \mu. \\ \text{we know that } \tau = \mu \frac{du}{dy} \therefore \mu = \frac{\tau}{\frac{du}{dy}}$$

Putting all the values, we have

$$231.5 = \mu \times \frac{3}{0.015} \text{ or, } 3\mu = 231.5 \times 0.015$$

$$\therefore \mu = \frac{231.5 \times 0.015}{3} = 1.1574 \text{ N-s/m}^2$$

$$\therefore \mu = 1.1574 \frac{\text{N-s}}{\text{m}^2}$$

(ii) Kinematic viscosity (ν)

$$\text{we know that } s \text{ (specific gravity)} = \frac{\text{wt Density of oil}}{\text{wt Density of water}}$$

$$\text{or, wt density of oil} = s \times \text{wt density of water} \\ = 0.95 \times 9.81 \frac{\text{KN/m}^3}{\text{KN/m}^3} = 9.3195$$

$$w = 9.3195 \times 1000 = 9319.5 \text{ N/m}^3$$

mass density or Density of oil $\rho = \frac{w}{g}$ [$\because w = \rho g$]

$$\text{or, } \rho = \frac{9319.5}{9.81} = 950$$

We know that Kinematic viscosity $\nu = \frac{\mu}{\rho}$

$$\text{or, } \nu(\text{nu}) = \frac{\text{Dynamic Viscosity}}{\text{Density of fluid}} = \frac{1.1574}{950}$$

$$= 0.00121 \text{ m}^2/\text{sec}$$

$$\therefore \nu(\text{nu}) = 0.00121 \text{ m}^2/\text{sec}$$

Problem: If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in m/sec at a distance y metre above the plate. determine the shear stress at $y=0$ and $y = 0.15\text{m}$. Take dynamic viscosity of fluid as 8.63 poise.

Solution: Data given.

$$u = \frac{2}{3}y - y^2, \frac{du}{dy} = \frac{2}{3} - 2y$$

$$\text{At } y=0, \frac{du}{dy} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

$$\text{At } y = 0.15 \quad \frac{du}{dy} = \frac{2}{3} - 2(0.15) = \frac{2}{3} - 0.30 \\ = 0.667 - 0.30 = 0.367$$

$$\text{Value of } \mu = 8.63 \text{ poise} = \frac{8.63}{10} \text{ SI units} = 0.863 \text{ N-s/m}^2$$

$$\left[\because 1 \text{ poise} = \frac{1}{10} \frac{\text{N-s}}{\text{m}^2} \right]$$

We know that shear stress $\tau = \mu \frac{du}{dy}$

(i) shear stress at $y=0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy} \right)_{y=0} = 0.863 \times 0.667 = 0.5756$$

$$\tau_0 = 0.5756 \text{ N/m}^2 \quad (\text{Ans})$$

(ii) Shear stress at $y = 0.15$

$$\tau_{y=0.15} = \mu \left(\frac{du}{dy} \right)_{y=0.15} = 0.863 \times 0.367 \\ = 0.3167 \text{ N/m}^2$$

$$\tau_{y=0.15} = 0.3167 \text{ N/m}^2$$

Problem: If the velocity distribution of a fluid over a plate is given by $u = \frac{3}{4}y - y^2$ where u is the velocity in metres per second at a distance of y metres above the plate. Determine the shear stress at $y = 0.15$ metre. Take dynamic viscosity of the fluid as $8.34 \times 10^{-4} \frac{\text{N-s}}{\text{m}^2}$

Solution: Data given.

$$u = \frac{3}{4}y - y^2, y = 0.15, \mu = 8.34 \times 10^{-4} \frac{\text{N-s}}{\text{m}^2}$$

$$\frac{du}{dy} = \text{velocity gradient} = \frac{3}{4} - 2y$$

velocity gradient at $y = 0.15 \text{ m}$

$$\left(\frac{du}{dy} \right) \text{ at } y = 0.15 = \frac{3}{4} - 2(0.15) = 0.75 - 0.30 \\ = 0.45 \text{ m/sec.}$$

Shear stress $\tau = \mu \frac{du}{dy}$

Putting all the values, we get.

Putting all the values we have $\frac{hp}{n^2} = 2^{120}$

$$\left[\frac{10}{5-N} = 8 \text{ police} \right] \quad \frac{2^{10}}{5-N} \cdot \frac{10}{8} =$$

$$0.50 - \frac{3}{2} \left(0.09 - \frac{2}{3} \right) = 0.09 \quad \text{or} \quad \frac{B_p}{n_p}$$

$$\frac{h}{L} \frac{\frac{G}{3} - \frac{G}{3}}{\frac{G}{3} - \frac{G}{3}} = \frac{\frac{h}{L} \frac{G}{3} - \frac{G}{3}}{\frac{G}{3} - \frac{G}{3}} = \frac{hp}{np}, \text{ so}$$

$$1 - \frac{h\frac{\epsilon}{\delta}}{\epsilon} - \frac{\epsilon}{\delta} = \frac{hp}{np} \cdot \frac{\epsilon/\delta}{h - h\frac{\epsilon}{\delta}} = \sigma_2$$

Solution: Data given.

• 2510d 8.0 80

over a flat plane of dimensions $l \times b$, $\frac{dy}{dt} = v$, $\frac{dv}{dt} = a$.
 The distance between the two surfaces is y , and the distance above
 the plate - determine the shear stress τ at a distance y from the
 plate. Assume dynamic viscosity η .

problem: The velocity distribution for flow

$$\frac{c_{al}}{\cancel{N} - N} \quad h_1^{01} \times 45t \cdot e = 2$$

$$\frac{34 \times 10}{N-562} \times 0.45 = 3.753 \times 10^{-4}$$

$$\tau = 0.8 \times 1.05 = 0.84 \text{ N/mm}^2$$

Problem: A flat plate of area $1.5 \times 10^6 \text{ mm}^2$ is pulled with a speed of 0.4 m/sec relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.

Solution: Data given

$$\text{Area of plate } A = 1.5 \times 10^6 \text{ mm}^2 = \frac{1.5 \times 10^6}{10^6} \text{ m}^2$$

$$\therefore A = 1.5 \text{ m}^2$$

speed of plate relative to another plate

$$du = 0.4 - 0 = 0.4$$

$$dy = \text{Distance between the plate} = \frac{0.15 \text{ mm}}{1000} = 0.00015 \text{ m.}$$

$$\mu = \text{Viscosity} = 1 \text{ poise} = \frac{1}{10} \frac{\text{N-s}}{\text{m}^2}$$

$$\text{We know that } \tau = \mu \frac{du}{dy} = \frac{1}{10} \times \frac{0.4}{0.00015}$$

$$\tau = 266.66 \text{ N/mm}^2$$

$$(1) \quad \tau = \frac{F}{A} = \frac{\text{Force}}{\text{Area}} \quad \therefore \text{Shear force} = \tau \times \text{Area}$$

$$F = 266.66 \times 1.5 = 399.99 = 400 \text{ N.}$$

$$(2) \quad \text{Power required to move the plate at the speed } 0.4 \text{ m/sec.} = F \times u = 400 \times 0.4 = 160 \text{ W}$$

$$P = 160 \text{ W}$$

Problem: Find the Kinematic viscosity of an oil having density 981 kg/m^3 . The shear stress at a point in oil is 0.2452 N/m^2 and velocity gradient at that point is 0.2 per second.

Solution: Data given.

$$\rho = \text{mass density} = \text{Density} = 981 \text{ kg/m}^3$$

$$\tau = \text{shear stress} = 0.2452 \text{ N/m}^2$$

$$\frac{du}{dy} = \text{velocity gradient} = 0.2/\text{sec}$$

$$\text{we know that } \tau = \mu \frac{du}{dy}$$

$$\text{or, } 0.2452 = \mu \times 0.2 \quad \therefore \mu = \frac{0.2452}{0.2} = 1.226 \frac{\text{N-s}}{\text{m}^2}$$

$$\mu = 1.226 \frac{\text{N-s}}{\text{m}^2}$$

$$\text{Kinematic viscosity } \nu(\text{in m}) = \frac{\mu}{\rho} = \frac{1.226}{981}$$

$$= 0.00124 \text{ m}^2/\text{sec}$$

$$= 0.00124 \times 10^4 \text{ cm}^2/\text{sec} \quad [\because 1 \text{ m}^2 = (10^2)^2 = 10^4 \text{ cm}^2]$$

$$= 12.40 \text{ cm}^2/\text{sec} = 12.40 \text{ Stoke}$$

$$\nu = 12.40 \text{ Stoke}$$

problem: Determine the specific gravity of a fluid having viscosity 0.05 Poise and Kinematic viscosity 0.035 Stokes.

Solution: Data given.

$$\mu = \text{viscosity} = 0.05 \text{ poise}, \nu = \text{kinematic viscosity} = 0.035 \text{ Stoke}$$

$$\mu = 0.005 \text{ Poise} = \frac{0.005}{10} = 0.0005 \text{ N s} \cdot \frac{\text{m}}{\text{m}^2}$$

$$\gamma = 0.035 \text{ Stoke} = 0.035 \text{ cm}^2/\text{sec} = \frac{0.035}{0.035 \times 10^{-4}} \text{ m}^2/\text{sec}$$

$$[\because 1 \text{ Stoke} = \text{cm}^2/\text{sec}]$$

$$\text{Kinematic viscosity } \nu = \frac{\mu}{\rho} = \frac{0.005}{\rho}$$

$$\therefore 0.035 \times 10^{-4} = \frac{0.005}{\rho} \therefore \rho = \frac{0.005}{0.035 \times 10^{-4}}$$

$$\text{or, } \rho = \frac{0.005 \times 10^4}{0.035} = \cancel{0.005} 1428.57 \text{ kg/m}^3$$

$$\text{specific gravity of liquid} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$(S) = \frac{1428.57}{1000} = 1.4285 = 1.43$$

$$(S) = 1.43 \quad (\text{Ans})$$

Problem :- Determine the viscosity of a liquid having Kinematic viscosity 6 Stokes and specific gravity 1.9

Solution :- Data given.

$$\nu = \text{Kinematic viscosity} = 6 \text{ Stokes} = 6 \frac{\text{cm}^2}{\text{sec}}$$

$$\nu = 6 \times 10^{-4} \text{ m}^2/\text{sec} \quad [\because 1 \text{ cm} = \frac{10^{-2}}{10^2} \text{ m}, 1 \text{ cm}^2 = \frac{10^{-4}}{10^4} \text{ m}^2]$$

$$\text{sp. gravity of liquid} = 1.9$$

Let the viscosity of liquid = μ .

Now sp. gravity of a liquid = $\frac{\text{Density of liquid}}{\text{Density of water}}$

$$1.9 = \frac{\text{Density of liquid}}{1000}$$

$$\therefore \text{Density of liquid} = 1000 \times 1.9 \\ = 1900 \text{ kg/m}^3$$

$$\text{We know that } \gamma(\text{nm}) = \frac{\mu}{l} \quad \text{or, } 6 \times 10^{-4} = \frac{\mu}{1900} \therefore \mu = 11400 \times 10^{-4}$$

$$\text{or, } \mu = 1.14 \text{ N/m}^2 \text{ or, } \mu = 1.14 \times 10^{-4} \text{ lb/in}$$

SURFACE TENSION :- Surface Tension

is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by σ . In m.s it is expressed as kgf/m while in SI units as N/m. Examples :- (i) Rain drops (A falling rain drop becomes spherical due to cohesion and Surface Tension).

- (ii) Birds can drink water from ponds.
- (iii) Collection of dust particles on water surface.
- (iv) Break up of liquid jets.

Capillarity: Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as Capillary rise while the fall of liquid surface is known as Capillary depression.

Let d = Diameter of the Capillary tube.

θ = Angle of Contact of the Water Surface.

h = Height of Capillary rise.

σ = Surface Tension force for unit length.

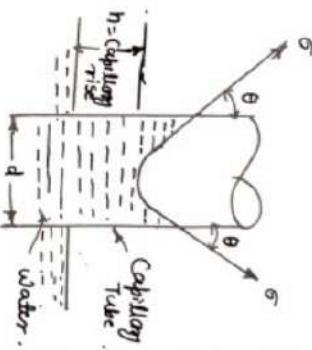
w = weight Density (ρg).

Now upward Surface tension force (Lifting force) = weight of the water column in the tube.

$$\pi d \cdot \sigma \cos\theta = \frac{\pi}{4} d^2 \rho g h \quad \therefore h = \frac{4\sigma \cos\theta}{w d}$$

For water and glass $\theta = 0$
Hence the Capillary rise of water in the glass tube

$$h = \frac{4\sigma}{w d}$$



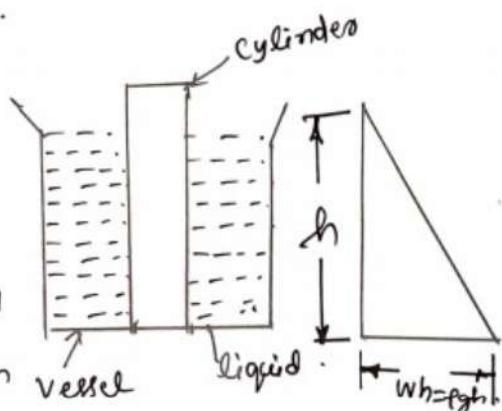
CHAPTER : 02 FLUID PRESSURE & ITS MEASUREMENTS.

FLUID PRESSURE :- When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom (and top) of the container. The force per unit area is called pressure. Pressure at a point is also called as intensity of pressure.

$$\text{Mathematically } (P) = \frac{F}{A}$$

Pressure Head of a liquid :- A liquid is subjected to pressure due to its own weight; this pressure increases as the depth of the liquid increases.

Let us consider a vessel containing liquid as shown in the figure. This liquid will exert pressure on vessel all sides and bottom of the vessel. Now let a cylinder be made to stand in liquid as shown.



Let h = height of liquid in the cylinder

A = Area of the cylinder base

w = Specific wt of the liquid.

P = Intensity of pressure.

Now total force/pressure force on the base of cylinder
= weight of the liquid in the cylinder

$$P \cdot A = wAh$$

[$\because Ah = \text{volume}$

$$w = \frac{W}{\text{volume}}$$

$$W = W \times \text{volume} = wAh$$

$$\text{or, } P = \frac{wAh}{A} = wh$$

$$P = \frac{F}{A} \therefore F = P \cdot A$$

$$\text{or, } P = wh = fgh.$$

As $P = wh$, the intensity of pressure in a liquid due to its depth will vary directly with depth.

Pressure head: As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a liquid pressure by the height of the free surface which would cause the pressure.

$$P = wh = fgh$$

$$\text{or, } h = \frac{P}{w} = \frac{P}{fg}$$

The height of the free surface above any point is known as the static pressure head at that point. The static pressure head is h . It is expressed in m, cm, mm etc.

Hence the intensity of pressure of a liquid may be expressed in two ways.
(1) As a force per unit area ($\text{in } N/m^2, N/mm^2 \text{ etc}$).
(2) As an equivalent static pressure head (in m, mm etc).

Problem :- Find the pressure at a depth of 15m below the free surface of water in a reservoir

Solution :- Data given.

$$h = \text{Depth of water} = 15\text{m}$$

$$w = \text{Specific weight of water} = 9.81 \text{ kN/m}^3$$

$$\text{We know that } p = wh = 9.81 \times 15 = 147.15 \frac{\text{kN}}{\text{m}^2}$$

$$P = 147.15 \frac{\text{kN}}{\text{m}^2} = 147.15 \text{ kPa}$$

$$[\because p = \frac{F}{A}]$$

Problem :- Find the height of water column corresponding to a pressure of 54 kN/m^2

Solution :- Data given.

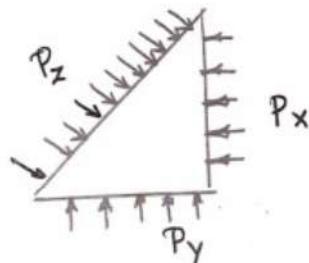
$$P = 54 \frac{\text{kN}}{\text{m}^2}, w = \text{specific wt of water} = 9.81 \frac{\text{kN}}{\text{m}^3}$$

$$h = \text{Height of water column} \quad P = wh$$

$$\text{or, } h = \frac{P}{w} = \frac{54}{9.81} \frac{\text{kN}}{\text{m}^2} = 5.5 \text{ m}$$

Statement of Pascal's Law :- This law states that "The intensity of pressure at any point in a fluid at rest, is the same in all directions."

$$\text{i.e. } P_x = P_y = P_z$$



Atmospheric pressure :- The pressure exerted by the atmosphere on a surface is called atmospheric pressure.

Atmospheric pressure is different at different heights above the earth sea level. The value of standard atmospheric pressure is taken as 1.01325 bars or 760mm of Hg.

Pressure Gauge: It is the pressure measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. In other words, the atmospheric pressure on the gauge scale is marked as zero. Gauge pressure is above the atmospheric pressure.

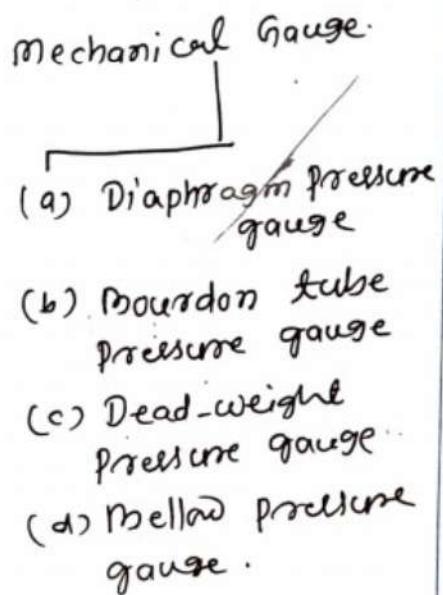
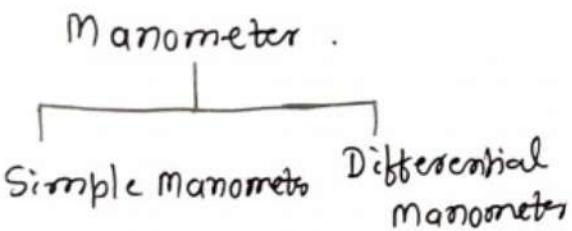
Vacuum Pressure: Vacuum pressure is defined as the pressure below the atmospheric pressure.

Absolute pressure: It is the pressure equal to the algebraic sum of atmospheric pressure and gauge pressure.

$$\therefore \text{Absolute pressure} = \text{Atmospheric pressure} + \text{Gauge pressure}$$

$$P_{\text{absolute}} = P_{\text{atmospheric}} + P_{\text{gauge}}$$

Measurement of Pressure:~ The pressure of a fluid is measured by the following devices.



Simple manometer:~ A simple manometer is a device used for measuring the pressure at a point in a fluid. It consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are
(i) piezometer (ii) U-tube manometer &
(iii) single column manometer.

Piezometer:~ It is the simplest form of manometer. It is used for measuring gauge pressure. One end of this manometer is connected to the point where pressure is to be measured and other end is

open to the atmosphere.

The rise of liquid gives

the pressure head

at that point. If

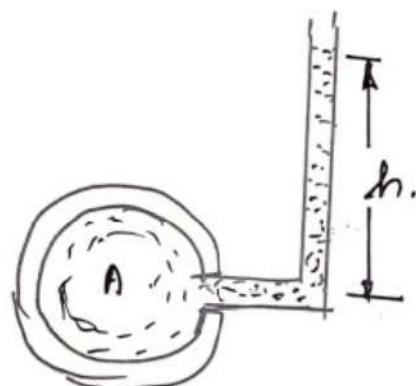
at a point A, the

height of liquid say

water is h in piezometer tube, then

$$\text{Pressure at } A = \rho gh \text{ N/m}^2$$

$$\text{Pressure at } A = \rho gh \text{ N/m}^2$$



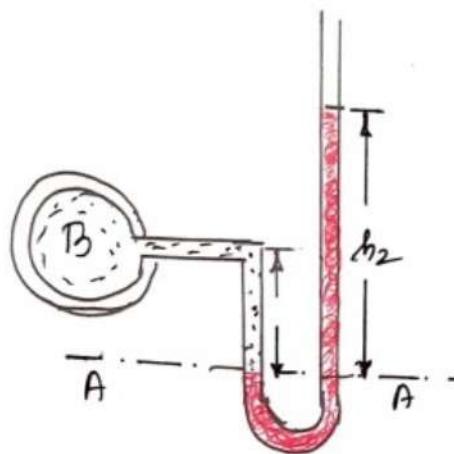
U-Tube Manometer :- It consists of glass tube bent in U-shape. One end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere.

The tube contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

For Gauge pressure :- From the figure

Let B is the point at which pressure is to be measured whose value is P.

The datum line is A-A.



For gauge pressure.

Let h_1 = Height of light liquid above datum line.
 h_2 = Height of heavy liquid above datum line.
 s_1 = sp. gravity of light liquid.
 s_2 = sp. gravity of heavy liquid.

$$\rho_1 = \text{Density of light liquid} = 1000 \times s_1$$

$$\rho_2 = \text{Density of heavy liquid} = 1000 \times s_2$$

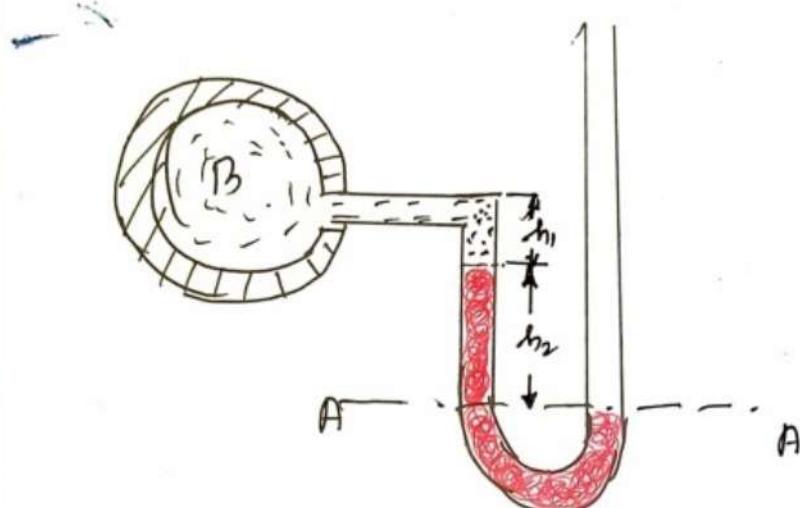
As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

$$\text{Pressure above A-A in the Left Column} \\ = P + \rho_1 \times g \times h_1$$

$$\text{Pressure above A-A in the right Column} \\ = \rho_2 \times g \times h_2$$

$$\text{Hence equating the two pressures} \\ P + \rho_1 g h_1 = \rho_2 g h_2 \therefore P = (\rho_2 g h_2 - \rho_1 g h_1)$$

For Vacuum Pressure : For measuring vacuum the level of the heavy liquid in the manometer will be shown in the figure.



Pressure above A-A in the Left Column

$$= \rho_2 gh_2 + \rho_1 gh_1 + P$$

Pressure head in the right Column

above A-A = 0

$$\text{Then } \rho_2 gh_2 + \rho_1 gh_1 + P = 0$$

$$\therefore P = -(\rho_2 gh_2 + \rho_1 gh_1)$$

Problem : The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the Left Limb is connected to a pipe in which a fluid of sp gravity 0.9 is

flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limb is 20cm.

Solution: Data given.

$$\text{sp gravity of fluid} = 0.9$$

$$\text{Density of fluid} = \rho_1 = s_1 \times 1000$$

$$= 0.9 \times 1000 = 900 \frac{\text{kg}}{\text{m}^3}$$

$$\text{sp gravity of mercury}$$

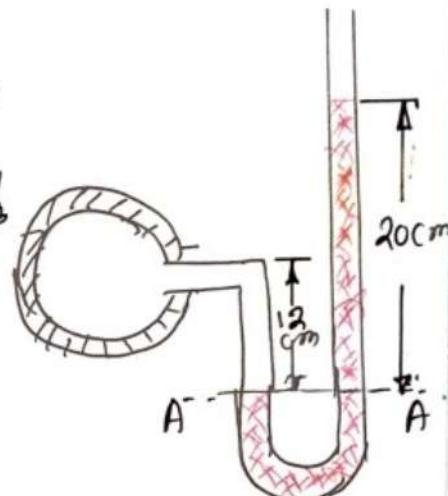
$$s_2 = 13.6$$

$$\text{Density of mercury}$$

$$\rho_2 = s_2 \times 1000$$

$$= 13.6 \times 1000$$

$$= 13600.00 \frac{\text{kg}}{\text{m}^3}$$



$$\text{Difference of mercury level} = h_2 = 20\text{cm} = \frac{20}{100} \\ = 0.2\text{m.}$$

Height of fluid from AA'

$$h_1 = 20 - 12 = 8\text{cm} = \frac{8}{100} = 0.08\text{m}$$

Let P = Pressure of fluid in pipe

Equating the pressure above AA', we have

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$P + 900 \times 9.81 \times 0.08 = 13600 \times 9.81 \times 0.2$$

$$\text{or } P + 706.32 = 26683.20 \therefore P = \underline{26683.20} -$$

$$P = 26683.2 - 706.32 = 25976.8 \frac{\text{N}}{\text{m}^2} = \frac{25976.8}{10^4} \\ = 2.5976 \frac{\text{N}}{\text{cm}^2} (10\text{m})$$

U-TUBE DIFFERENTIAL MANOMETER: ~ we know

that Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes.

Let us consider a differential manometer of U-tube type. Here there are two points A and B at different levels and also contains liquids of different sp. gravity. These points are connected to the U-tube manometer. From figure 1

Let the pressure at A and B are P_A & P_B .

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

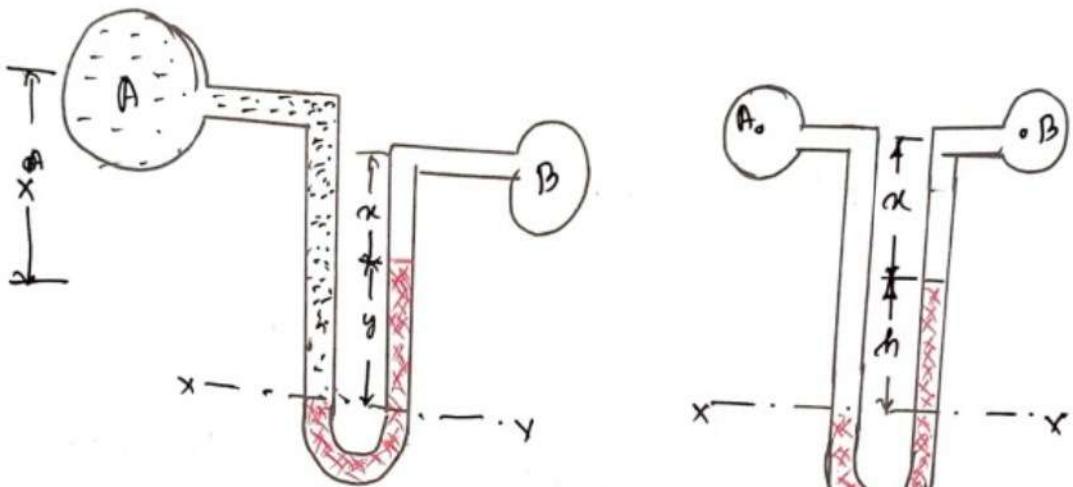
x = Distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.



Two pipes of different levels.
figure 1.

A & B are at the
same level.
figure 2

Pressure above x-x in the Left limb = $\rho_1 g(h+x) + P_A$
where P_A = pressure at A.

Pressure above x-x in the right limb =
 $\rho_2 g x h + P_2 g x y + P_B$

where P_B = pressure at B.

Equaling the two pressure, we have

$$\rho_1 g(h+x) + P_A = \rho_2 g x h + \rho_2 g y + P_B$$

$$P_A - P_B = \rho_2 g x h + \rho_2 g y - \rho_1 g(h+x)$$

$$= h \times g (\rho_2 - \rho_1) + \rho_2 g y - \rho_1 g x$$

Difference of pressure at A and B

$$= h \times g (\rho_2 - \rho_1) + \rho_2 g y - \rho_1 g x$$

From figure (2) the two points A and B are at the same level and contains the same liquid of density ρ_1 .

Pressure above x-x in right limb =

$$\rho_2 g x h + \rho_1 g x x + P_B$$

Pressure above $x-x$ in Left limb = $\rho_1 g x (h+x) + P_A$

Equating the two pressure.

$$\rho_g \times g \times h + \rho_1 g x + P_B = \rho_1 g x (h+x) + P_A$$

$$P_A - P_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g (h+x)$$

$$P_A - P_B = g \times h (\rho_g - \rho_1)$$

BOURDON TUBE PRESSURE GAUGE

Bourdon tube pressure gauge is used for measuring high as well as low pressure. A simple form of this gauge is shown in the figure. In this case, the pressure

elements consists of a metal tube of approximately elliptical cross-section. This tube is bent in the form of a segment of a circle and responds to pressure changes. When one end of the tube which is attached to the gauge case is connected to the source of pressure, the internal pressure causes the tube to expand. whereby circumferential stress ie hoop tension is set up. The free end of the tube moves and is in turn connected by suitable levers to a rack, which engages with a small pinion mounted on

CHAPTER : 03 HYDROSTATICS.

Hydrostatic Pressure :- The term 'Hydrostatic' means the study of pressure, exerted by a liquid at rest. The direction of such pressure is always perpendicular to the surface, on which it acts.

Total pressure :- It is defined as the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always at right angle (or normal) to the surface.

Centre of pressure :- It is defined as the point of application of the total pressure on the surface.

Total pressure on a Horizontally immersed surface

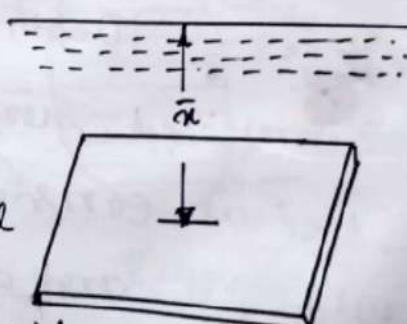
Let us consider a plane horizontal surface immersed in a liquid

Let A = Area of the immersed surface

\bar{x} = Depth of the horizontal surface from the liquid level in metre.

w = Specific weight of the liquid.

P = Total pressure on the surface = weight of the liquid above immersed surface.



$P = \text{Specific weight of liquid} \times \text{Volume of liquid}$

$$[w = \frac{W}{V} : w = wv]$$

or, $P = \text{specific weight of liquid} \times \text{Area of Surface} \times \text{Depth of liquid}$

$$P = w A \bar{x} = \varrho g A \bar{x} \text{ KN.}$$

Total pressure on a vertically immersed surface

Let us consider a plane vertical surface

immersed in a liquid as shown in the figure.

First of all, let us

divide the whole

immersed surface into

a number of small

parallel strips as shown in the figure.

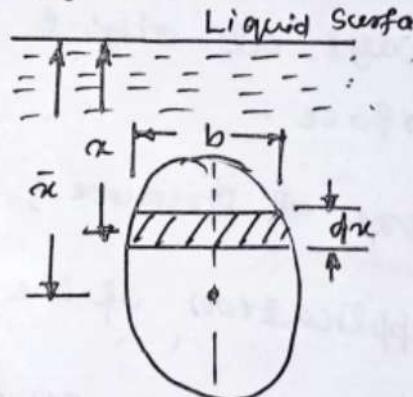
Let w = specific weight of the liquid.

A = Total Area of the immersed surface.

\bar{x} = Depth of centre of gravity of the immersed surface from the liquid surface.

Let us consider a strip of thickness dx , width b and at a depth x from the free surface of the liquid.

We know that, intensity of pressure on the strip, and area of the strip = wx .



and Area of the strip = $b \cdot dx$.

∴ Pressure on the strip = $P = \text{intensity of pressure} \times \text{Area}$

$$\text{or } P = w \cdot x \cdot b \cdot dx.$$

Now total pressure on the surface

$$P = \int w \cdot x \cdot b \cdot dx = w \int x \cdot b \cdot dx.$$

But $\int x \cdot b \cdot dx$ = Moment of the surface area about the liquid level.

$$= A\bar{x}$$

$$\therefore P = w A\bar{x} = \rho g A\bar{x}$$

Centre of Pressure of a vertically immersed surface.

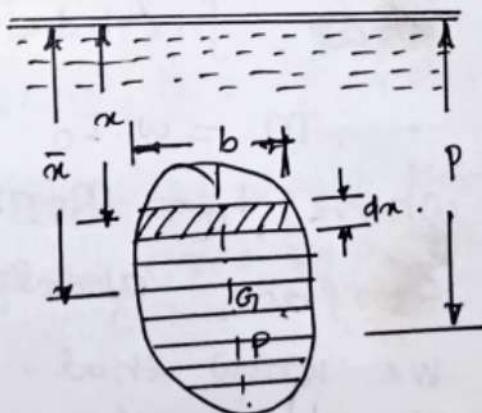
Let us consider a plane surface immersed vertically in a liquid as shown in the figure.

First of all, let us divide the whole immersed surface into a number of small parallel strips as shown in the figure.

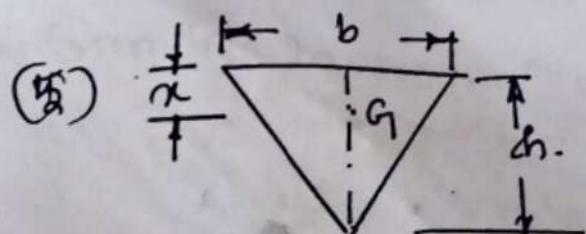
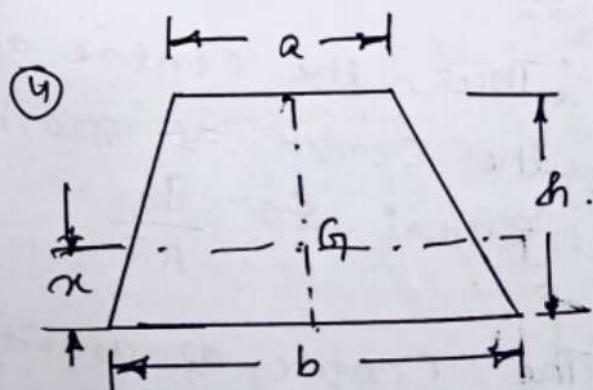
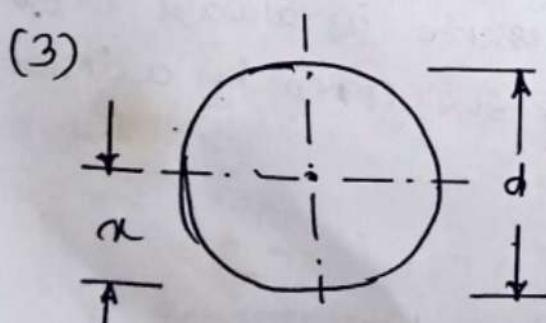
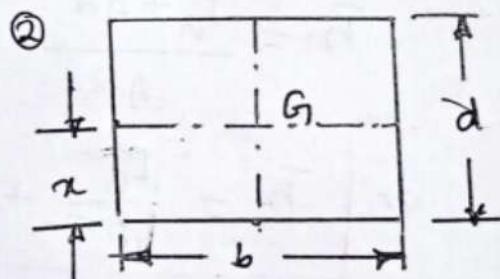
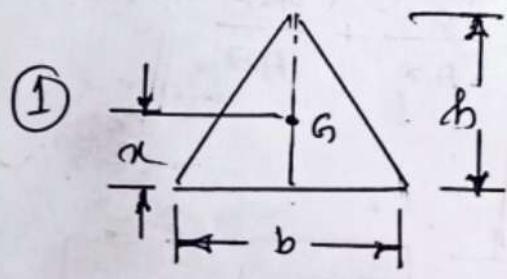
Let w = specific weight of the liquid.

A = Area of immersed surface.

\bar{x} = Depth of centre of gravity of the immersed surface from the liquid surface.



S.L No	Name of figure.	C.G from base.	Area	I about an axis passing through C.G and parallel to the base.	I about base.
1.	Triangle	$\alpha = \frac{b}{3}$	$\frac{bb}{2}$	$\frac{bb^3}{36}$	$\frac{bh^3}{12}$
2.	Rectangle	$\alpha = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
3.	Circle	$\alpha = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	-
4.	Trapezium	$\alpha = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2}h$	$\frac{a^2+4ab+b^2}{3b(a+b)} \times h^2$	-
5.	Triangle	$\alpha = \frac{2b}{3}$	$\frac{b^3}{36}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$



Problem :- A rectangular tank 4m long 2m wide contains water up to a depth of 2.5m. Calculate the pressure on the base of the tank.

Solution :- Data given

$$l = 4\text{m}, b = 2\text{m}, \bar{x} = 2.5\text{m}$$

We know that, base area of the tank

$$A = l \times b = 4 \times 2 = 8\text{m}^2$$

and pressure on the base of the tank

$$P = wA\bar{x} = fgA\bar{x} = 1000 \times 9.81 \times 8 \times 2.5$$

$$P = 196200 \text{ N} = 196.2 \text{ kN}$$

Problem :- A tank 3m x 4m contains 1.2m deep oil of specific gravity 0.8. Find (i) intensity of pressure at the base of the tank. and (ii) total pressure on the base of the tank.

Solution :- Data given

$$A = \text{size of tank} = 3\text{m} \times 4\text{m} = 12\text{m}^2$$

$$\bar{x} = \text{Depth of oil} = 1.2\text{m}$$

$$s = \text{specific gravity of oil} = 0.8$$

$$(w) = \text{specific weight of oil} = 9.81 \times 0.8 = 7.848 \text{ kN/m}^3$$

$$[\because s = \frac{\text{sp wt of oil}}{\text{sp wt of water}} \text{ or, wt of oil} = \frac{s \times \text{wt of water}}{\text{water}} = \frac{0.8}{1} \times 9.81 \text{ J.}]$$

$$\therefore w = 9.81 \times 0.8 = 7.848 \text{ kN/m}^3. \quad (\text{Ans})$$

(ii) Total pressure on the base of the tank

$$P = wA\bar{x} = 7.848 \times 12 \times 1.2 = 113.011 \text{ kN} \quad (\text{Ans})$$

i) Intensity of pressure at the base of tank

$$P = wh = 7.848 \times 1.2 = 9.42 \text{ kN/m}^2$$

Problem: An isosceles triangular plate of base 3 metres and altitude 3 metres is immersed vertically in water. Determine the total pressure and centre of pressure of the plate.

Solution: Data given

Base width (b) = 3 m. Altitude (h) = 3 m.

Surface area of the triangular plate
 $= \frac{1}{2} b h = \frac{1}{2} \times 3 \times 3 = \frac{9}{2} = 4.5 \text{ m}^2$

\bar{x} = Depth of centre of gravity of the plate from the water surface $= \frac{3}{3} = 1 \text{ m}$.

Total pressure on the plate

$$P = w A \bar{x} = 9.81 \times 4.5 \times 1 = 44.14 \text{ kN}$$

Centre of pressure

We know I_G = moment of inertia of the

triangular section about its centre of gravity and parallel to water surface.

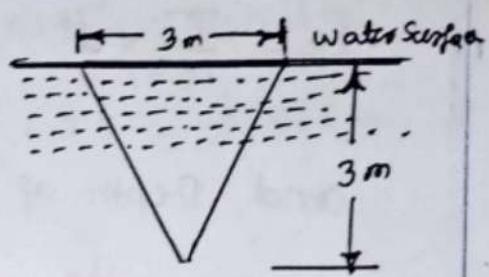
$$I_G = \frac{bh^3}{36} = \frac{3 \times (3)^3}{36} = 2.25 \text{ m}^4$$

and Depth of centre of pressure from the water surface.

$$\bar{h} = \frac{l_0}{A\bar{x}} + \bar{x} = \frac{2.25}{0.5 \times 1} + 1$$

$$= 0.5 + 1$$

$$\therefore \boxed{h = 1.5 \text{ m.}}$$



Problem :- A Circular gate of 2m diameter is immersed vertically in an oil of specific gravity 0.84 as shown in the figure. Find the oil pressure on the gate and position of the centre of pressure on the gate.

Solution :- Data given

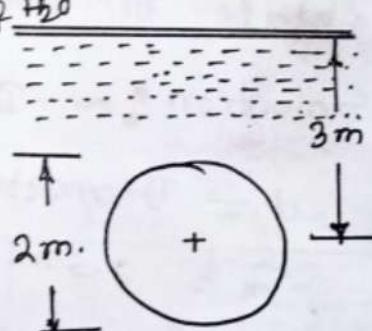
$$d = \text{Diameter of circular gate} = 2\text{m.}$$

$$s = \text{specific gravity of oil} = 0.84$$

$$\text{Specific weight of oil} = s \times \text{sp. wt. of } H_2O$$

$$\text{or, } 0.84 \times 9.81 = 8.24 \frac{\text{KN}}{\text{m}^3}$$

$$\bar{x} = 3\text{m}$$



We know that, Surface area

$$\text{of the circular gate} = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (2)^2 = 3.1416 \text{ m}^2$$

$$\text{Total pressure of the gate } P = w A \bar{x}$$

$$\text{or, } P = 8.24 \times 3.1416 \times 3 = 77.66 \text{ KN.}$$

Position of the centre of pressure.

$I_G = \text{M.I. of Circular section about its C.G}$
and parallel to the oil surface.

$$I_a = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times (2)^4 = 0.7853 \text{ m}^4$$

and Depth of centre of pressure from the surface.

$$\bar{h} = \frac{I_a}{A\bar{x}} + \bar{x} = \frac{0.7853}{3.1416 \times 3} + 3 = 0.2499 + 3$$

$$\boxed{\bar{h} = 0.0833 + 3 = 3.08 \text{ m.} \quad (\text{Ans})}$$

Problem. Figure shows a circular plate of diameter 1.2m placed vertically in water in such a way that the centre of the plate is 2.5m below the free surface of water. Determine (i) Total pressure on the plate (ii) position of centre of pressure.

Solution: Data given

d = Diameter of the plate = 1.2m

$$\bar{x} = 2.5 \text{ m.}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (1.2)^2 = 1.13 \text{ m}^2$$

(i) Total pressure (P)

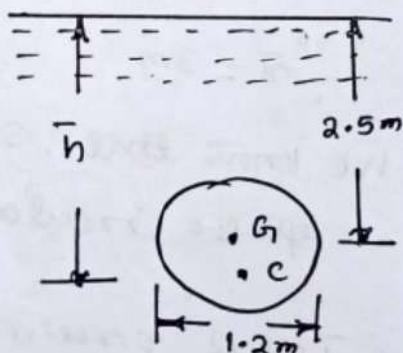
We know that $P = w A \bar{x}$

$$= 9.81 \times 1.13 \times 2.5 = 27.7 \text{ kN.}$$

$$\boxed{P = 27.7 \text{ kN}}$$

(ii) Position of centre of pressure, \bar{h}

$$\bar{h} = \frac{I_a}{A\bar{x}} + \bar{x}$$



$$\text{where } I_G = m \cdot 1 = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times (1.2)^4 = 0.1018 \text{ m}^4$$

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{0.1018}{1.13 \times 2.5} + 2.5 = 2.536 \text{ m.}$$

$$\boxed{\bar{h} = 2.536 \text{ m.}}$$

Problem :- A rectangular plate 3m long and 1m wide is immersed vertically in water in such a way that its 3metres side is parallel to the water surface and is 1metre below it. Find (i) Total pressure on the plate (ii) Position of Centre of Pressure.

Solution :- Data given

b = width of the ~~plane~~ rectangular plane surface = 3m
 $\bar{x} = 1 \text{ m.}$

$l = \text{Depth of the } " "$

$$A = \text{Area of rectangular plane surface} = 3 \times 1 = 3 \text{ m}^2$$

$$\bar{x} = 1 + \frac{1}{2} = 1 + 0.5 = 1.5$$

(i) Total pressure

$$P = wA\bar{x} = 9.81 \times 3 \times 1.5$$

$$\boxed{P = 44.145 \text{ kN}}$$

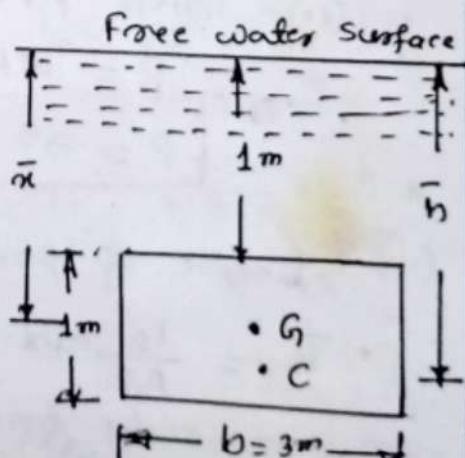
(ii) Centre of Pressure

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

$$\text{But } I_G = \frac{bd^3}{12} = \frac{3 \times (1)^3}{12} = 0.25 \text{ m}^4$$

$$\bar{h} = \frac{0.25}{3 \times 1.5} + 1.5 = 0.055 + 1.5$$

$$\boxed{\therefore \bar{h} = 1.555 \text{ m}}$$



Problem: An isosceles triangular plate of base of 3m and altitude 3m is immersed vertically in an oil of specific gravity 0.8. The base of the plate coincides with the free surface of oil. Determine
 (i) Total pressure on the plate (ii) centre of pressure

Solution: Data given.

$$b = \text{base of the plate} = 3\text{m}, h = \text{height of plate} = 3\text{m}$$

$$\text{Area} = A = \frac{b \times h}{2} = \frac{3 \times 3}{2} = 4.5 \text{ m}^2$$

$$s = \text{specific gravity of oil} = 0.8$$

$$\bar{x} = \text{The distance of C.G from the free surface of oil} = \frac{h}{3} = \frac{3}{3} = 1\text{m}$$

$$\text{we know that } s = \frac{\text{wt Density of oil}}{\text{wt Density of water}}$$

$$\begin{aligned} \text{wt Density of oil} &= s \times \text{Density of water} \\ &= 0.8 \times 1000 = \\ &= 0.8 \times 9.81 = 7.848 \end{aligned}$$

(i) Total pressure on the plate,

$$P = wA\bar{x} = 7.848 \times 4.5 \times 1 = 35.3$$

$$P = 35.3 \text{ kN}$$

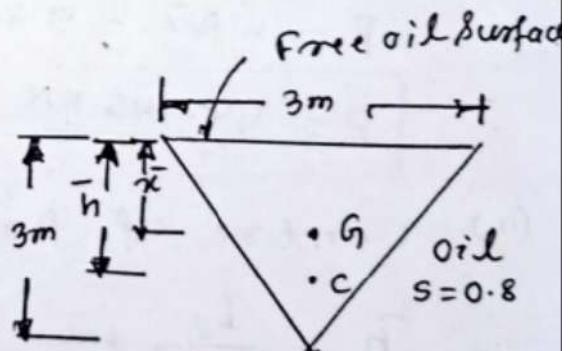
(ii) Centre of pressure

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

$$\text{For isosceles triangle } I_G = \frac{bh^3}{36} = \frac{3 \times 3^3}{36} = 2.25$$

$$\bar{h} = \frac{2.25}{4.5 \times 1} + 1 = 0.50 + 1 = 1.50 \text{ m}$$

$$\bar{h} = 1.50 \text{ m.}$$



Problem :- Determine the total pressure on a circular plate of diameter 1.5m which is placed vertically in water in such a way that the centre of the plate is 3m below the free surface of water. Find the position of centre of pressure.

Solution :- Data given:

$$d = \text{Diameter of the } \odot \text{ circular plate} = 1.5\text{m}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (1.5)^2 = 1.767 \text{ m}^2$$

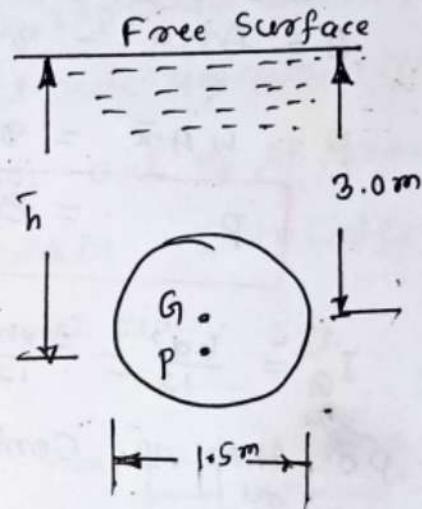
$$\bar{x} = 3.0 \text{m.}$$

(i) Total pressure (P)

$$\text{we know that } P = wA\bar{x}$$

$$= 9.81 \times 1.767 \times 3 \\ = 52.0028 \text{ kN}$$

$$P = 52.0028 \text{ kN}$$



(ii) position of centre of pressure, \bar{h}

$$\bar{h} = \frac{l_a}{A\bar{x}} + \bar{x} \quad \text{where } l_a = \frac{\pi d^4}{64} = \frac{\pi}{64} \times (1.5)^4 \\ = 0.2486 \text{ m}$$

$$\bar{h} = \frac{l_a}{A\bar{x}} + \bar{x} = \frac{0.2486}{1.767 \times 3} + 3 = \frac{0.2486}{5.301} + 3$$

$$\bar{h} = 0.0468 + 3 = 3.0468 \text{ m}$$

Problem :- A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the

- Plane surface when its upper edge is horizontal and (a) coincides with water surface.
 (b) 2.5 m below the free water surface.

Solution: Data given

$$b = \text{width of plane surface} = 2m$$

$$d = \text{Depth of plane surface} = 3m$$

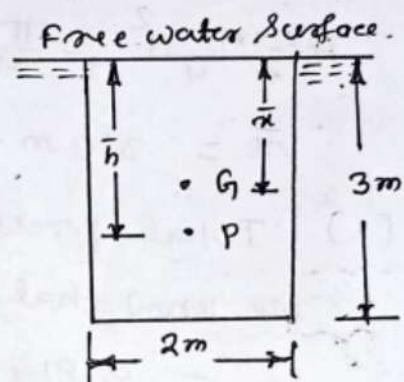
- (a) Upper edge coincides with water surface.

$$\bar{x} = \frac{3}{2} = 1.5$$

$$A = \text{Area} = 3 \times 2 = 6m^2$$

$$P = w A \bar{x} = 9.81 \times 6 \times 1.5 \\ P = 88.29 \text{ kN}$$

$$I_G = \frac{bd^3}{12} = \frac{2 \times (3)^3}{12} = 4.50 \text{ m}^4$$



$$\text{position of Centre of Pressure } h = \frac{I_G}{A \bar{x}} + \bar{x}$$

$$= \frac{4.50}{6 \times 1.5} + 1.5 = \frac{4.50}{9.00} + 1.5 = 0.50 + 1.5 = 2.0 \text{ m}$$

$$\therefore h = 2.0 \text{ m}$$

- (b) Upper edge is 2.5 m below water surface

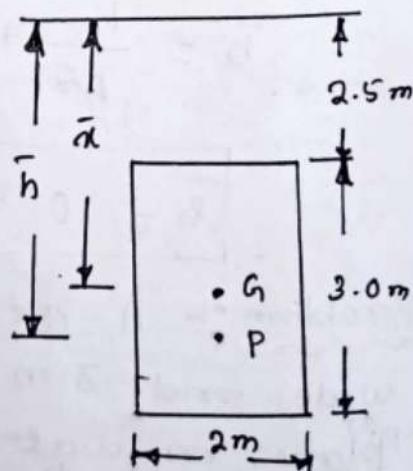
$$\text{Total pressure } (P) = w A \bar{x}$$

$$= 9.81 \times 6 \times (2.5 + \frac{3}{2})$$

$$= 9.81 \times 6 \times (2.5 + 1.5)$$

$$= 9.81 \times 6 \times 4$$

$$P = 235.44 \text{ kN}$$



centre of pressure is given by $\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$

where $I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$, $\bar{x} = 2.0 \text{ m}$

$$\bar{h} = \frac{4.5}{6 \times 4} + 2.0$$

or, $\bar{h} = \frac{4.5}{6 \times 4} + 4 = 0.187 + 4 = 4.187 \text{ m}$

$$\boxed{\bar{h} = 4.187 \text{ m}}$$

Problem: Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4m and altitude 4m when it is immersed vertically in an oil of sp. gravity 0.9. The base of the plate coincides with the free surface of oil.

Solution:

Data given

$$b = \text{Base of plate} = 4 \text{ m}$$

$$h = \text{Height of plate} = 4 \text{ m}$$

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2} \times 4 \times 4 = 8 \text{ m}^2$$

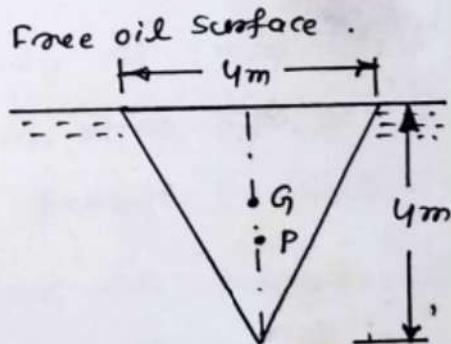
$$\text{sp. gravity of oil} = 0.9$$

We know that (S) Specific gravity = $\frac{\text{Density of oil}}{\text{Density of water}}$

$$\text{or, Density of oil} = S \times \text{Density of water}$$

$$= 0.9 \times 1000 = 900 \text{ kg/m}^3$$

\bar{x} = The Distance of C.G from the free surface of oil = $\frac{1}{3} \times 4 = 1.33 \text{ m}$.



$$h = 1.33 \text{ m.}$$

$$\begin{aligned}\text{Total pressure on the plate (P)} &= w A \bar{x} = \rho g A \bar{x} \\ &= 900 \times 9.81 \times 8 \times 1.33 = 93940.56 \text{ N} \\ P &= 93.940 \text{ kN}\end{aligned}$$

Centre of pressure \bar{h} from free surface of oil is given by $\bar{h} = \frac{I_G}{A \bar{x}} + \bar{x}$

$$\begin{aligned}I_G &= \text{m. l. of triangular section about C.G.} \\ &= \frac{bh^3}{36} = \frac{4 \times (4)^3}{36} = 7.11 \text{ m}^4\end{aligned}$$

$$\bar{h} = \frac{7.11}{8 \times 1.33} + 1.33 = 0.668 + 1.33 = 1.99 \text{ m}$$

$$\bar{h} = 1.99 \text{ m.}$$

Archimedes Principle

states " whenever a body is immersed wholly or partially in a fluid , it is buoyed up (ie lifted up) by a force equal to the weight of the fluid displaced by the body .

OR

Another definition.

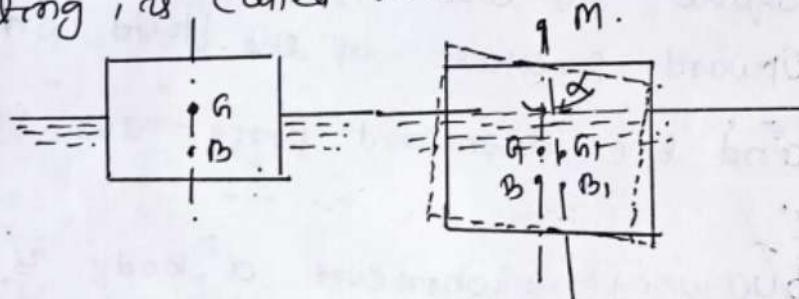
In other words , " whenever a body is immersed wholly or partially in a fluid , the resultant force acting on it , is equal to the difference between the upward pressure of the fluid on its bottom and the downward force due to gravity .

Buoyancy ... whenever a body is immersed wholly or partially in a fluid it is subjected to an upward force which tends to lift it up . This tendency for an immersed body to be lifted up in the fluid , due to an upward force opposite to action of gravity is known as buoyancy . The force tending to lift up the body under such conditions is known as buoyant force or force of buoyancy .

Centre of Buoyancy:~ The point of application of the force of buoyancy on the body is known as the centre of buoyancy. It is always the centre of gravity of the volume of fluid displaced.

META CENTER:

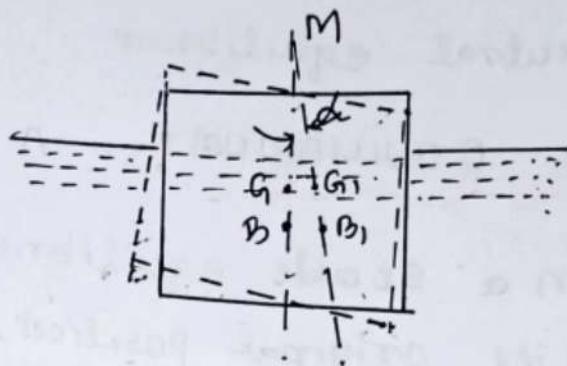
whenever a body, floating in a liquid, is given a small angular displacement, it starts oscillating about some point. This point, about which the body starts oscillating, is called metacentre.



In other words, the metacentre may also be defined as the intersection of the line passing through the original centre of buoyancy (B) and centre of gravity (G) of the body, and the vertical line through the centre of buoyancy (B') as shown in the figure.

METACENTRIC HEIGHT:~ The distance between the centre of gravity of a floating body and the metacentre (ie distance GM) is shown in

the figure is called metacentric height.



FLOATATION

FLOATATION: ~ we see that whenever a body is placed over a liquid, either it sinks down or floats on the liquid, is subjected to the following two forces. (i) Gravitational force & (ii) Upthrust of the liquid.

Here the two forces act opposite to each other. A little consideration will show that, if the gravitational force is more than the upthrust of the liquid, the body will sink down. But if the gravitational force is less than the ~~upthrust~~ upthrust of the liquid, the body will float.

Conditions of Equilibrium of a Floating body.

A body is said to be in equilibrium, when it remains in a steady state, while floating in a liquid. Following are the three conditions of equilibrium of a floating body.

(a) Stable equilibrium, (b) Unstable equilibrium
& (c) Neutral equilibrium.

(a) STABLE EQUILIBRIUM: ~ A body is said to be in a stable equilibrium, if it returns back to its original position, when given a small angular displacement. This happens when the metacentre (M) is higher than the centre of gravity (G) of the floating body.

(b) Unstable equilibrium: ~ A body is said to be in an unstable equilibrium, if it does not return back to its original position and heels farther away, when given a small angular displacement. This happens when the metacentre (M) is lower than the centre of gravity (G) of the floating body.

(c) Neutral Equilibrium: ~ A body is said to be in a neutral equilibrium, if it occupies a new position and remains at rest in this new position, when given a small angular displacement. This happens when the metacentre (M) coincides with the centre of gravity (G) of the floating body.

CHAPTERS 4.0 KINEMATICS OF FLOW

TYPES OF FLOW: - According to different considerations fluid flows may be classified in several ways. They are given below.

- (i) steady and unsteady (ii) uniform and non uniform flow, (iii) Laminar and Turbulent flow
- (iv) Compressible and incompressible flow (v) Rotational and irrotational flow.

(i) STEADY FLOW: - A flow, in which the quantity of liquid flowing per second is constant, is called a steady flow. In other words a steady flow may be defined as that in which at any point in the flowing fluid velocity, Acceleration, Density, Pressure and temp are independent of time i.e $\frac{\partial v}{\partial t} = 0$

$$\frac{\partial \phi}{\partial t} = 0, \quad \frac{\partial a}{\partial t} = 0$$

Example: Flow of fluid through a Constant diameter pipe at a constant flow rate is a steady type of flow

Unsteady Flow: - A flow, in which the quantity of liquid flowing per second is not constant, is called a unsteady flow.

In this type of flow, velocity, pressure or density at a point changes with respect to time. Mathematically $\frac{\partial v}{\partial t} \neq 0$ $\frac{\partial p}{\partial t} \neq 0$

$$\frac{\partial p}{\partial t} \neq 0, \quad \frac{\partial a}{\partial t} \neq 0$$

Example : Flow of fluid through a uniform dia pipe at non-uniform rate.

(2) Uniform Flow : ~ A flow, in which the velocities of liquid particles at all sections of a pipe or channel are equal is

Called uniform flow

Example : ~ Flow of liquids under pressure through long pipe lines of constant diameter is uniform flow.

Non Uniform Flow : ~ A flow, in which the velocities of liquid particles at all sections of the pipe or channel are not equal, is called a non-uniform flow.

Example : ~ Flow of liquids under pressure through long pipe lines of varying diameters is non-uniform flow.

(3) LAMINAR FLOW : ~ When fluid flows, each liquid particle has a definite path and

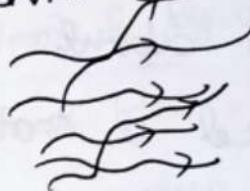
The paths of individual particles do not cross each other is called a laminar flow.



It is also called as streamline flow

Example: ~ Very smooth surface pipe with high pressure.

Turbulent Flow: ~ In this type of flow of fluid each liquid particles do not have a definite path and the paths of individual particles also cross each other is called as turbulent flow



Example: ~ Very Rough surfaces pipe with high pressure.

④ Compressible Flow: ~ A fluid is said to be compressible if the volume and density of the fluid changes from point to point or in other words volume and density are not constant. Mathematically $f \neq \text{constant}$

Example: ~ All the gases are generally considered to have compressible flow.

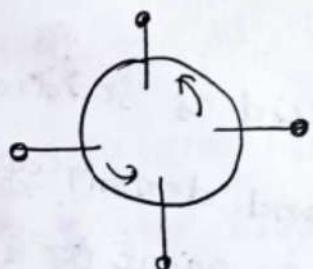
Incompressible Flow: ~ A fluid is said to be incompressible (shape changes but volume does not change).

if the volume and density of the flowing fluid does not change from point to point ie volume and density is always constant.

Example :- All the liquids are generally considered to have incompressible flow.

(5)

Rotational Flow :- A flow, in which the fluid particles also rotate (ie have some angular velocity) about their own axes while flowing is called a rotational flow. ^{Example} In a rotational flow, if a match stick is thrown on the surface of the moving fluid, it will rotate about its axis as shown in the figure.



Rotational Flow



irrotational flow

irrotational flow :- A flow is said to be irrotational if the fluid particles while moving in the direction of flow do not rotate about their own axes.

Example :- In an irrotational flow if a match stick is thrown on the surface

of the moving fluid, it does not rotate about its axis but retain its original orientation.

(6) One Dimensional Flow: A flow, in which the streamlines of its moving particles may be represented by straight lines, is called one dimensional flow. It may be $x-x$, $y-y$ or $z-z$ direction.

Two-Dimensional Flow: A flow, whose streamlines may be represented by a curve is called a two dimensional flow. A curved streamline will be along any two mutually perpendicular directions.

Three Dimensional Flow: A flow, whose streamlines may be represented in space, i.e. along three mutually perpendicular directions, is called three-dimensional flow.

RATE OF FLOW OR DISCHARGE (Q): Rate of flow or discharge is defined as the quantity of a liquid flowing per second through a section of pipe or a channel. It is generally denoted by Q.

For an incompressible fluid (liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. But for compressible flow (ie for gases), the rate of flow is usually expressed as the weight of fluid flowing across the section.

Let us consider liquid flowing through a pipe

A = Area of the cross-section of the pipe.

V = Average velocity of the liquid

∴ Discharge $Q = A \times V$ (Area \times Average Veloc.)

(i) For liquid $Q = A \times V = m^2 \times m/sec = m^3/sec$
or, litres/sec.

Note: 1 litres = 1000 cm^3 or, $\frac{1}{1000} \text{ m}^3$

(ii) For gases Q is in kgf/sec or $\frac{\text{Newton}}{\text{sec}}$

CONTINUITY EQUATION

The Continuity equation is actually mathematically statement of the principle of conservation of mass. This principle states that mass can neither be created nor destroyed. On the basis of this principle the continuity equation is derived.

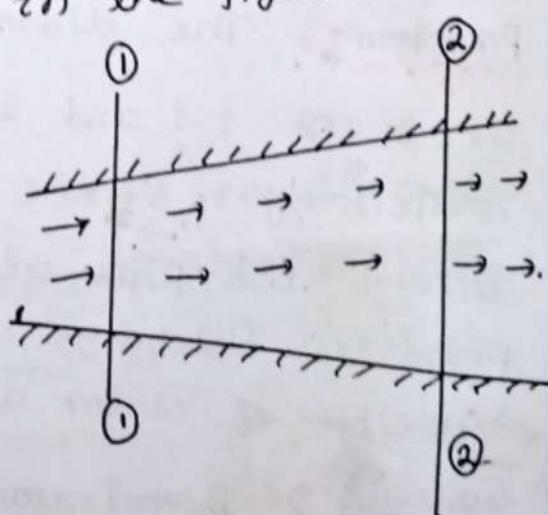
If an incompressible liquid is continuously flowing through a pipe or a channel whose cross sectional area may or may not be constant the quantity of liquid passing per second is the same at all sections. This is known as the equation of a liquid flow. It is the 1st and fundamental equation of flow.

Let us consider two cross-sections of a pipe as shown in the figure.

A_1 = cross-sectional area
of the pipe at section 1-1

v_1 = velocity of the fluid
at section 1-1

ρ_1 = Density of the fluid
at section 1-1



Similarly A_2 , v_2 and ρ_2 are the corresponding values at section 2-2

Then total quantity of fluid passing through section 1-1 = $\rho_1 A_1 v_1$

and total quantity of fluid passing through section 2-2 = $\rho_2 A_2 v_2$

According to law of conservation of mass
Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\therefore \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This equation is applicable both compressible and incompressible fluids and is called Continuity equation.

In case of incompressible fluids $\rho_1 = \rho_2$

Then Continuity equation becomes $A_1 v_1 = A_2 v_2$

Problem: The diameter of a pipe at the sections 1-1 and 2-2 are 200mm and 300mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 4 m/sec. Find (i) Discharge through the pipe and (ii) Velocity of water at section 2-2

Solution: D_1 = Diameter of the pipe at

$$\text{Section 1-1} = 200\text{mm} = \frac{200}{1000} = 0.2\text{m.}$$

$$\text{Area} = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = \text{Velocity} = 4 \text{ m/sec.}$$

D_2 = Diameter of the pipe at section 2-2

$$= 300 \text{ mm} = \frac{300}{1000} = 0.3 \text{ m}$$

$$A_2 = \text{Area} = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0707 \text{ m}^2$$

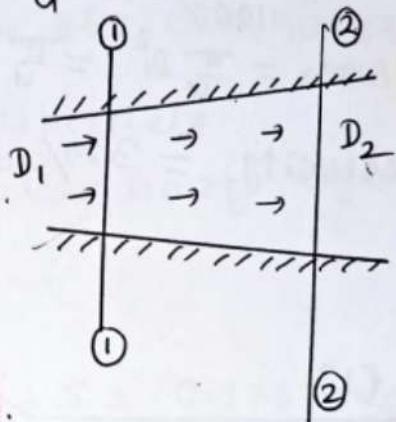
(i) Discharge through

the pipe Q

Now using the formula

$$Q = A_1 V_1 = 0.0314 \times 4$$

$$Q = 0.1256 \text{ m}^3/\text{sec.}$$



(ii) Velocity of water at section 2-2

We know from Continuity Equation

$$A_1 V_1 = A_2 V_2$$

Putting all the values, we have

$$0.0314 \times 4 = 0.0707 \times V_2 \therefore V_2 = \frac{0.0314 \times 4}{0.0707} = 1.77 \text{ m/sec.}$$

$$\therefore V_2 = 1.77 \text{ m/sec}$$

Problem: A pipe (1) 450mm in diameter branches into two pipes (2) and (3) of diameters 300mm and 200mm respectively as shown in the figure. If the average velocity in 450mm diameter pipe is 3m/sec. Find (i) Discharge through 450mm diameter

Type (ii) Velocity in 200 mm diameter pipe is
the average velocity in 300 mm pipe is 2.5 m/sec.

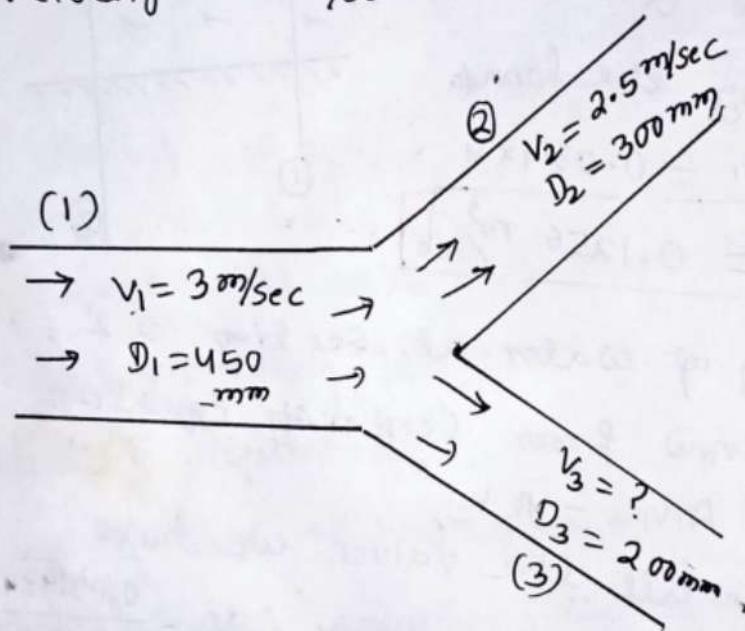
Solution :- Data given

D_1 = Diameter of the pipe having size 450 mm

$$= \frac{450}{1000} = 0.45 \text{ m}$$

$$A_1 = \text{Area} = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.45)^2 = 0.159 \text{ m}^2$$

$$V_1 = \text{Velocity} = 3 \text{ m/sec.}$$



D_2 = Diameter of branch pipe having

$$\text{size} = 300 \text{ mm} = \frac{300}{1000} = 0.3 \text{ m.}$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0707 \text{ m}^2$$

$$V_2 = \text{Velocity} = 2.5 \text{ m/sec.}$$

D_3 = Diameter of branch pipe having

$$\text{size} = 200 \text{ mm} = \frac{200}{1000} = 0.2 \text{ m}$$

$$A_3 = \text{Area} = \frac{\pi}{4} D_3^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$$

(i) Discharge through pipe(1)

$$Q_1 = A_1 V_1 \text{ putting all the values, we have } Q_1 = 0.159 \times 3 = 0.477 \text{ m}^3/\text{sec.}$$

(ii) Velocity in pipe of diameter 200mm i.e. V_3

Let Q_1 , Q_2 and Q_3 be the discharge in pipes 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

$$Q_2 = A_2 V_2 = 0.0707 \times 2.5 = 0.1767 \text{ m}^3/\text{sec.}$$

$$\therefore 0.477 = 0.1767 + Q_3 = 0.1767 + V_3 A_3$$

$$\text{or, } A_3 V_3 = 0.477 - 0.1767 = 0.300$$

$$\text{or, } V_3 = \frac{0.300}{A_3} = \frac{0.300}{0.0314} = 9.55 \text{ m/sec.}$$

$$\therefore V_3 = 9.55 \text{ m/sec.}$$

Problem: A 25cm diameter pipe carries oil of sp. gravity 0.9 at a velocity of 3m/sec. At another section the diameter is 20cm. Find the velocity at this section and also mass rate of flow of oil.

Solution: Data given

$$\text{At section 1, } D_1 = \text{Diameter} = 25\text{cm} = \frac{25}{100} = 0.25\text{m.}$$

$$A_1 = \text{Area} = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.25)^2 = 0.0491 \text{ m}^2$$

$$V_1 = \text{Velocity} = 3 \text{ m/sec}$$

At Section 2

$$D_2 = 20 \text{ cm} = \frac{20}{100} = 0.2 \text{ m}$$

$$\begin{aligned} \text{Area} = A_2 &= \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.2)^2 \\ &= 0.0314 \text{ m}^2 \end{aligned}$$

$$V_2 = ?$$

Mass rate of flow of oil = ?

Applying continuity equation at Section 1 & 2.

$$A_1 V_1 = A_2 V_2 \quad \text{or}, \quad V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0491 \times 3}{0.0314}$$

$$V_2 = 4.69 \text{ m/sec}$$

We know that mass rate of flow of oil
= mass density $\times Q = \rho \times A_1 V_1$

$$\text{Specific gravity of oil} = \frac{\text{Density of oil}}{\text{Density of water}}$$

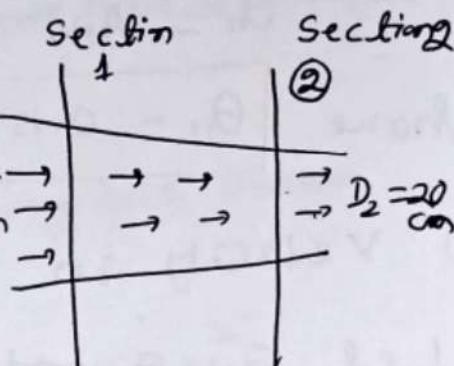
$$\begin{aligned} \text{Density of oil} &= \text{sp. gravity of oil} \times \text{Density of water} \\ &= 0.9 \times 1000 \text{ kg/m}^3 = 900 \text{ kg/m}^3 \end{aligned}$$

$$\text{Mass rate of flow} = \rho \times A_1 V_1$$

$$= 900 \times 0.049 \times 3 = 132.30 \text{ kg/sec}$$

$$\boxed{\text{Mass rate of flow} = 132.30 \text{ kg/sec}}$$

Problem: A 30cm diameter pipe, conveying water, branches into two pipes of diameters 20cm and 15cm respectively. Gt. the average



Velocity in the 30cm diameter pipe 2.5m/sec. Find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/sec.

Solution: Data given.

$$D_1 = \text{Diameter of pipe} = 30\text{cm} = \frac{30}{100} = 0.3\text{m}$$

$$A_1 = \text{Area} = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0707\text{m}^2$$

$$V_1 = 2.5\text{ m/sec}$$

$$D_2 = \text{Diameter of pipe} = 20\text{cm}$$

$$= \frac{20}{100} = 0.2\text{m}$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314\text{ m}^2$$

$$V_2 = 2\text{ m/sec}$$

$$D_3 = \text{Diameter of pipe} = 15\text{cm} = \frac{15}{100} = 0.15\text{m}$$

$$A_3 = \text{Area} = \frac{\pi}{4} D_3^2 = \frac{\pi}{4} \times (0.15)^2 = 0.0176\text{ m}^2$$

(i) Discharge in pipe 1 or Q_1

(ii) Velocity in pipe of diameter 15cm or V_3

(iii) Velocity in pipe 3 respectively.

Let Q_1 , Q_2 and Q_3 are discharges in pipes 1, 2 and 3 respectively.

According to continuity equation

Then

$$Q_1 = Q_2 + Q_3$$

The discharge Q_1 in pipe 1 is given by $= A_1 V_1$

$$Q_1 = A_1 V_1 = 0.0707 \times 2.5 = 0.1767 \text{ m}^3/\text{sec}$$

$$\boxed{Q_1 = 0.1767 \text{ m}^3/\text{sec}}$$

(ii) we know that $G_1 = G_2 + G_3$

$$\text{or, } A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$\text{or, } A_3 V_3 = A_1 V_1 - A_2 V_2 \text{ or, } V_3 = \frac{A_1 V_1 - A_2 V_2}{A_3}$$

$$\text{or, } V_3 = \frac{0.0707 \times 2.5 - 0.0314 \times 2}{0.0176} = \frac{0.1767 - 0.0628}{0.0176}$$

$$\text{or, } V_3 = \frac{0.1139}{0.0176} = 6.47 \text{ m/sec}$$

$$V_3 = 6.47 \text{ m/sec}$$

EXPLAIN ENERGY OF FLOWING LIQUID

Different Types of Heads or Energies of a liquid in motion.

These are mainly three types of energies or heads of flowing liquids.

(i) Potential Head or Potential Energy

It is the energy possessed by a liquid particle by virtue of its configuration or position above some suitable datum line. It is denoted by Z .

(ii) Velocity Head or Kinetic Energy

It is the energy possessed by a liquid particle by virtue of its motion or velocity of flowing liquid. It is measured by $\frac{V^2}{2g}$ where V is the mean velocity of flowing liquid and g is the acceleration due to gravity ($g = 9.81 \frac{\text{m}}{\text{s}^2}$)

(iii) Pressure Head or Pressure Energy

It is the energy possessed by a liquid particle

by virtue of its existing pressure. It is denoted by $\frac{P}{\omega}$ where 'P' is the pressure and ' ω ' is the weight density of the liquid and $\omega = \rho g$

TOTAL HEAD OF A LIQUID PARTICLE IN MOTION

The total head of a liquid particle in motion is the sum of its potential head, kinetic head and pressure head. Mathematically

$$H \text{ (Total head)} = z + \frac{v^2}{2g} + \frac{P}{\omega} \text{ m of liquid}$$

TOTAL ENERGY OF A LIQUID PARTICLE IN MOTION

The total energy of a liquid particle in motion is the sum of its potential energy, kinetic energy and pressure energy. Mathematically

$$\text{Total Energy (E)} = z + \frac{v^2}{2g} + \frac{P}{\omega} \frac{\text{Nm}}{\text{kg}} \text{ of liquid}$$

Problem: In a pipe of 90mm diameter water is flowing with a mean velocity of 2 m/sec and at a gauge pressure of 350 KN/m². Determine the total head, if the pipe is 8 metres above the datum line. Neglect friction.

Solution: Data given.

$$\text{Diameter of the pipe} = 90\text{mm}$$

$$\text{Pressure} = P = 350 \text{ KN/m}^2, \text{ Velocity of water} \\ = 2 \text{m/sec.}$$

$$z = \text{Datum Head} = 8\text{m} ..$$

$$w = \text{specific wt of H}_2\text{O} = 9.81 \text{ KN/m}^3$$

$$\text{Total head of water, } H = z + \frac{v^2}{2g} + \frac{P}{\omega}$$

Putting all the values, we have.

CHAPTER: 5.0 ORIFICES, NOTCHES & WEIRS

ORIFICE: An orifice is an opening having a closed perimeter, made in the walls or the bottom of a tank or a vessel containing fluid through which the fluid may be discharged. This opening is of any type of cross-section such as circular, triangular, rectangular etc.

USES: It is used for the measuring the rate of flow of fluid or discharge.

CLASSIFICATION OF ORIFICE

There are many types of orifices. They may be classified on the basis of their size, shape and shape of the upstream edges and the discharge.

Conditions.

1. According to the size: The orifices may be classified as (a) Small orifice
(b) Large orifice.

An orifice is termed small when its dimensions are small compared to the head causing flow. The orifice is large if the dimensions are comparable with the head causing flow.

$$H = 8 + \frac{(2)^2}{2 \times 9.81} + \frac{350}{9.81} = 43.88 \text{ m}$$

$$H = 43.88 \text{ m}$$

Problem: - Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0m/sec.

Find the total head or total energy per unit weight of the water at a cross section which is 5m above the datum line.

Solution: - Data given.

$$D = \text{Diameter of pipe} = 5\text{cm} = \frac{5}{100} = 0.05 \text{m}$$

$$P = \text{pressure} = 29.43 \frac{\text{N}}{\text{cm}^2} = 29.43 \frac{\text{N}}{10^4 \text{m}^2} \\ = 29.43 \times 10^4 \text{ N/m}^2$$

$$V = \text{Velocity} = 2.0 \text{ m/sec.}$$

$$Z = \text{Datum Head} = 5 \text{ m.}$$

$$\text{Total head} = \text{Datum head} + \text{kinetic head} \\ + \text{pressure head.}$$

$$\text{Pressure head} = \frac{P}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$$

$$[\because \rho \text{ for water} = 1000 \text{ kg/m}^3]$$

$$\text{Kinetic head} = \frac{V^2}{2g} = \frac{(2)^2}{2 \times 9.81} = 0.204 \text{ m}$$

$$\text{Total head} = 5 + 0.204 + 30 = 35.204 \text{ m}$$

$$H = 35.204 \text{ m}$$

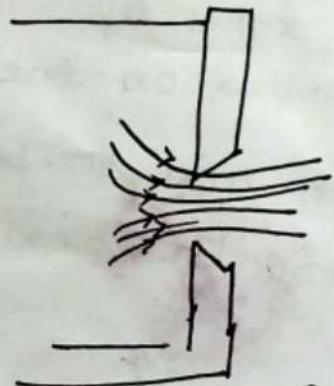
Problem: - A pipe, through which water is flowing, is having diameters 20cm and 10cm at the cross-sections 1 and 2 respectively.

The velocity of water at section 1 is given 4.0m/sec. Find the velocity head at section 2 and also rate of discharge.

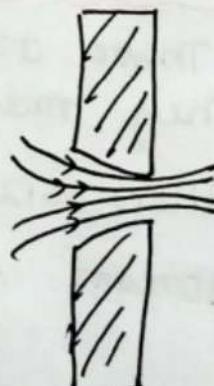
2. According to the shape : The orifice may be classified as (i) Circular orifice
(ii) Rectangular orifice (iii) Square orifice
(iv) Triangular orifice.

Out of these Circular and Rectangular orifices are most commonly used. It depends upon their cross-sectional areas.

3. According to the shape of upstream edge
The orifices may be classified as (i) sharp edged orifice (ii) bell mouthed orifice.



Sharp edged orifice.



Bell mouthed orifice.

4. According to nature of discharge.
The orifices are classified

as (i) Free discharging orifice
(ii) Submerged orifice.

(a) Fully Submerged Partially Submerged

Solution Data given.

$$D_1 = \text{Diameter of bigger Pipe} = 20\text{cm} = \frac{20}{100} = 0.2\text{m}$$

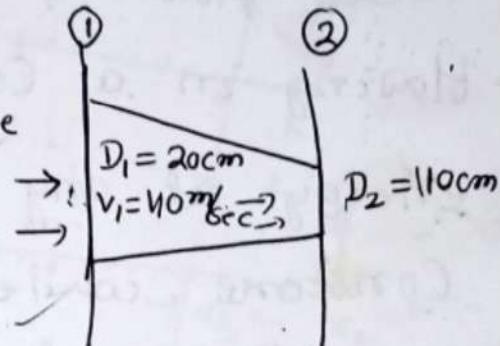
$$A_1 = \text{Area} = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{m}^2$$

$$V_1 = 4.0 \text{ m/sec}$$

$D_2 = \text{Diameter of smaller pipe}$

$$= 10\text{cm} = \frac{10}{100} = 0.1\text{m}$$

$$A_2 = \text{Area} = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785 \text{m}^2$$



Applying Continuity equation at sections 1 & 2

$$A_1 V_1 = A_2 V_2 \therefore V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314 \times 4}{0.00785} = 16.0 \text{ m/sec}$$

$$(i) \text{ Velocity head at section (1)} = \frac{V_1^2}{2g} = \frac{(4)^2}{2 \times 9.81}$$

$$= 0.815 \text{ m}$$

$$(ii) \text{ Velocity head at section (2)} = \frac{V_2^2}{2g} = \frac{(16)^2}{2 \times 9.81}$$

$$= \frac{256}{2 \times 9.81} = 13.047 \text{ m}$$

(iii) Rate of discharge = $A_1 V_1$ or $A_2 V_2$

$$= 0.0314 \times 4 = 0.1256 \text{ m}^3/\text{sec} = 0.1256 \times 1000 = 125.6 \text{ litres/sec}$$

$$\boxed{Q = 0.1256 \text{ m}^3/\text{sec} \text{ or, } 125.6 \text{ litres/sec}}$$

$$1 \text{ lit} = 1000 \text{ cm}^3$$

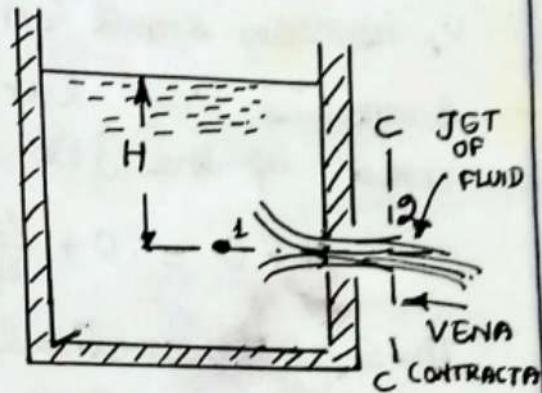
$$1 \text{ lit} = \frac{1}{1000} \text{ m}^3$$

Jet of Water :- The continuous stream of a liquid that comes out or flows out of an orifice is known as the jet of water.

FLOW THROUGH AN ORIFICE :- Let us consider a tank fitted with a circular orifice in one of its sides as shown in the figure. Let 'H' be the head of the liquid above the centre of the orifice. While the liquid flows through the orifice it forms a jet of liquid whose area of cross-section is less than

that of orifice. The area of jet of fluid goes on decreasing and at a section C-C, the area is minimum. This section is approximately

at a distance of half of diameter of the orifice. At this section the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called as Vena-contracta. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.



STATEMENT OF BERNOULLI'S THEOREM

It states that in a steady, ideal flow of an incompressible fluid, flowing in a continuous stream, the total energy at any point of the fluid is constant, while the particles moves from one point to another. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy.

Mathematically Bernoulli's theorem is written as

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

Explanation OF Bernoulli's Theorem

Let us consider flow of an incompressible fluid through a non-uniform tapering pipe as shown in the figure.

It consists of two sections LL and MM of the pipe. We assume that the pipe is running full and there is a continuity of flow between the two sections.

Let A_1 = Area of cross-section of pipe at 'inlet' section LL

V_1 = Velocity of liquid at LL section.

Let us consider two points 1 and 2 as shown in the figure. Point 1 is inside the tank and point 2 at the Vena-Contracta. Here the flow is steady and at a constant head H. Applying Bernoulli's theorem at section 1 and 2, we have

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$, $P_1 = P_2 = P_a$ (P_a = Atmospheric pressure)

~~$\therefore \frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$~~ , Now $\frac{P_1}{\rho g} = H$
 $\frac{P_2}{\rho g} = 0$ (Atmospheric pressure)

V_1 is very small in comparison to V_2 as area of tank is very large as compared to the area of the jet of liquid. $V_1 = 0$

$$H + 0 = 0 + \frac{V_2^2}{2g} : V_2^2 = 2gH \therefore V_2 = \sqrt{2gH}$$

This is theoretical velocity. Actual velocity will be less than this value.

$$\therefore V_2 = \sqrt{2gH}$$

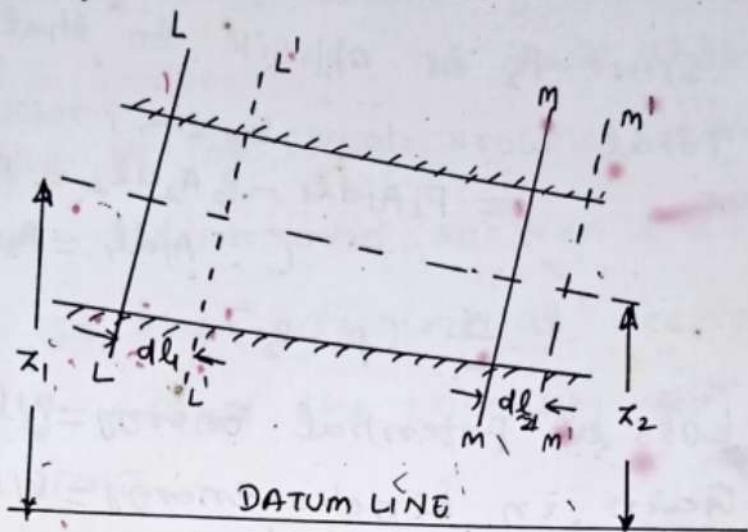
This equation is known as Torricelli's theorem.

ORIFICES COEFFICIENTS

- There are three important orifice coefficients. They are
- (a) Co-efficient of contraction (C_c)
 - (b) Co-efficient of velocity (C_v)
 - (c) Co-efficient of discharge (C_d)

P_1 = Pressure at LL section.

x_1 = Height of LL above datum line.



A_2, V_2, P_2, x_2 are the area of cross-section, velocity of liquid, pressure and height of MM above the datum at outlet section.

Let the liquid between the two sections LL and MM move LL and MM through a very small lengths dl_1 and dl_2 .

dl_1 = Distance of movement of fluid from LL to LL'

dl_2 = Distance of movement of fluid from MM to MM'

Let ω = Specific wt of the fluid

$W = \omega V$ is the weight of the liquid between

LL and LL'. Since the flow is continuous,

$$W = \omega A_1 dl_1 = \omega A_2 dl_2$$

Since specific weight (ω) = $\frac{\text{Weight of the fluid}}{\text{Volume of the fluid}}$

Weight of the fluid = $\omega \times \text{Volume of the fluid}$

$$= \omega \times A_1 dl_1$$

$$A_1 dl_1 = \frac{W}{\omega} \quad (1) \quad \text{similarly } A_2 dl_2 = \frac{W}{\omega} \quad (2)$$

Work done against pressure force at LL moving the liquid to LL' = Force \times distance = $P_1 A_1 dl_1$

$$\therefore P = \frac{F}{A} \text{ or, } F = PA$$

Similarly work done by pressure at Mm
in moving the liquid $m'm' = -P_2 A_2 dl_2$

Since P_2 is opposite to that of P_1

Total work done by the pressure

$$= P_1 A_1 dl_1 - P_2 A_2 dl_2 = A_1 dl_1 (P_1 - P_2).$$

[$A_1 dl_1 = A_2 dl_2$ from eq(182)].

$$= \frac{W}{w} (P_1 - P_2)$$

Loss of Potential Energy $= W(z_1 - z_2)$

Gain in Kinetic Energy $= W\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right)$

$$= \frac{W}{2g} (v_2^2 - v_1^2)$$

From the law of conservation of energy

Total gain in energy = Total loss

Gain in kinetic energy = Loss of potential
Energy + work done by the pressure.

$$\frac{W}{2g} (v_2^2 - v_1^2) = W(z_1 - z_2) + \frac{W}{w} (P_1 - P_2)$$

or, Cancelling W on both sides, we have

$$\frac{(v_2^2 - v_1^2)}{2g} = (z_1 - z_2) + \frac{1}{w} (P_1 - P_2)$$

$$\text{or, } z_1 - z_2 + \frac{P_1}{w} - \frac{P_2}{w} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\text{or, } z_1 + \frac{P_1}{w} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{w} + \frac{v_2^2}{2g}$$

which proves Bernoulli's equation.

Co-efficient of Contraction: The ratio of the area of the jet at vena-contracta to the area of the orifice is known as Co-efficient of Contraction. It is denoted by C_c .

$$\text{Then } C_c = \frac{a_c}{a}$$

where a_c = Area of jet at vena contracta
 a = Area of orifice.

The value of C_c varies from 0.61 to 0.63
 In general, the value of C_c may be taken as 0.62

Co-efficient of Velocity: The ratio of actual velocity (V) of the jet at Vena-Contracta to the theoretical velocity (V_{th}) of jet. It is denoted by C_v .

$$C_v \text{ mathematically } C_v = \frac{V}{V_{th}} = \frac{V}{\sqrt{2gh}}$$

where V = Actual velocity of jet at Vena Contracta

V_{th} = Theoretical velocity

The value of C_v varies from 0.95 to 0.99.
 For sharp-edged orifices the value of C_v is taken as 0.98.

Co-efficient of Discharge: The ratio of actual discharge (Q) through an orifice to the theoretical discharge (Q_{th}) is known as Co-efficient of discharge. It is denoted by

$$C_d = \frac{Q}{Q_{th}}$$

Problem: ~ The water is flowing through a pipe having diameters 20cm. and 10cm at Sections 1 and 2 respectively. The rate of flow through the pipe is 35 litres/sec. The section 1 is 6m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm^2 . Find the intensity of pressure at section 2.

Solution: ~ Data given.

$$\text{At section 1. } D_1 = 20\text{cm.} = \frac{20}{100} = 0.2\text{m.}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$$

$$P_1 = 39.24 \text{ N/cm}^2 = 39.24 \frac{\text{N}}{(0.01)^2 \text{ m}^2} = 39.24 \frac{\text{N}}{10^{-4} \text{ m}^2}$$

$$= 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0\text{ m.}$$

At section 2

$$D_2 = 10\text{cm} = \frac{10}{100} = 0.1\text{m.}$$

$$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times (0.1)^2 \\ = 0.00785 \text{ m}^2$$

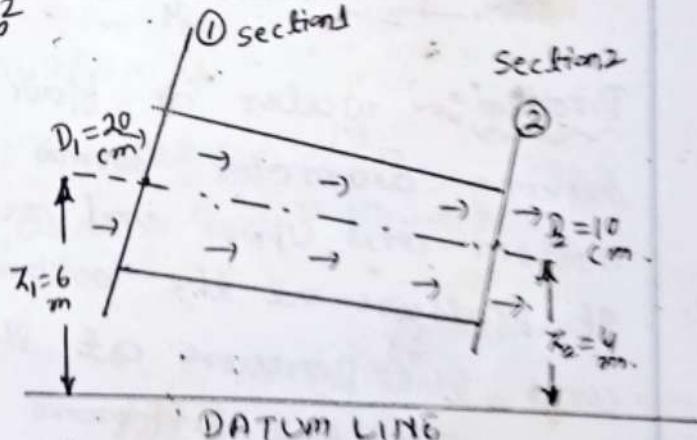
$$z_2 = 4\text{m}, P_2 = ?$$

$$\text{Rate of flow } Q = 35 \text{ lit/sec} = \frac{35}{1000} = 0.035 \text{ m}^3/\text{sec}$$

We know that $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456 \text{ m/sec.}$$



$$\left[\begin{array}{l} 1 \text{ lit} = 1000 \text{ cm}^3 \\ 1 \text{ lit} = \frac{1}{1000} \text{ m}^3 \end{array} \right]$$

Applying Bernoulli's equation at Section 1 & 2

$$\text{We get } \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Putting all the values, we have

$$\text{or, } \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or, } \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

$$\text{or, } 40.0 + 0.063 + 6 = \frac{P_2}{1000 \times 9.81} + 1.012 + 4.0$$

$$\text{or, } 46.063 = \frac{P_2}{1000 \times 9.81} + 5.012$$

$$\text{or, } \frac{P_2}{1000 \times 9.81} = 46.063 - 5.012 = 41.051$$

$$\text{or, } P_2 = 41.051 \times 1000 \times 9.81 = 402710.31 \frac{\text{N}}{\text{m}^2}$$

$$\text{or, } P_2 = \frac{402710.3}{10^4} = 40.27 \frac{\text{N}}{\text{cm}^2}$$

Problem: water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is $24.525 \frac{\text{N}}{\text{cm}^2}$ and the pressure at the upper end is $9.81 \frac{\text{N}}{\text{cm}^2}$. Determine the difference in datum head if the rate of flow through pipe is 40 lit/sec.

Solution: Data given

$$\text{At Section 1, } D_1 = 300 \text{ mm} = \frac{300}{1000} = 0.3 \text{ m.}$$

$$P_1 = 24.525 \frac{\text{N}}{\text{cm}^2} = 24.525 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0707 \text{ m}^2$$

At Section 2

$$D_2 = 200 \text{ mm} = \frac{200}{1000} = 0.2 \text{ m}$$

Where Q = Actual discharge
 Q_{th} = Theoretical discharge
 The value of C_d varies from 0.62 to 0.65

RELATION BETWEEN THE ORIFICE COEFFICIENTS.

We know that $C_d = \frac{Q(\text{Actual})}{Q(\text{Theoretical})}$

$$\text{But } Q = A \times V$$

$$C_d = \frac{\text{Actual Area} \times \text{Actual Velocity}}{\text{Theoretical Area} \times \text{Theoretical Velocity}}$$

$$= \frac{\text{Actual Area}}{\text{Theoretical Area}} \times \frac{\text{Actual Velocity}}{\text{Theoretical Velocity}}$$

$$C_d = C_c \times C_v$$

[where
 C_c = coefficient of contraction
 C_v = co-efficient of velocity]

Problem : An orifice 50mm in diameter is discharging water under a head of 10 metres. If $C_d = 0.6$ and $C_v = 0.97$. Find (i) Actual discharge (ii) Actual velocity of the jet at vena-contracta.

Solution : Diameter of the orifice $= d = 50\text{mm}$

$$d = \frac{50}{1000} = 0.05\text{m}$$

$$\text{Area of the orifice } a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2 \\ = 0.001963 \text{ m}^2$$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi (0.3)^2}{4} = 0.0314 \text{ m}^2$$

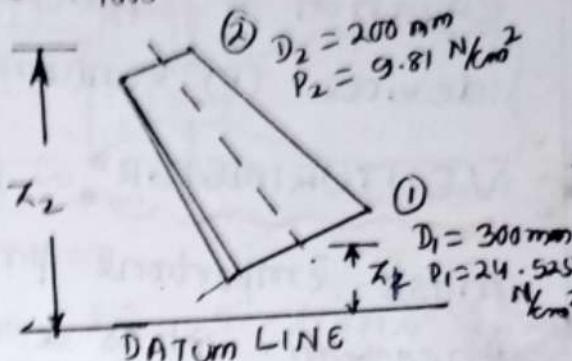
$$P_2 = 9.81 \text{ N/cm}^2 = 9.81 \times \frac{\text{N}}{10^4 \text{ m}^2} = 9.81 \times 10^4 \text{ N/m}^2$$

$$Q = \text{Rate of flow} = 40 \text{ l/sec} = \frac{40}{1000} = 0.04 \text{ m}^3/\text{sec}$$

We know that $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.04}{0.0314} = 0.565 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{0.0314} = 1.274 \text{ m/sec}$$



Applying Bernoulli's equation at

sections (1) & (2) we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or, } \frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{(0.565)^2}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

$$\text{or, } 25.0 + 0.0162 + z_1 = 10.00 + 0.0827 + z_2$$

$$\text{or, } 25.016 + z_1 = 10.082 + z_2$$

$$\text{or, } z_2 - z_1 = 25.016 - 10.082 = 14.93 \text{ m}$$

Difference in datum head $= z_2 - z_1 = 14.93 \text{ m}$ (Ans)

Head $H = 10\text{m}$, $C_d = 0.6$, $C_v = 0.97$

(i) We know that $C_d = \frac{\text{Actual Discharge}}{\text{Theoretical Discharge}}$

But Theoretical discharge = aV
 $= \text{Area of orifice} \times \text{Theoretical velocity}$
 $= a \times \sqrt{2gH}$
 $= 0.001963 \times \sqrt{2 \times 9.81 \times 10} = 0.001963 \times \sqrt{196.20}$
 $= 0.001963 \times 14.007 = 0.02749 \text{ m}^3/\text{sec.}$

Actual Discharge = $C_d \times \text{Theoretical discharge}$

Actual Discharge = $0.6 \times 0.02749 = 0.01649 \text{ m}^3/\text{sec}$

(Ans)

(ii) Actual Velocity:

We know that $C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$

Actual velocity = $C_v \times \text{Theoretical velocity}$
 $= 0.97 \times \sqrt{2gH}$
 $= 0.97 \times \sqrt{2 \times 9.81 \times 10} = 0.97 \times \sqrt{196.20}$
 $= 0.97 \times 14.007 = 13.58 \text{ m/sec}$

Actual velocity = 13.58 m/sec] (Ans)

Problem :- The head of water over an orifice of diameter 40mm is 10m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$.

PRACTICAL APPLICATION OF BERNOULLI'S EQUATION.

Bernoulli's equation is applied in all problems of incompressible fluid flow. This equation is applicable to the following measuring devices. (i) Venturiometer & (ii) Pitot-tube

VENTURI METER: A venturiometer is one of the most important practical application of Bernoulli's theorem. It is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three parts (P)

(i) Converging part (ii) Throat & (iii) Divergent part. The basic principle of venturiometer is that by reducing the cross-sectional area of the flow passage, a pressure difference is created and the measurement of pressure difference is used to determine discharge through pipe.

Let us consider a venturiometer fitted in a horizontal pipe through which some liquid is flowing as shown in the figure. Let a_1 and a_2 be the cross sectional areas at the inlet and throat section (ie section 1 & 2) of the venturiometer.

Let p_1 = intensity of pressure at inlet section.

v_1 = velocity of flow rate at section 1.

a_1 = Area at inlet section.

z_1 = Potential head.

d_1 = Diameter at inlet section.

Solution :- Data given :-

$$H = \text{Head of water} = 10 \text{ m}$$

$$d = \text{Diameter of orifice} = 40 \text{ mm} = \frac{40}{1000} = 0.040 \text{ m}$$

$$\alpha = \text{Area of orifice} = \frac{\pi d^2}{4} = \frac{\pi}{4} (0.040)^2 \\ = 0.001257 \text{ m}^2$$

$$C_d = 0.6, C_v = 0.98$$

(i) we know that $C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$

But Theoretical discharge = $V_{th} \times \text{Area of orifice}$.

$$= \sqrt{2gH} \times 0.001257$$

$$= \sqrt{2 \times 9.81 \times 10} \times 0.001257$$

$$= 14 \times 0.001257 = 0.0179 \text{ m}^3/\text{sec}$$

\therefore Actual Discharge = $C_d \times \text{Theoretical discharge}$

$$= 0.6 \times 0.0179 = 0.0107 \text{ m}^3/\text{sec}$$

(ii) we know that $C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$

$$\text{Actual velocity} = C_v \times \text{Theoretical velocity}$$

$$= 0.98 \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 10}$$

$$= 0.98 \times 14 = 13.72 \text{ m/sec.}$$

Actual velocity = 13.72 m/sec

Problem :- The head of water over the centre of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 litres/sec. Find the Coefficient of discharge.

P_2, V_2, a_2, z_2 and d_2

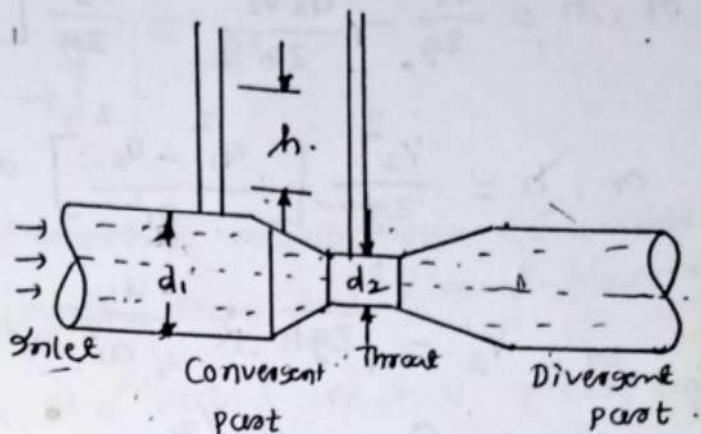
are the corresponding values at section 2

we assume that

flowing fluid is

incompressible

and there is



no loss of energy between the sections 1 & 2 of the venturimeter.

Applying Bernoulli's equation at sections 1 and 2

$$\text{we have } \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \dots (1)$$

As pipe is horizontal, hence $z_1 = z_2$

$$\text{or, } \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\text{or, } \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g} \right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \dots (2)$$

In the above expression $\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g} \right)$ is the difference between the pressure heads at sections 1 and 2, which is known as Venturi head and it is denoted by h .

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \dots (3)$$

If Q represents the discharge through the pipe then by continuity equation $a_1 V_1 = a_2 V_2$

$$\text{or, } V_1 = \frac{a_2 V_2}{a_1}$$

Putting this value in eq(3) we have

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1} \right)^2}{2g} = \frac{V_2^2}{2g} - \frac{a_2^2 V_2^2}{2g a_1^2}$$

Solution

Data given.

$$d = \text{Diameter of orifice} = 20\text{mm} = \frac{20}{1000} = 0.020\text{m}$$

$$A = \text{Area of the orifice} = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (0.020)^2 = 0.000314 \text{ m}^2$$

$$H = \text{Head of water} = 1\text{m.}$$

$$Q = \text{Actual Discharge} = 0.85 \text{ lit/sec} = \frac{0.85}{1000} = 0.00085 \text{ m}^3/\text{sec.}$$

$$\text{We know that Theoretical velocity} = V_{th} = \sqrt{2gH}$$

$$V_{th} = \sqrt{2 \times 9.81 \times 1} = \sqrt{19.62} = 4.429 \text{ m/sec.}$$

$$\text{Theoretical Discharge, } Q_{th} = V_{th} \times \text{Area of orifice.}$$

$$= 4.429 \times 0.000314 = 0.00139 \text{ m}^3/\text{sec.}$$

$$\text{We know that } C_d = \text{Co-efficient of discharge}$$

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{0.00085}{0.00139} = 0.61$$

$$\therefore C_d = 0.61$$

NOTCH

NOTCH :~ A notch may be defined as an opening provided in the side of a tank or vessel such that the liquid surface in the tank is below the top edge of the opening. Notches are generally made of metallic plates. They are also provided in narrow channels (particularly in laboratory).

USES: It is used for measuring the rate of flow of a liquid through a small channel or a tank.

TYPES OF NOTCH :~ There are several

types of notches, depending upon their shapes. However, the following are

important from subject point of view

(1) According to the shape of opening

(i) Rectangular notch

(ii) Triangular notch or V-notch

(iii) Trapezoidal notch and

(iv) Stepped notch.

(2) According to the effect of the sides on the nappe

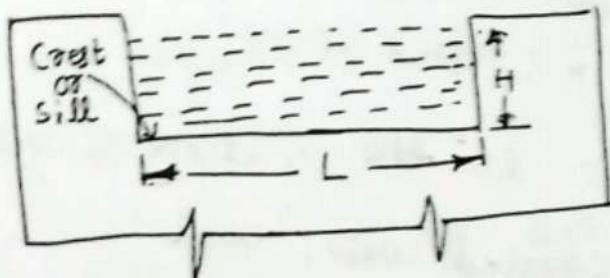
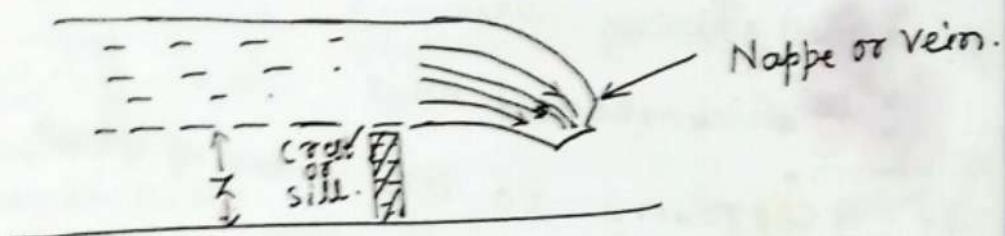
(a) Notch with end contraction (b) Notch without WEIR.

A weir is a concrete or masonry structure built across a river (stream) it is placed in an open channel over which the flow occurs.

It is built across a river in order to raise the level of water on the upstream side and to allow the excess water to flow over its entire length. A weir

is similar to a small dam. It is generally in the form of vertical wall.

USES:- It is used for measuring the rate of flow of water in a river



Nappe or vein: ~ The sheet of water flowing through over weir or notch is known as the nappe or vein.

Crest or sill: ~ The top of the weir over which the water flows is known as sill or crest.

CLASSIFICATION OF WEIR: There are different types of weirs depending upon their shapes, nature of discharge, width of crest or nature of crest.

1. According to shape

(i) Rectangular weir

(ii) Cippoletti weir

2. According to nature of discharge

(i) ordinary weir

(ii) submerged weir

3. According to the width of crest

(i) Narrow - crested weir

(ii) Broad - crested weir

4. According to the nature of crest

(i) Sharp - crested weir, and

(ii) Ogee - weir.

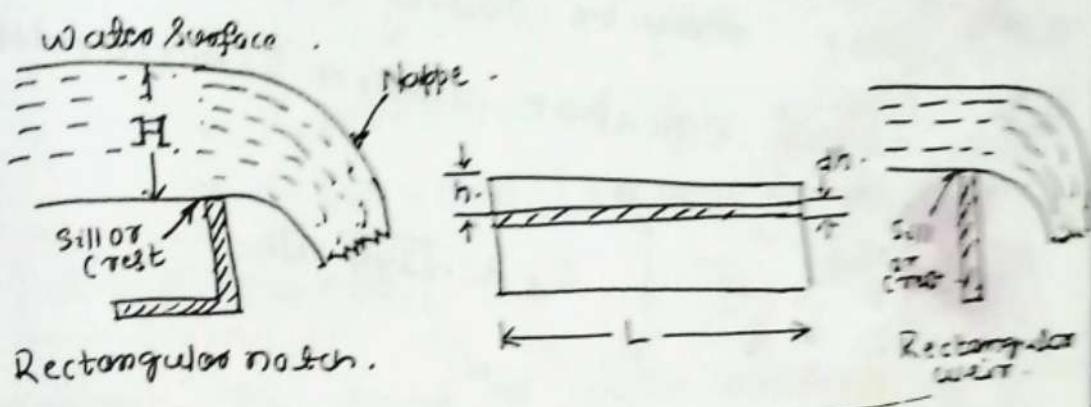
DIFFERENCE BETWEEN NOTCH AND WEIR

The main difference between a notch and a weir is that the notch is of small size, but the weir is of a bigger one. Moreover a notch is usually made in a metallic plate, whereas a weir is usually made of masonry or concrete.

DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

The expression for discharge over a rectangular notch or weir is the same.

Let us consider a rectangular notch or weir provided in a channel carrying water as shown in the figure.



Let H = Height of water above sill of the notch.

L = Length of notch or weir.

C_d = Co-efficient of discharge.

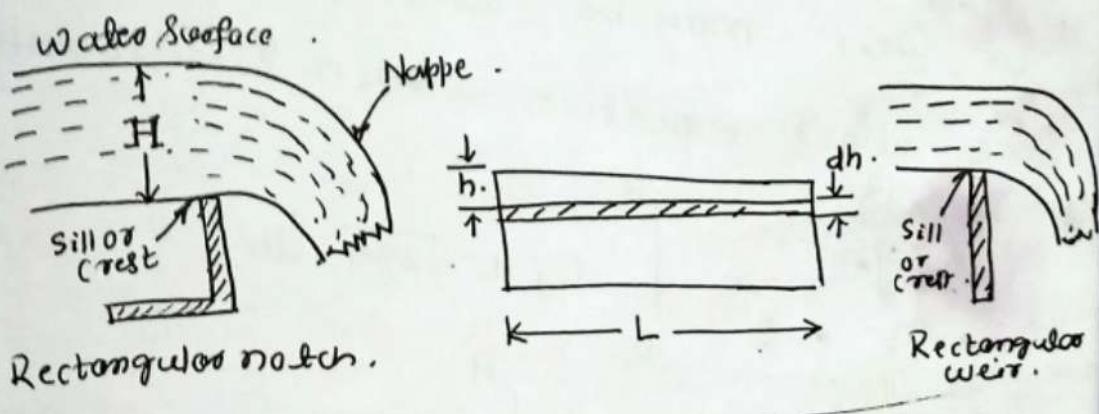
for finding the discharge of water flowing over the weir or notch, let us consider a horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in the figure

Area of strip = $L \times dh$.

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consider a horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in the figure

Area of strip = $L \times dh$.

We know that theoretical velocity of water flowing through strip $\sqrt{2gh}$

The discharge through the strip is given by $dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$

$$\text{or, } dQ = C_d \times L dh \times \sqrt{2gh} \quad (1)$$

The total discharge over the notch or weir may be found out by integrating the above equation within limits 0 and H

$$\int_0^Q dQ = \int_0^H C_d \cdot L \cdot \sqrt{2gh} \cdot dh$$

$$\begin{aligned} \text{or, } Q &= C_d L \sqrt{2g} \int_0^H \sqrt{h} dh \\ &= C_d L \sqrt{2g} \left[\frac{h^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^H \\ &= C_d L \sqrt{2g} \left[\frac{h^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^H = \frac{2}{3} C_d L \sqrt{2g} \left[h^{\frac{3}{2}} \right]_0^H \end{aligned}$$

$$Q = \frac{2}{3} C_d L \sqrt{2g} (H)^{\frac{3}{2}}$$

problem: Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300mm. Take $C_d = 0.60$

Solution: Data given

L = Length of the notch = 2 m

H = Head over notch = 300 mm = 0.30 m
= $\frac{300}{1000} = 0.30 \text{ m}$

C_d = Co-efficient of discharge = 0.60

$$Q = \text{Discharge} = \frac{2}{3} C_d \times L \sqrt{2g} \left[H \right]^{3/2}$$

$$= \frac{2}{3} \times 0.60 \times 2 \times \sqrt{2 \times 9.81} [0.30]^{1.5}$$

$$= 0.666 \times 0.60 \times 2 \times \sqrt{19.62} \times [0.30]$$

$$= 0.666 \times 0.60 \times 2 \times 4.4294 \times 0.164 = 0.580 \text{ m}^3/\text{sec}$$

$$Q = 0.580 \text{ m}^3/\text{sec}$$

Problem: The head of water over a rectangular notch is 900 mm. The discharge is 300 litres/sec. Find the length of the notch, when $C_d = 0.62$.

Solution: Data given.

H = Head of water = 900 mm = $\frac{900}{1000} = 0.90 \text{ m}$

Q = Discharge = 300 litres/sec = $\frac{300}{1000} = 0.3 \text{ m}^3/\text{sec}$

C_d = Co-efficient of discharge = 0.62.

C_d = Co-efficient of discharge = 0.62.

Let Length of notch or weir = L

Using equation $Q = \frac{2}{3} C_d L \sqrt{2g} (H)^{3/2}$

$$0.3 = \frac{2}{3} \times 0.62 \times L \sqrt{2 \times 9.81} \times (0.90)^{1.5}$$

$$= \frac{2}{3} \times 0.62 \times \sqrt{19.62} \times (0.90)^{1.5}$$

$$= 0.666 \times 0.62 \times L \times 4.429 \times 0.853$$

INTRODUCTION: A pipe is a closed conduit having circular cross-sectional section and used to carry water or any other fluid. The flow in a pipe is termed pipe flow only when the fluid completely fills the cross section and there is no free surface of fluid. When the pipe is running full, the flow is under pressure, but if the pipe is not running full, the flow is not under pressure. This is seen in the case of sewer pipes, culverts etc.

[Sewer pipes: It means channel for carrying water and refuse from drain].

Generally the fluid flowing in a pipe is always subjected to resistance due to shear forces between fluid particles and the boundary walls of the pipe and between the fluid particles themselves. resulting from the viscosity of the fluid. The resistance to the flow of fluid is in general known as frictional

resistance. In generally the flow of fluid in a pipe may be either laminar or turbulent.

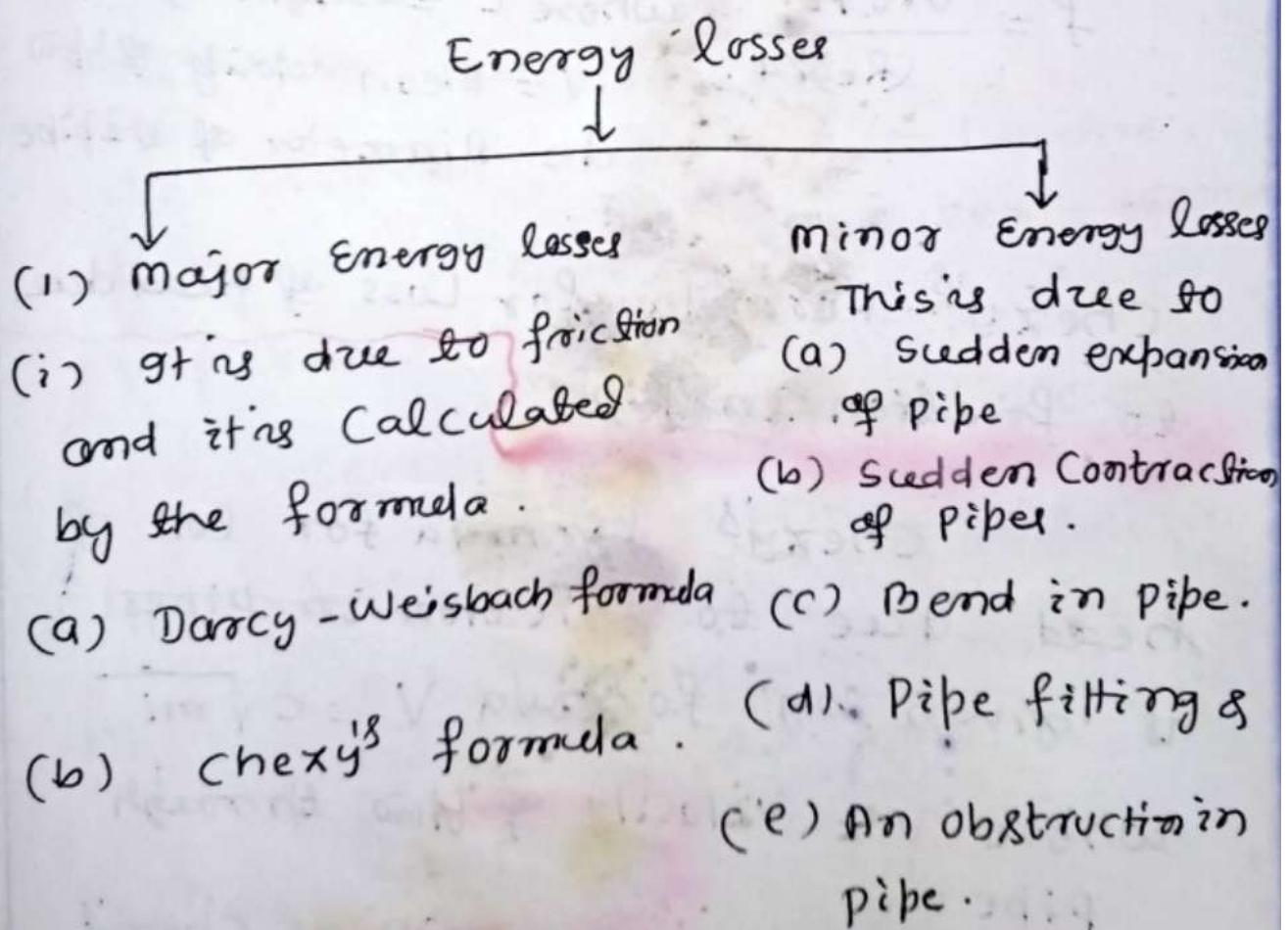
LOSS OF HEAD (ENERGY) IN PIPES

When water flows in a pipe, it creates some resistance to its motion, whose effect is to reduce the velocity and ultimately the head of water available. Actually there are many types of losses, but the major losses is due to frictional resistance of a pipe depends upon the roughness of the inside of the pipe. We know from experimentally that more the roughness of the inside surface of the pipe, we know from experimentally that more the roughness of the inside surface of the pipe, greater will be the resistance. This friction

is known as fluid resistance and the resistance is known as frictional resistance.

LOSS OF ENERGY IN PIPES.

When water flows in a pipe, it experiences some resistance to its motion, due to which its velocity and ultimately the head of water available is reduced. This loss of energy (or head) is classified as follows.



LOSS OF ENERGY (HEAD) DUE TO FRICTION

The loss of head (Energy) in pipes due to friction is calculated from Darcy weisbach equation and is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \cdot g}$$

where h_f = Loss of head due to friction.

f = Co-efficient of friction.

It is a function of Reynold's number

$$= \frac{16}{Re} \text{ for } Re < 2000 \quad [\text{Laminar flow}]$$

$$f = \frac{0.0791}{(Re)^{1/4}} \quad \text{where } L = \text{Length of the pipe}$$

V = mean velocity of flow

d = Diameter of the pipe.

Chezy's Formula for Loss of Head due to friction in pipes.

Chezy's formula for loss of head due to friction in pipes is given by formula $V = C \sqrt{m_i}$

where V = Velocity of flow through pipe.

C is a constant known as Chezy's constant.

$$\text{and } \frac{h_f}{L} = i$$

i is loss of head ~~due~~ per unit length of pipe.

$$m = \text{Hydraulic mean depth} = \frac{A}{P} = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$$

Problem: Find the loss of head, due to friction in a pipe of 1 metre diameter and 15 km long. The velocity of water in the pipe is 1 metre/sec. Take coefficient of friction is 0.005.

Solution: Data given.

$$d = \text{Diameter of the pipe} = 1 \text{ metre}$$

$$L = \text{Length of the pipe} = 15 \text{ km} = 15000 \text{ m}$$

$$V = \text{velocity of water} = 1 \text{ m/sec}$$

$$f = \text{co-efficient of friction} = 0.005$$

h_f = Loss of head due to friction

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g} = \frac{4 \times 0.005 \times 15000 \times (1)^2}{1 \times 2 \times 9.81}$$

$$h_f = 15.29 \text{ m}$$

Problem 2 On a pipe of diameter 350mm and length 75m water is flowing at a velocity of 2.8 m/sec. Find the head lost due to friction using (i) Darcy - weisbach formula

(ii) chezy's formula for which $C = 55$

Assume Kinematic viscosity of water as 0.012 stoke.

Solution. Data given

$$d = \text{Diameter of the pipe} = 350\text{mm}$$

$$= \frac{350}{1000} = 0.350\text{m.}$$

$$L = \text{Length of the pipe} = 75\text{m.}$$

$$V = \text{Velocity of the fluid} = 2.8 \text{ m/sec.}$$

$$C = \text{chezy's Constant} = 55$$

$$\nu = \text{Kinematic viscosity of water}$$

$$= 0.012 \text{ stoke} = 0.012 \text{ cm}^2/\text{sec}$$

$$= 0.012 \times 10^{-4} \text{ m}^2/\text{sec.}$$

(i) Darcy - weisbach formula is given by

$$h_f = \frac{4 \cdot f \cdot l \cdot V^2}{d \times 2g}$$

where $f = \text{Co-efficient of friction}$.

$$Re = \frac{V \times d}{\nu} = \frac{2.8 \times 0.35}{0.012 \times 10^{-4}} = 8.167 \times 10^5$$

$$f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.0791}{(8.167 \times 10^5)^{1/4}} = 0.00263$$

Head Lost due to friction

$$h_f = \frac{4 \cdot f l v^2}{d \times 2g} = \frac{4 \times 0.00263 \times 75 \times (2.8)^2}{0.35 \times 2 \times 9.81}$$

$$h_f = 0.9 \text{ m}$$

(ii) Chezy's formula is given by $V = C\sqrt{m i}$

$$\text{where } C = 55, m = \frac{A}{P} = \frac{\pi d^2}{4 \pi d} = \frac{d}{4} = \frac{0.35}{4}$$

$$\text{or, } m = 0.0875 \text{ m.}$$

Putting all the values, we have

$$\Rightarrow 2.8 = 55 \sqrt{0.0875 \times i}$$

$$\text{or, } \frac{2.8}{55} = \sqrt{0.0875 \times i} \text{ or, } \left(\frac{2.8}{55}\right)^2 = 0.0875 \times i^2$$

$$\text{or, } 0.0025 = 0.0875 \times i \therefore i = \frac{0.0025}{0.0875} = 0.0296$$

$$\text{But } i = \frac{h_f}{L} = 0.0296 \text{ m, } \frac{h_f}{75} = 0.0296.$$

$$\therefore h_f = 0.0296 \times 75 = 2.22 \text{ m}$$

$$\therefore h_f = 2.22 \text{ m}$$

Problem :- Find the diameter of a pipe of length 2000m when the rate of flow of water through the pipe is 200 litres/sec. and the head lost due to friction is 4m. Take the value of $C = 50$ in Chezy's formula.

Solution: Data given

$$l = \text{length of the pipe} = 2000\text{m}$$

$$Q = \text{Rate of flow} = 200 \text{ lit/sec} = \frac{200}{1000} \\ = 0.2 \text{ m}^3/\text{sec}$$

$$h_f = \text{Head lost due to friction} = 4\text{m.}$$

$$C = \text{Value of Chezy Constant} = 50$$

d = Let the diameter of the pipe

$$\text{We know that } Q = A \cdot V \text{ or, } V = \frac{Q}{A} = \frac{0.2}{\frac{\pi d^2}{4}} = \frac{0.2 \times 4}{\pi d^2}$$

$$\text{or, } V = \frac{0.2 \times 4}{\pi d^2}$$

$$m = \text{Hydraulic mean depth} = \frac{d}{4}$$

i = Loss of head per unit length

$$i = \frac{h_f}{L} = \frac{4}{2000} = 0.002.$$

We know from Chezy's formula

$$V = C \sqrt{mi} \quad \text{or, } \frac{0.2 \times 4}{\pi d^2} = 50 \sqrt{\frac{d}{4} \times 0.002}$$

$$\text{or, } \frac{0.2 \times 4}{\pi \times 50 \times d^2} = \sqrt{\frac{d}{4} \times 0.002}$$

$$\text{or, } \frac{0.00509}{d^2} = \sqrt{\frac{d}{4} \times 0.002}$$

Squaring both sides, we get

$$\frac{d}{4} \times 0.002 = \left(\frac{0.00509}{d^2} \right)^2 = \frac{0.0000259}{d^4}$$

$$\text{or, } d^5 = \frac{0.0000259 \times 4}{0.002} = 0.0518$$

$$\text{or, } d = \sqrt[5]{0.0518} = (0.0518)^{\frac{1}{5}} = 0.553\text{m}$$

$$\therefore d = 0.553 \times 1000 = 553\text{mm}$$

Problem: Water is flowing through a pipe 1500m long with a velocity of 0.8m/sec. What should be the diameter of the pipe, if the loss of head due to friction is 8.7m. Take f for the pipe as 0.01.

Solution: Data given

$$l = \text{length of the pipe} = 1500\text{m}$$

$$v = \text{velocity of water} = 0.8\text{m/sec.}$$

$$d = \text{Diameter of the pipe} = ?$$

$$h_f = \text{Head lost due to friction} = 8.7\text{m}$$

$$f = \text{Co-efficient of friction} = 0.01$$

$$\text{We know that } h_f = \frac{4flv^2}{d \times 2g}$$

Putting all the values, we have

$$8.7 = \frac{4 \times 0.01 \times 1500 \times (0.8)^2}{d \times 2 \times 9.8}$$

$$\text{or, } 8.7 \times d \times 2 \times 9.8 = 4 \times 0.01 \times 1500 \times (0.8)^2$$

$$\text{or, } d = \frac{4 \times 0.01 \times 1500 \times (0.8)^2}{8.7 \times 2 \times 9.8} = 0.225\text{m} = 0.225 \times 1000$$

$$d = 225\text{mm}$$

Problem : Find the loss of head, due to friction, in a pipe of 500mm diameter and 1.5 kilometres long. The velocity of water in the pipe is 1 m/sec. Take coefficient of friction = 0.005.

Solution : Data given.

$$d = \text{Diameter of the pipe} = 500\text{mm} = \frac{500}{1000} = 0.5\text{m}$$

$$f = 0.005$$

$$l = \text{Length of the pipe} = 1.5\text{km} = 1500\text{m}$$

$$v = \text{Velocity of water} = 1\text{m/sec.}$$

$$h_f = \text{Head loss due to friction} = ?$$

$$h_f = \frac{4f l v^2}{d \times 2g} = \frac{4 \times 0.005 \times 1500 \times (1)^2}{0.5 \times 2 \times 9.8} = 3.06\text{m}$$

$$h_f = 3.06\text{m}$$

Problem : - in a pipe of 300mm diameter and 800m length an oil of specific gravity 0.8 is flowing at the rate of $0.45\text{ m}^3/\text{sec}$. Find

(i) Head lost due to friction.

(ii) Power required to maintain the flow

Take Kinematic viscosity of oil as 0.3 Stoke .

Solution :- Data given .

$$d = \text{Diameter of the Pipe} = 300 \text{ mm} = \frac{300}{1000} = 0.3 \text{ m}.$$

$$l = \text{Length of the pipe} = 800 \text{ m}.$$

$$s = \text{specific gravity of oil} = 0.8.$$

$$\nu = \text{Kinematic viscosity of oil} = 0.38 \text{ stoke}$$

$$= 0.3 \text{ cm}^2/\text{sec} = 0.3 \times 10^{-4} \text{ m}^2/\text{sec}.$$

$$Q = \text{Discharge} = 0.45 \text{ m}^3/\text{sec}.$$

(i) Head lost due to friction, $= h_f$.

$$\text{we know that } Q = A V \therefore V = \frac{Q}{A}.$$

$$\text{or, } V = \frac{Q}{\frac{\pi d^2}{4}} = \frac{0.45}{\frac{\pi \times (0.3)^2}{4}} = 6.363 \text{ m/sec.}$$

$$Re = \text{Reynold's number} = \frac{V \times D}{\nu} = \frac{6.363 \times 0.3}{0.3 \times 10^{-4}}$$

$$= 6.363 \times 10^4$$

$$f = \text{Co-efficient of friction} = \frac{0.0791}{(6.363)^{1/4}}$$

$$= \frac{0.0791}{(6.363 \times 10^4)^{1/4}} = 0.00498$$

$$h_f = \frac{4 f l V^2}{d \times 2g} = \frac{4 \times 0.00498 \times 800 \times (6.363)^2}{0.3 \times 2 \times 9.81}$$

$$h_f = 109.61 \text{ m.}$$

(ii) Power required, P

Power required to maintain the flow where $w = 0.8 \times 9.81 = 7.848 \text{ KN/m}^3$

$$P = w Q h_f, Q = 0.45 \text{ m}^3/\text{sec.}$$

$$h_f = 109.61, Q = 0.45 \text{ m}^3/\text{sec.}$$

$$P = 7.848 \times 0.45 \times 109.61 = 387.09 \text{ kW}$$

$$\therefore P = 387.09 \text{ kW}$$

MINOR ENERGY (HEAD) LOSSES

The loss of Head or Energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases.

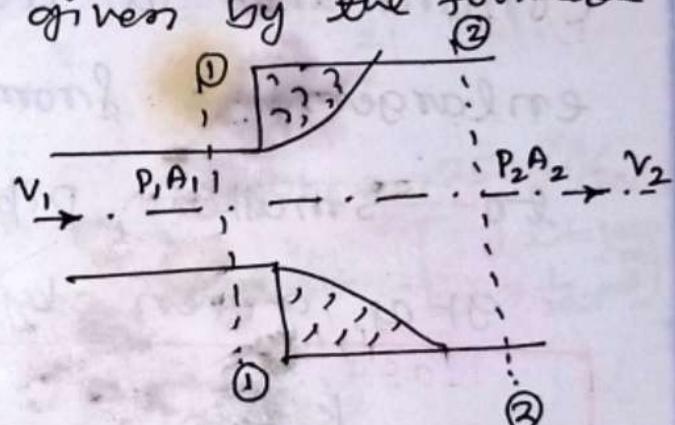
- (i) Loss of head due to sudden enlargement.
- (ii) Loss of head due to sudden Contraction.
- (iii) Loss of head due to obstruction in the pipe.
- (iv) Loss of head at the entrance to a pipe.
- (v) Loss of head at the exit of a pipe.
- (vi) Loss of head due to bend in the pipe.
- (vii) Loss of head in various Pipe fittings.

LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT:

Let us consider liquid flowing through a pipe which has sudden enlargement. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy (or head).

Loss of Energy (Head) due to sudden enlargement is given by the formulae

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

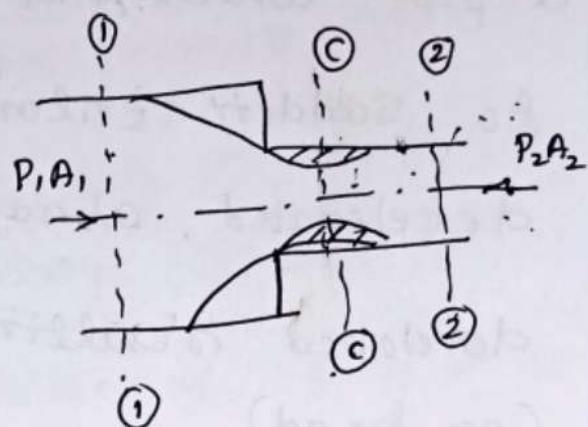


LOSS OF HEAD DUE TO SUDDEN CONTRACTION

Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in the figure. Let two sections 1-1 and 2-2 before and after contraction. As the liquid flows from larger pipe to smaller pipe, the area of flow goes on decreasing and becomes

minimum at a section c-c as shown in the figure. The section c-c is called Vena - Contracta.

After section c-c a sudden enlargement of the area takes place.



The loss of head due to sudden contraction is actually due to sudden enlargement from Vena - Contracta to smaller pipe

It is given by the formulae

$$h_c = \frac{K V_2^2}{2g} = 0.375 \frac{V_2^2}{2g} \quad \therefore C_c = \frac{A_c}{A_2}$$

$$K = \left[\frac{1}{C_c} - 1 \right]^2$$

If the value of C_c is not given then the head loss due to contraction is taken as $h_c = \frac{0.5 V_2^2}{2g}$ if the value of C_c is assumed to be equal to 0.62.

$$\text{Then } K = \left[\frac{1}{0.62} - 1 \right]^2 = 0.375$$

Problem: Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 lit/sec

Solution: Data given

$$D_1 = \text{Diameter of smaller pipe} = 200 \text{ mm} = \frac{200}{1000} = 0.20 \text{ m}$$

$$\text{Area} = A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$$

$$D_2 = \text{Diameter of larger pipe} = 400 \text{ mm} = \frac{400}{1000} = 0.40 \text{ m}$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.4)^2 = 0.1257 \text{ m}^2$$

$$Q = \text{Discharge through the pipe} = 250 \text{ lit/sec} \\ = \frac{250}{1000} = 0.25 \text{ m}^3/\text{sec} \quad \left[\because 1 \text{ lit} = \frac{1000}{cm^3} = \frac{1}{1000} \text{ m}^3 \right]$$

$$\text{We know that } Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.0314} = 7.96 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.1257} = 1.99 \text{ m/sec.}$$

Loss of head due to sudden enlargement is given by $h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81}$

$$h_e = \frac{(5.97)^2}{2 \times 9.81} = 1.816 \text{ m of water}$$

Problem :- The rate of blow of water through a horizontal pipe is $0.25 \text{ m}^3/\text{sec}$. The diameter of the pipe which is 200mm is suddenly enlarged to 400mm . The pressure intensity in the smaller pipe is 11.772 N/cm^2 . Determine (i) Loss of head due to sudden enlargement. (ii) pressure intensity in the larger pipe (iii) power lost due to enlargement.

Solution :- Data given $Q = 0.25 \text{ m}^3/\text{sec}$

$$D_1 = \text{Diameter of the smaller pipe} = 200\text{mm}$$

$$= \frac{200}{1000} = 0.20\text{m}$$

$$\text{Area of the smaller pipe} = \frac{\pi}{4} \times (D_1)^2 = \frac{\pi}{4} \times (0.20)^2$$

$$= 0.03141\text{m}^2$$

$$D_2 = \text{Diameter of the larger pipe} = 400\text{mm}$$

$$= \frac{400}{1000} = 0.40\text{m}$$

$$\text{Area of the larger pipe} = \frac{\pi}{4} \times (D_2)^2$$

$$= \frac{\pi}{4} \times (0.40)^2 = 0.1257\text{m}^2$$

$$\text{Pressure in smaller pipe} = 11.772 \frac{\text{N}}{\text{cm}^2}$$

$$= 11.772 \frac{\text{N}}{10^4 \text{m}^2} = 11.772 \times 10 \frac{\text{N}}{\text{m}^2}$$

Now velocity in smaller pipe $v_1 = \frac{Q}{A_1}$

$$\text{or, } v_1 = \frac{0.25}{0.0314} = 7.96 \text{ m/sec}$$

Velocity in larger pipe $v_2 = \frac{Q}{A_2} = \frac{0.25}{0.1257} = 1.99 \text{ m/sec}$

(i) Loss of head due to sudden enlargement.

$$h_e = \frac{(v_1 - v_2)^2}{2g} \quad \text{putting all the values,}$$

$$\text{we get } h_e = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = \frac{(5.97)^2}{2 \times 9.81}$$

$$= \frac{35.6409}{2 \times 9.81} = 1.816 \text{ m.}$$

$$h_e = 1.816 \text{ m}$$

(ii) Let P_2 = pressure intensity in larger pipe

Then applying Bernoulli's equation before
and after the sudden enlargement.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

Since the pipe is horizontal $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\text{or, } \frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{(7.96)^2}{2 \times 9.81} - \frac{(1.99)^2}{2 \times 9.81} - 1.816$$

$$= 12 + 3.229 - 0.208 - 1.816$$

$$\text{Or}, \frac{P_2}{\rho g} = 15.229 - 2.0178 = 13.21$$

$$\text{Or}, P_2 = 13.21 \times \rho g = 13.21 \times 1000 \times 9.81$$

$$\text{Or}, P_2 = 12.96 \text{ N/cm}^2$$

(iii) Power lost due to sudden enlargement

$$P = \frac{\rho g Q h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000}$$

$$\therefore P = 4.453 \text{ kW}$$

Problem :- Calculate the discharge through a pipe of diameter 200mm when the difference of pressure head between the two ends of a pipe 500m apart is 4m of water. Take the value of ' f ' = 0.009 in the formula $h_f = \frac{4f l v^2}{d \times 2g}$

Solution :- Data given

$$d = \text{Diameter of the Pipe} = 200\text{mm} = \frac{200}{1000} = 0.20\text{m}$$

$$l = \text{Length of the Pipe} = 500\text{m}$$

$$h_f = \text{Difference of pressure head} = 4\text{m of water}$$

$$f = 0.009$$

We know that $h_f = \frac{4 \cdot f \cdot l \cdot v^2}{d \cdot g}$

$$\text{or, } 4.0 = \frac{4 \times 0.009 \times 500 \times v^2}{0.2 \times 2 \times 9.81} = 4.587 v^2$$

$$\text{or, } v^2 = \frac{4.0}{4.587} = 0.872 \text{ or, } v = \sqrt{0.872}$$

$$\text{or, } v = 0.933 \text{ m/sec}$$

∴ Discharge $Q = \text{Area} \times \text{Velocity}$

$$\text{or, } Q = \frac{\pi}{4} d^2 \times 0.933 = \frac{\pi}{4} \times (0.20)^2 \times 0.933$$

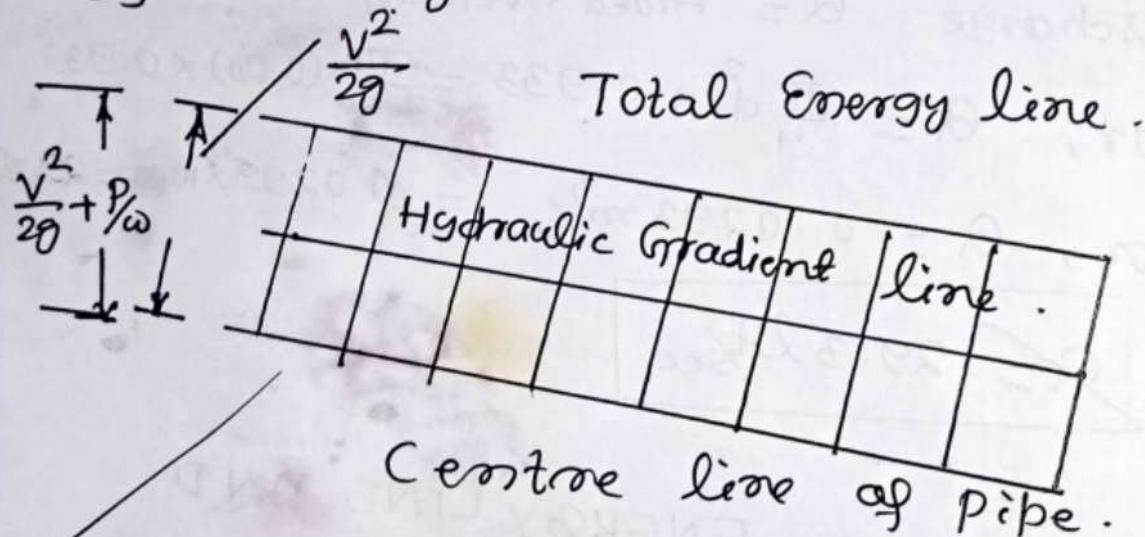
$$\text{or, } Q = 0.0293 \text{ m}^3/\text{sec} = 0.0293 \times 1000 = 29.3 \text{ l/sec}$$

$$\therefore Q = 29.3 \text{ l/sec}$$

✓ EXPLAIN TOTAL ENERGY LINE AND HYDRAULIC GRADIENT LINE.

Hydraulic Gradient line is an imaginary line which indicates the pressure head at different sections conveying fluid in between two tanks. The line shows the piezometric head at different sections of the pipe line of piezometric heads or pressure heads ($P/\rho g$) of a liquid flowing

in a pipe, be plotted as vertical ordinates on the centre line of the pipe, then the line joining the tops of such ordinates is known as hydraulic gradient line. It is ~~symbol~~
Symbolically written as (H.G.L)



TOTAL ENERGY LINE :- If the sum

of pressure head and velocity heads $(\frac{P}{\rho} + \frac{V^2}{2g})$ of a liquid flowing in a pipe, be plotted as vertical ordinates on the centre line of the pipe, then the line joining the tops of such

✓ In other words, the total energy line lies, over the hydraulic gradient, by an amount equal to the velocity heads as shown in the diagram.

$$FE = 2, \text{ so } \frac{V_1}{g} = 2$$

$$\frac{V_1}{g} = \frac{2}{\sin \theta}$$