LECTURE NOTES

ON

CONTROL SYSTEM ENGG. (TH-3)

DIPLOMA COURSES

6TH SEMESTER

ELECTRICAL ENGINNERING

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DEPT. OF ELECTRICAL ENGINEERING

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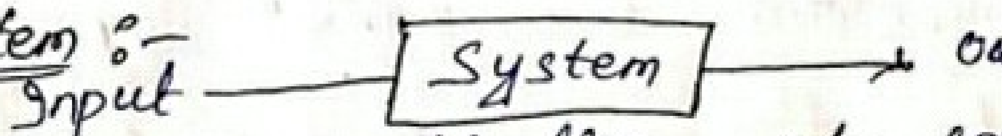
* System :-

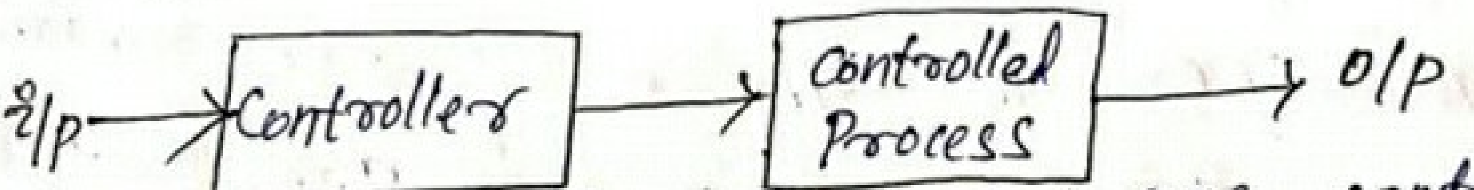


* It is the combination of components that act together to perform the desired task.

* Control System :-
It means that the quantity of intrest in a machine or mechanism is controlled according to the desired manner.

* Types of control system :-
There are 2 types of control system.
(a) open loop control system (non-feedback system)
(b) closed loop " " (feed back system)

(a) Open loop :-
In open loop control system the control action is independent of the desired output.
* In this system the output is not comparre with the reference input. So it is called as non-feedback control system.



The main component of OLCS are controller and controlled process.
The controlled may be amplifier, phase shifter, filter etc. modulator, zener diode, depends upon the system.
(An input is apply to the controller and controlled process work and we get desired output)
Examples :- Automatic washing m/c, traffic light controll, auto-~~pilot~~ ~~system~~.

→ Error ratio $= \dfrac{E(S)}{R(S)}$

→ We know,

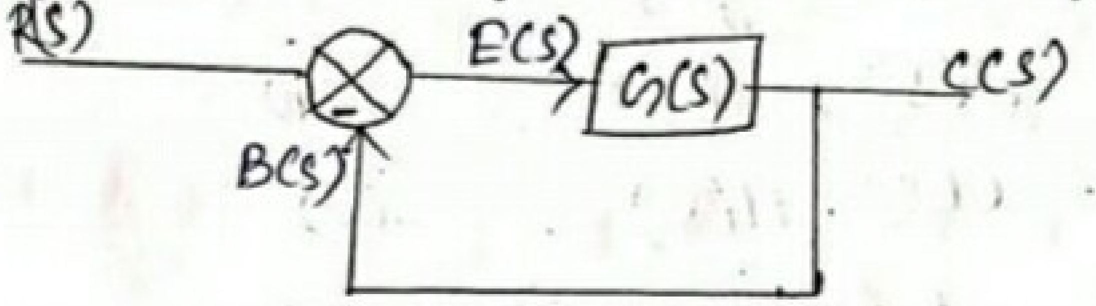
$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S).H(S)}$$

→ Put the value of $C(S) = E(S).G(S)$,

$$\frac{E(S).\cancel{G(S)}}{R(S)} = \frac{\cancel{G(S)}}{1+G(S).H(S)}$$

$$\boxed{\text{Error Ratio} \Rightarrow \frac{E(S)}{R(S)} = \frac{1}{1 \pm G(S).H(S)}}$$

* <u>Unit feedback system</u> :- $(H(S) = 1)$



$$\boxed{\Rightarrow T.F = \frac{C(S)}{R(S)} = \frac{G(S)}{1 \pm G(S)}}$$

<u>Dt - 19/12/19</u>

* <u>Block diagram</u> Re-presentation :-

→ A block may be present single component or a group of component but each block is complitely characterised by a transfer function.

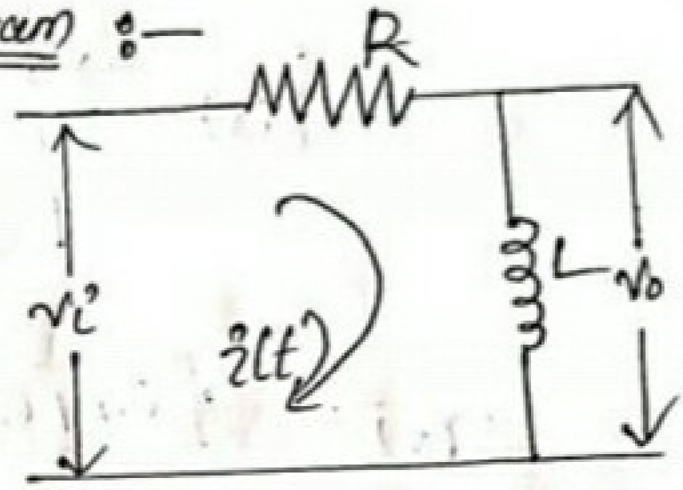
→ The T.F. is an expression with relates O/P to I/P in 's' domain.

→ T.F. does not give any information about the internal structure of the system.

→ Once we determine the T.F., then we can represent the system by the block diagram

→ Block diagram are single line diagram, i.e. the flow of system variable from one block to another block is represented by a single line.

→ <u>How to draw the block diagram</u> :-



→ $v_i = Ri(t) + L\frac{di(t)}{dt}$

$\Rightarrow V_i(S) = RI(S) + LSI(S)$

$\Rightarrow \boxed{V_i(S) = (R+SL)I(S)}$

→ $v_0 = \frac{L\,di}{dt}$

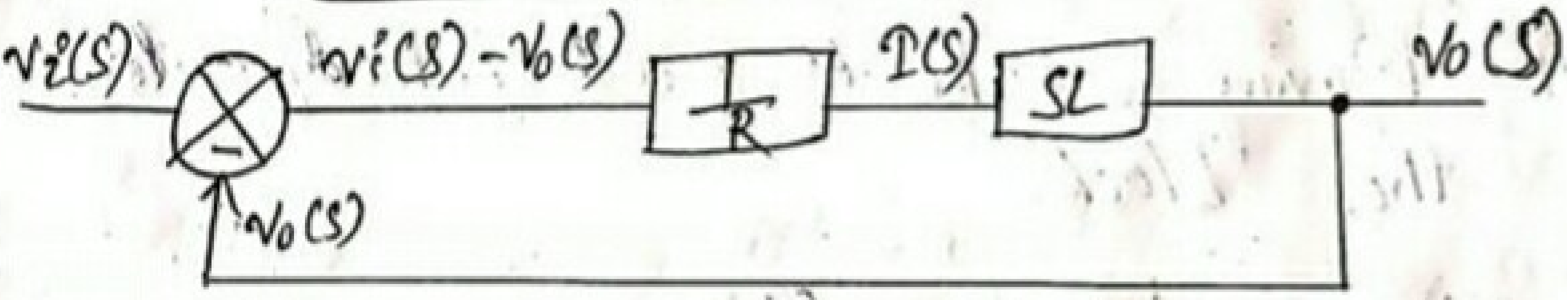
$\boxed{\Rightarrow v_0(S) = SLI(S)}$

→ $\boxed{T.F. = \frac{v_0(S)}{v_i(S)} = \frac{SL}{R+SL}}$

-o-

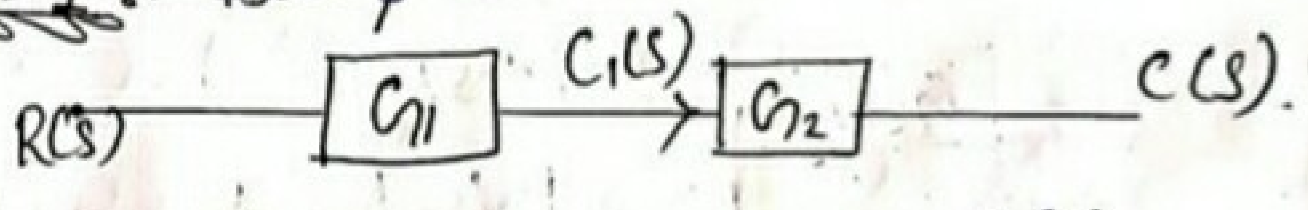
→ $i(t) = \frac{v_i - v_0}{R}$

$\Rightarrow \boxed{I(S) = \frac{1}{R}\left[v_i(S) - v_0(S)\right]}$



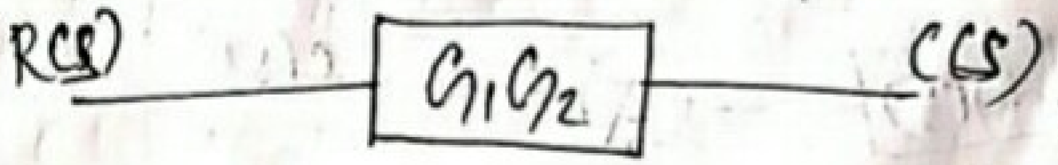
⇒ <u>Block diagram reduction</u>:-

1) <u>Rule - 1</u>:- Blocks in cascade connection;

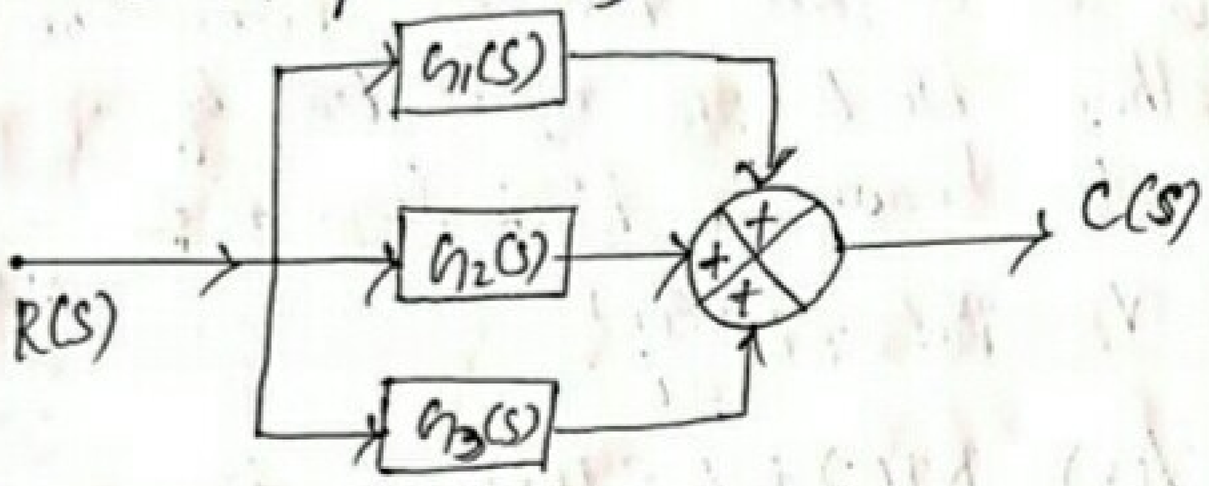


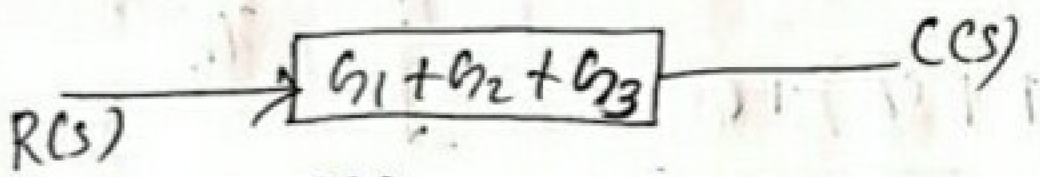
$T.F. \Rightarrow G_1 = \frac{C_1(S)}{R(S)}$ & $G_2 = \frac{C(S)}{C_1(S)}$

$\Rightarrow \frac{\cancel{C_1(S)}}{R(S)} \times \frac{C(S)}{\cancel{C_1(S)}} = \frac{C(S)}{R(S)} = G_1 G_2$



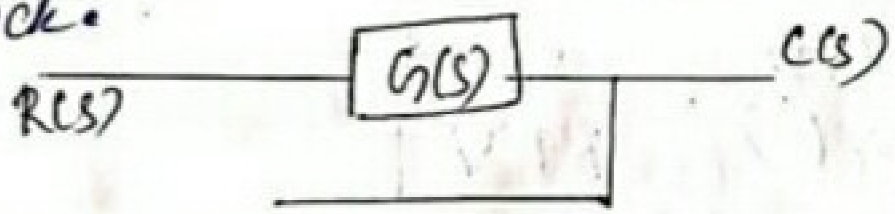
2. Rule-2:- Blocks in parallel;

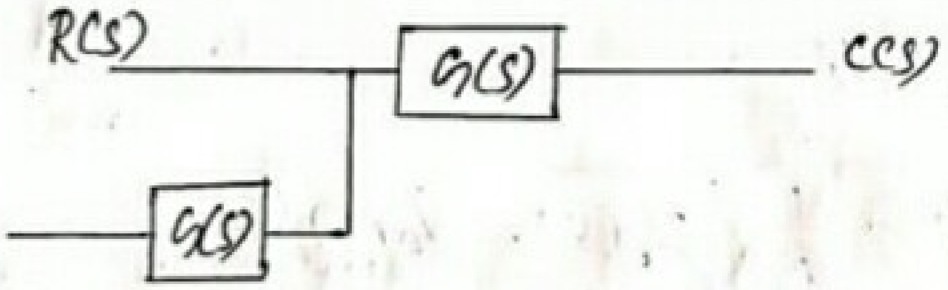




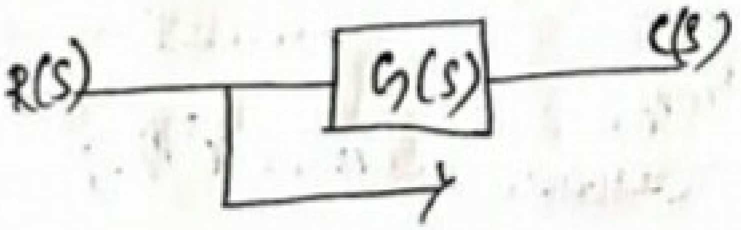
$$T.F. \Rightarrow \frac{C(s)}{R(s)} = G_1 + G_2 + G_3$$

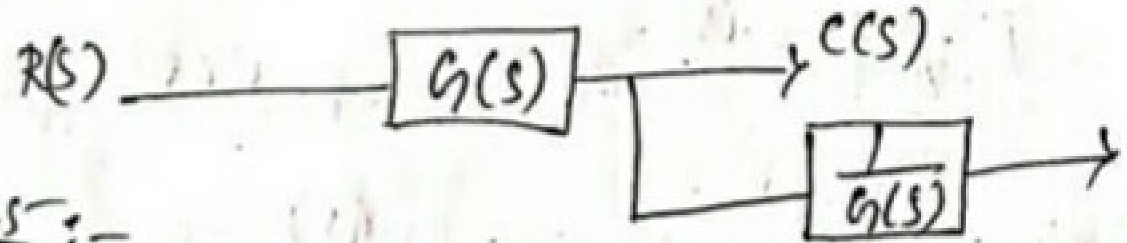
3. Rule-3 :- Moving a take off point a-head of a block.





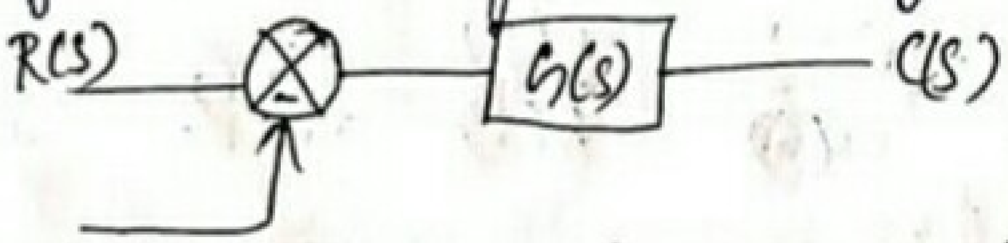
4) Rule 4:- Moving a take off point behind/after the block.

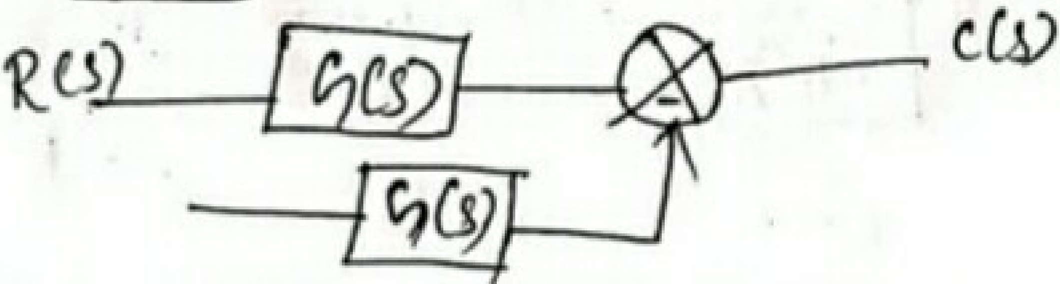




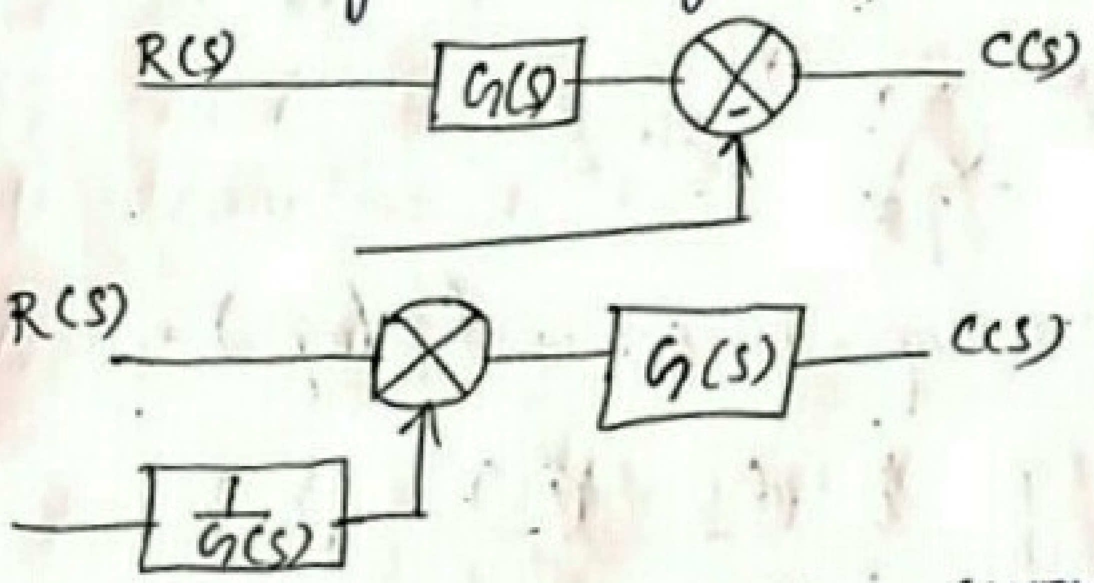
Rule-5 :-

5. Moving a summing point beyond the block.

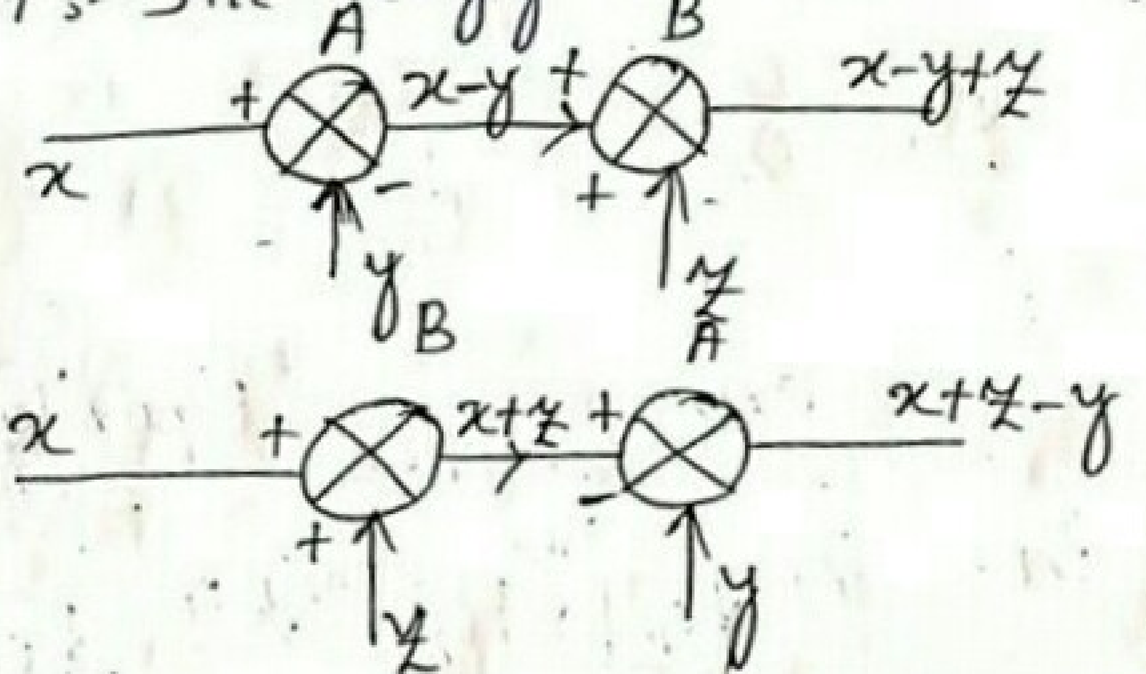




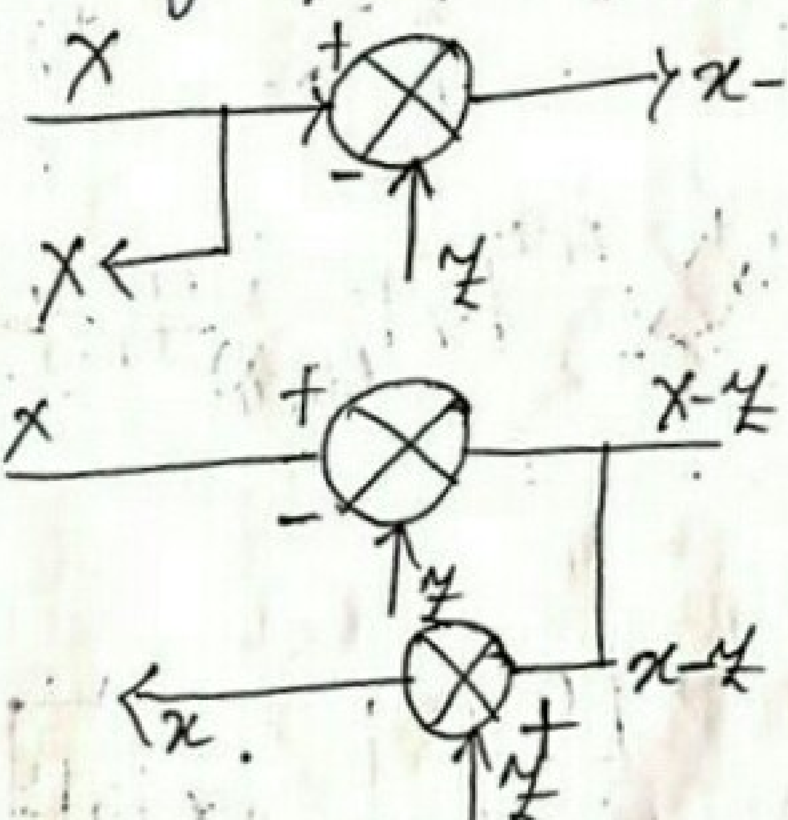
6. Rule - 6 :- Moving a summing point ahead of a block



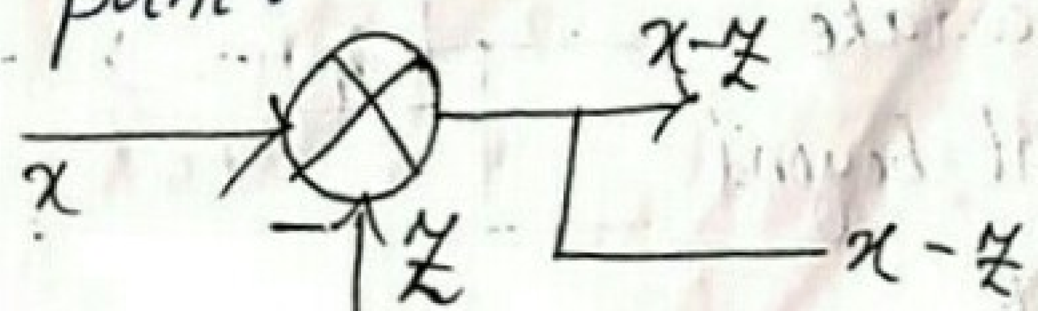
7. Rule-7 :- Interchanging of two summing point.

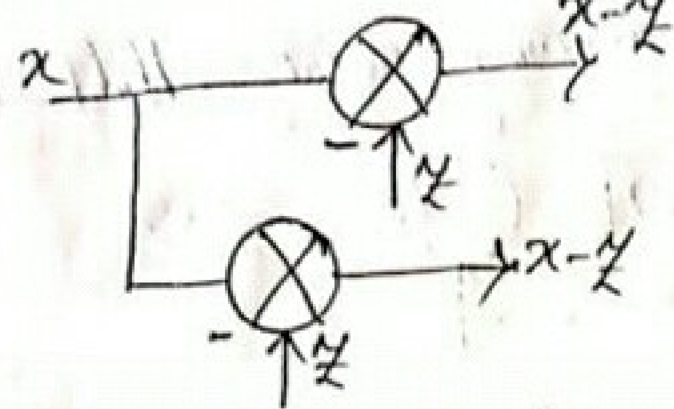


8. Rule-8 :- Moving a take off point beyond a summing point.

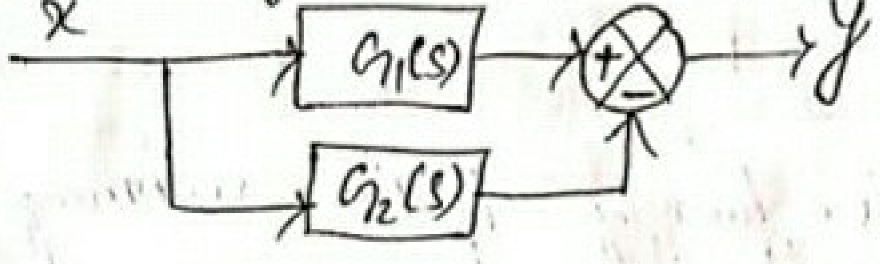


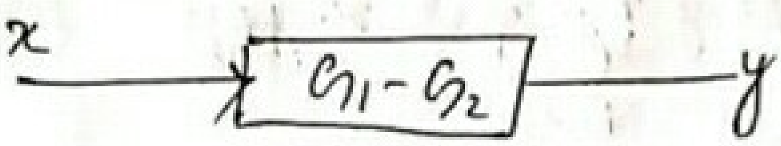
9. Rule-9 :- Moving a take off point ahead of summing point.

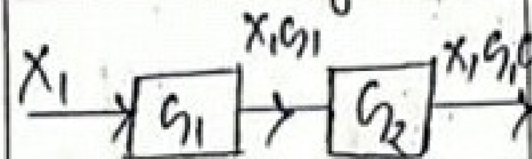
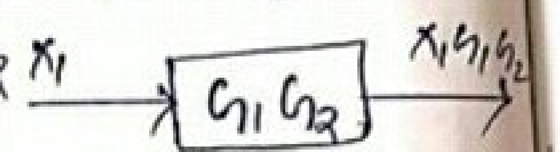
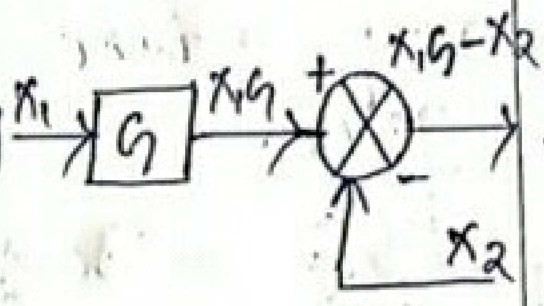
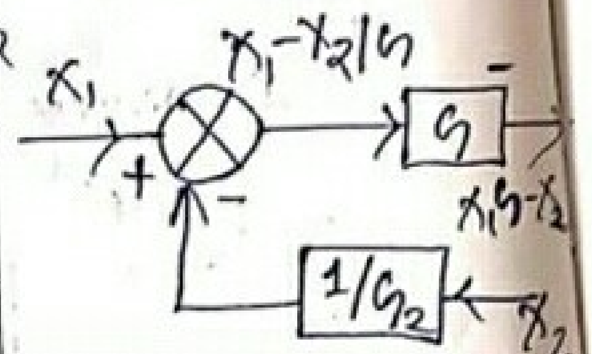
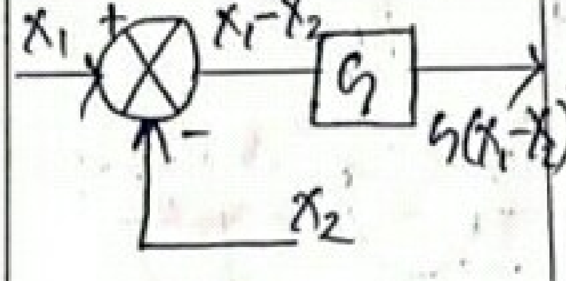
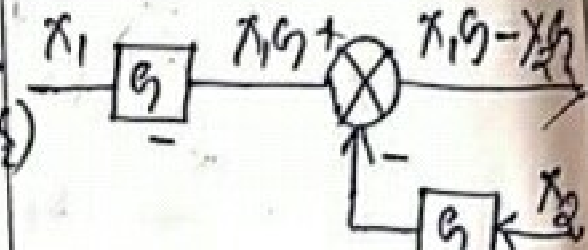
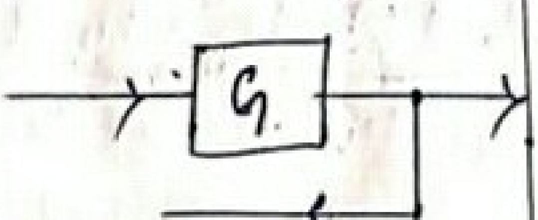
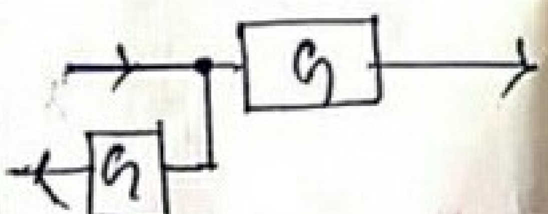
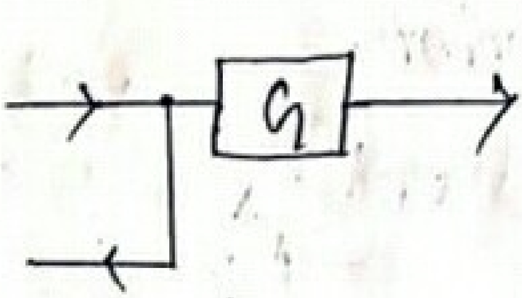
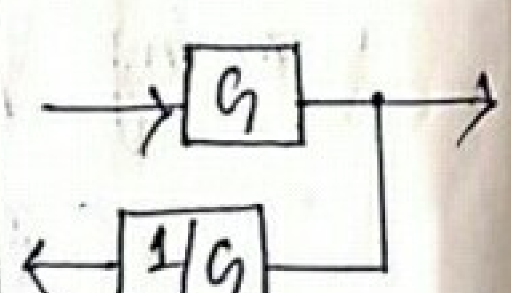


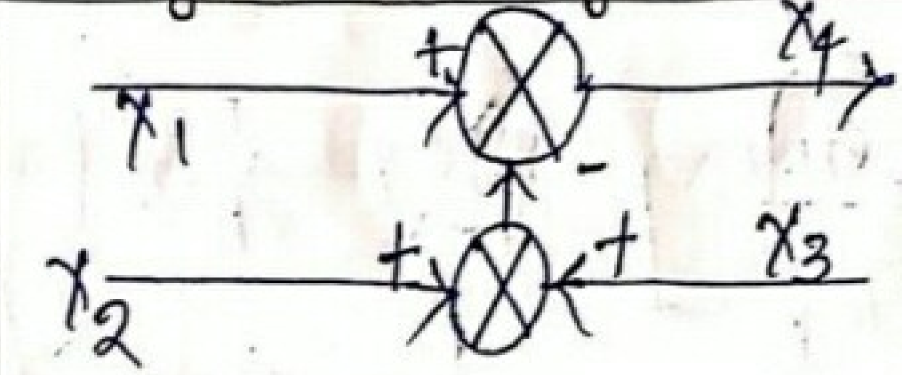
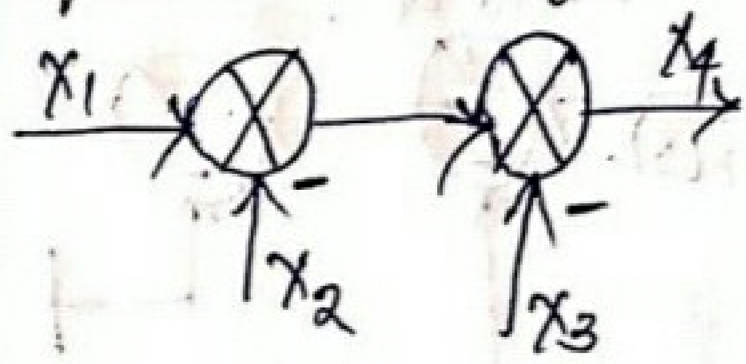
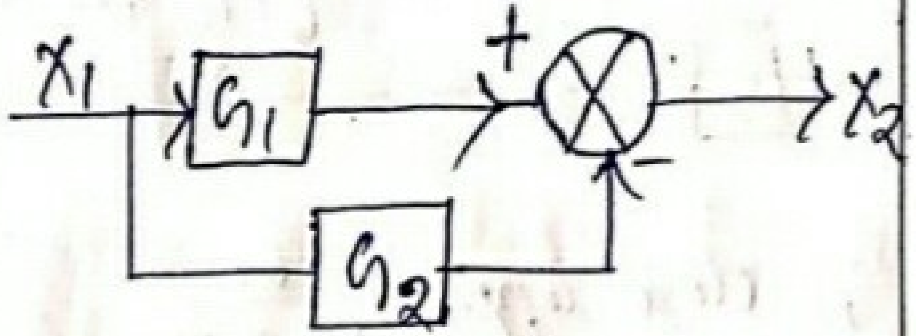
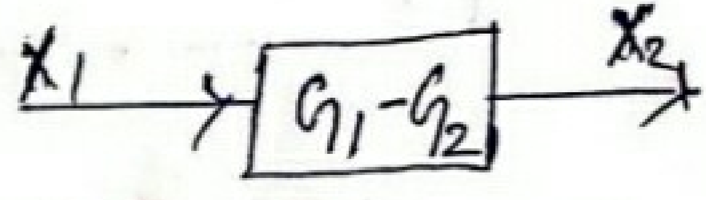
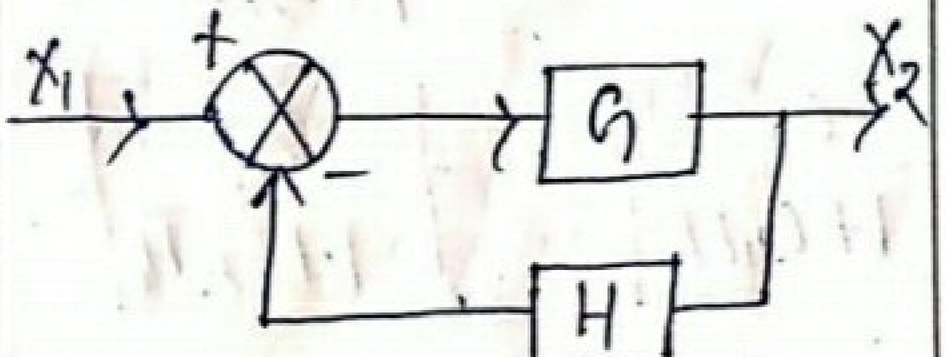
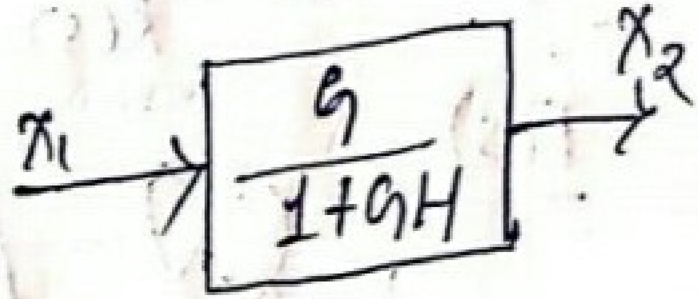


10. <u>Rule-10</u> (Eleminating a forward loop) :-

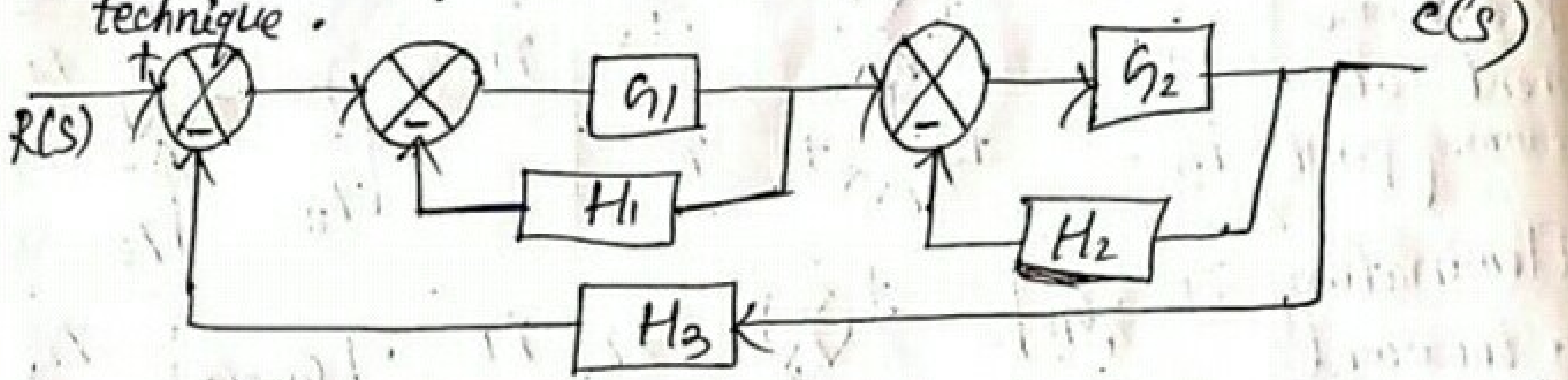




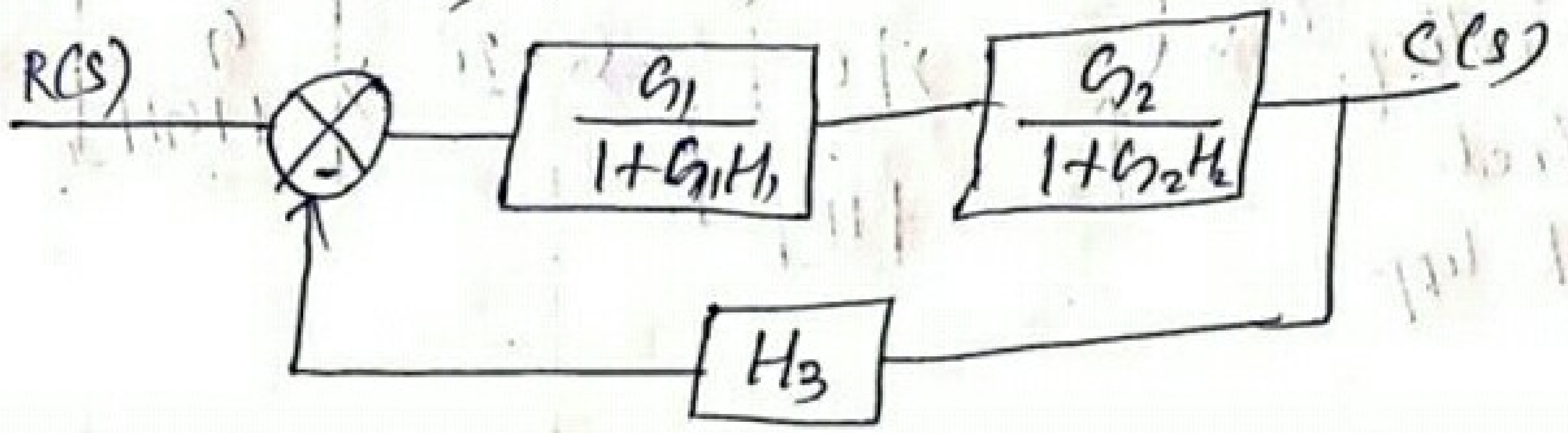
SL. No.	Rule	Original diagram	Equivalent diagram
01.	Block in cascade		
02.	Moving a summing point ahead of a block		
03.	Moving a summing point beyond a block		
04.	Moving a take off point ahead of a block		
05	Moving a take off point beyond a block		

Sl No.	Rule	Original diagram	Equivalent diagram
06.	Rearrangement of summing point		
07.	Eliminating a forward loop.		
08.	Eliminating a feed-back loop		

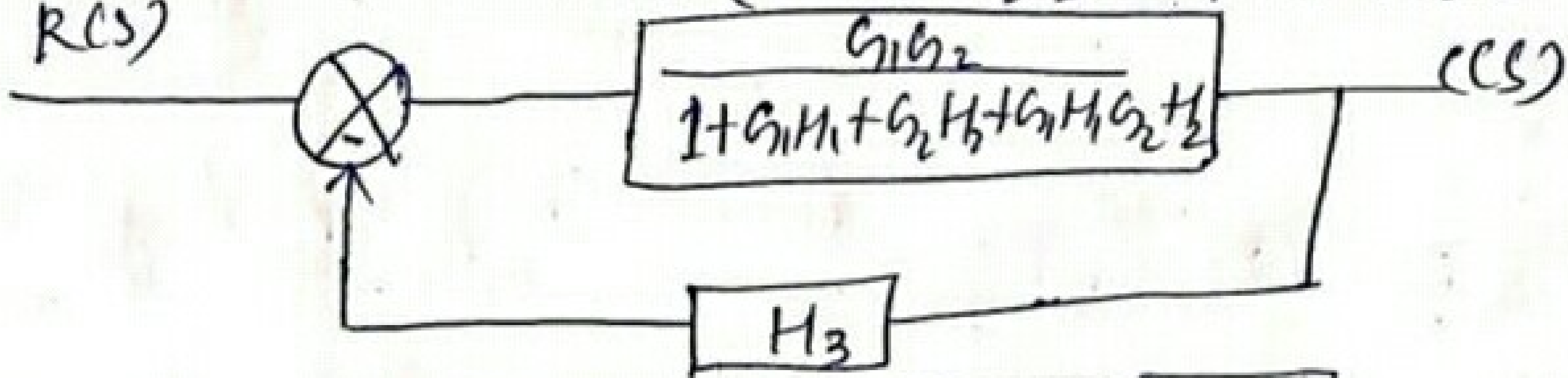
Q1) Derive the transfer function using block reduction technique.



$$T.F. = \frac{C(s)}{R(s)} \quad , \quad \text{close lopp} = \frac{G}{1 \pm GH}$$



$$\frac{G_1}{1+G_1H_1} \times \frac{G_2}{1+G_2H_2} = \frac{G_1G_2}{1+G_2H_2+H_1G_1+G_1H_1G_2H_2}$$



R(S)

$$\frac{\dfrac{G_1G_2}{1+G_1H_1+G_2H_2+G_1H_1G_2H_2}}{1+\dfrac{G_1G_2}{1+G_1H_1+G_2H_2+G_1H_1G_2H_2} \times H_3}$$

C(S)

R(S)

$$\frac{\dfrac{G_1G_2}{\cancel{1+G_1H_1+G_2H_2+G_1H_1G_2H_2}}}{\dfrac{1+G_1H_1+G_2H_2+G_1H_1G_2H_2+G_1G_2H_3}{\cancel{1+G_1H_1+G_2H_2+G_1H_1G_2H_2}}}$$

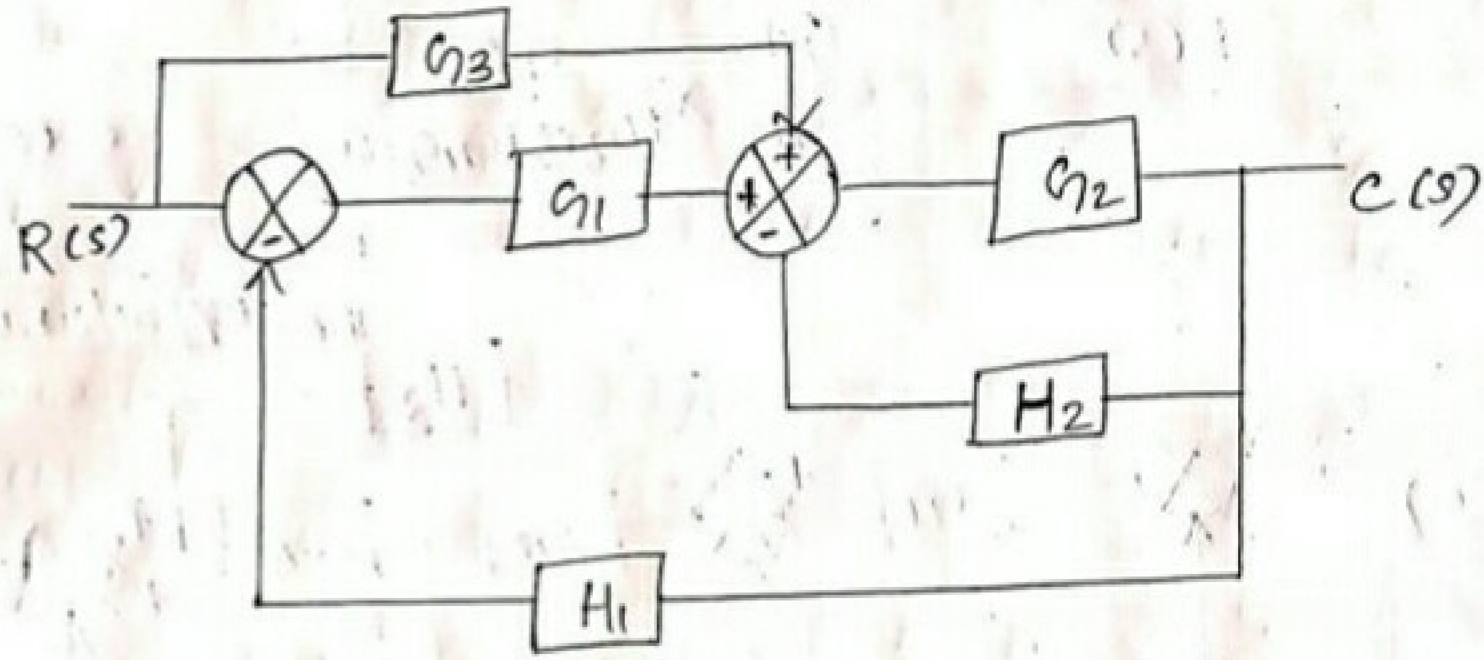
C(S)

R(S)

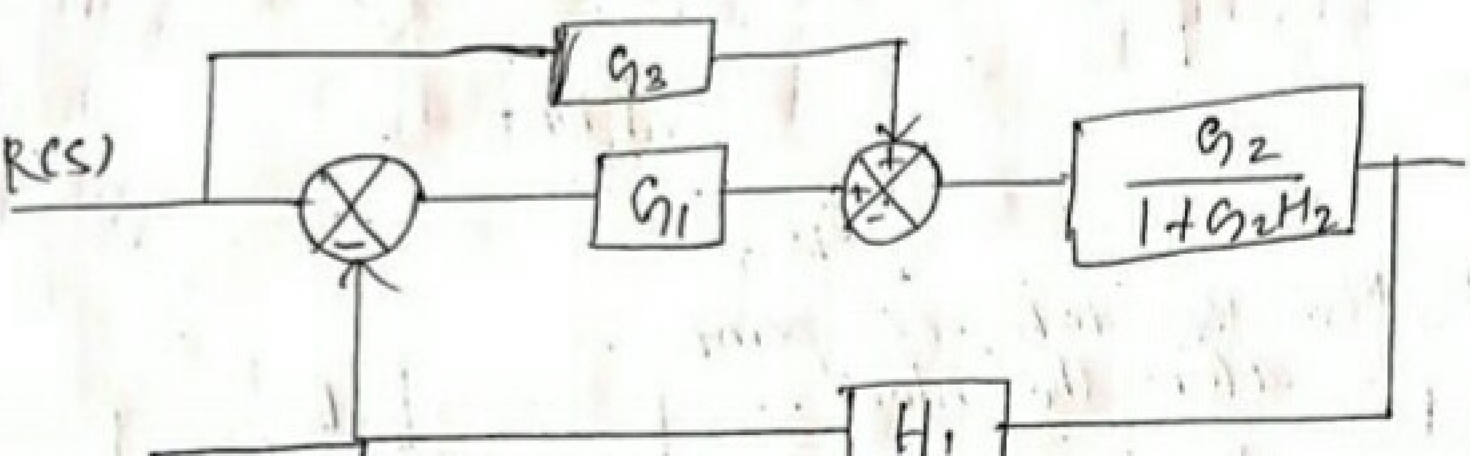
$$\frac{G_1G_2}{1+G_1H_1+G_2H_2+G_1H_1G_2H_2+G_1G_2H_3}$$

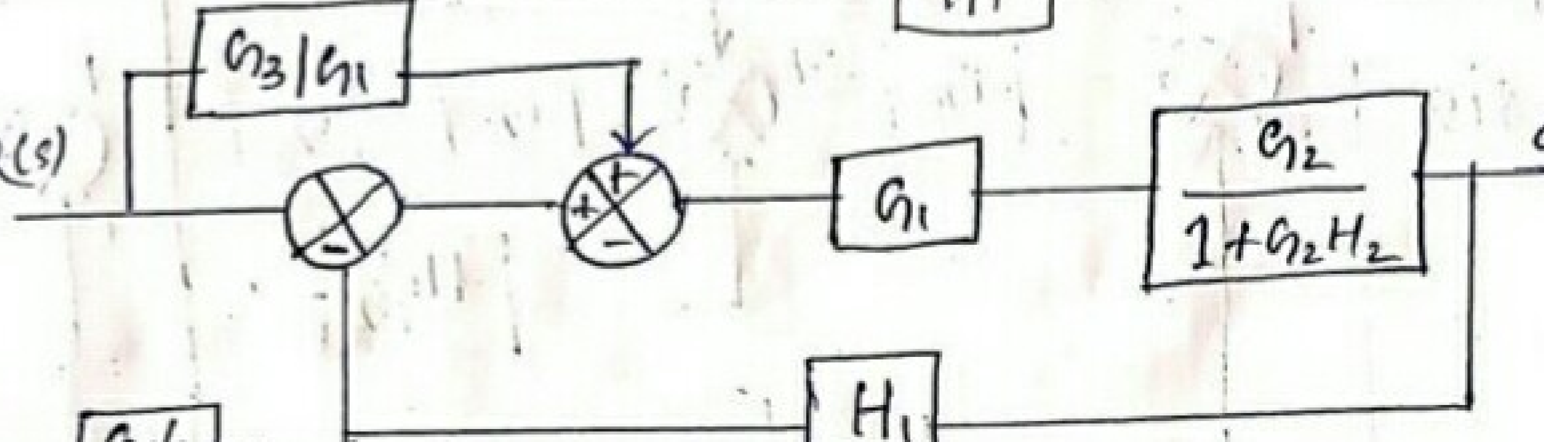
C(S)

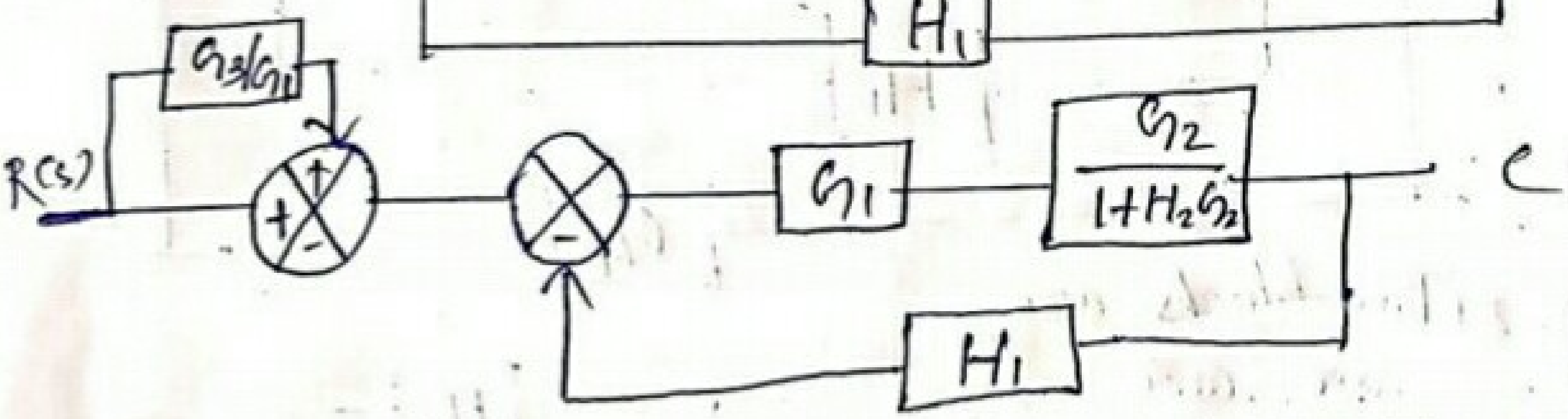
Q)

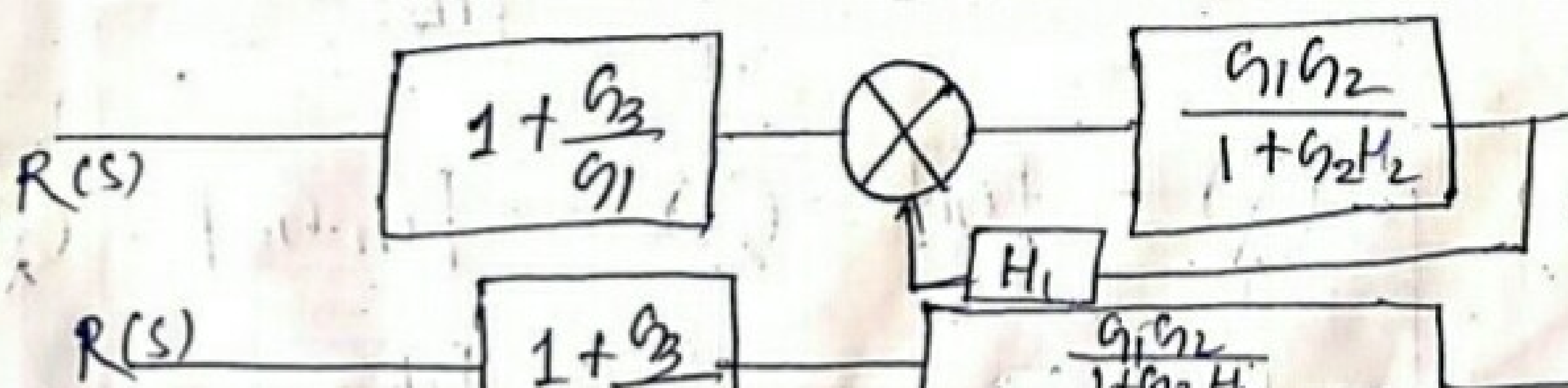

G3
R(s)
G1
G2
C(s)
H2
H1

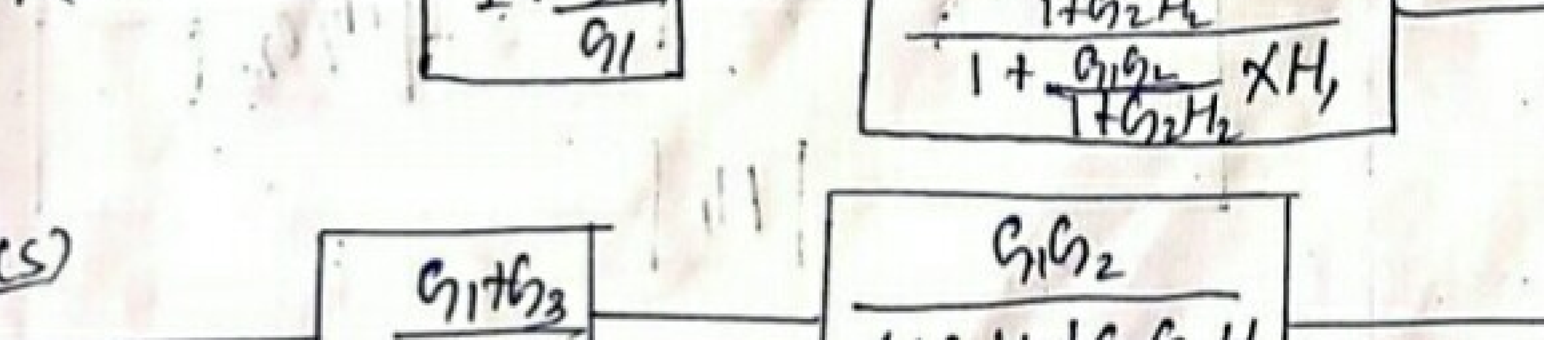
Soln

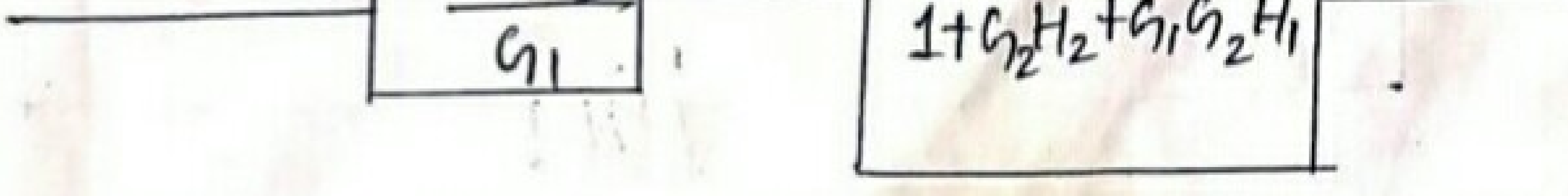

G3
R(s)
G1
$\frac{G_2}{1+G_2H_2}$
H1

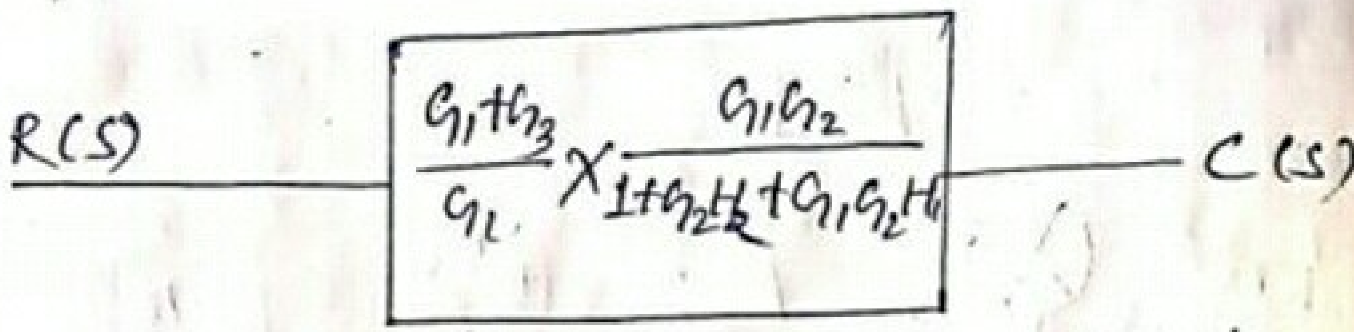

G3/G1
R(s)
G1
$\frac{G_2}{1+G_2H_2}$
H1


G3/G1
R(s)
G1
$\frac{G_2}{1+H_2G_2}$
C
H1


R(s)
$1+\frac{G_3}{G_1}$
$\frac{G_1G_2}{1+G_2H_2}$
H1

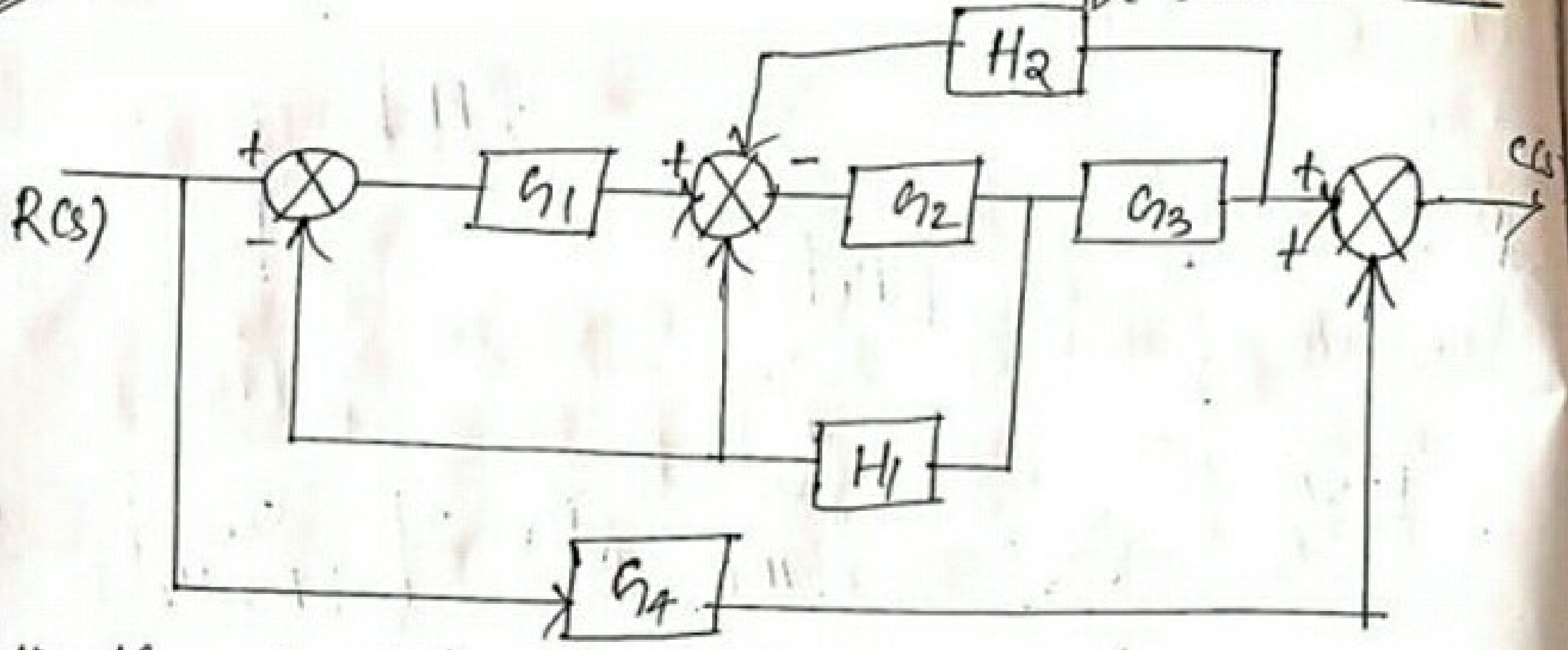

R(s)
$1+\frac{G_3}{G_1}$
$\frac{\frac{G_1G_2}{1+G_2H_2}}{1+\frac{G_1G_2}{1+G_2H_2}\times H_1}$


R(s)
$\frac{G_1+G_3}{G_1}$
$\frac{G_1G_2}{1+G_2H_2+G_1G_2H_1}$



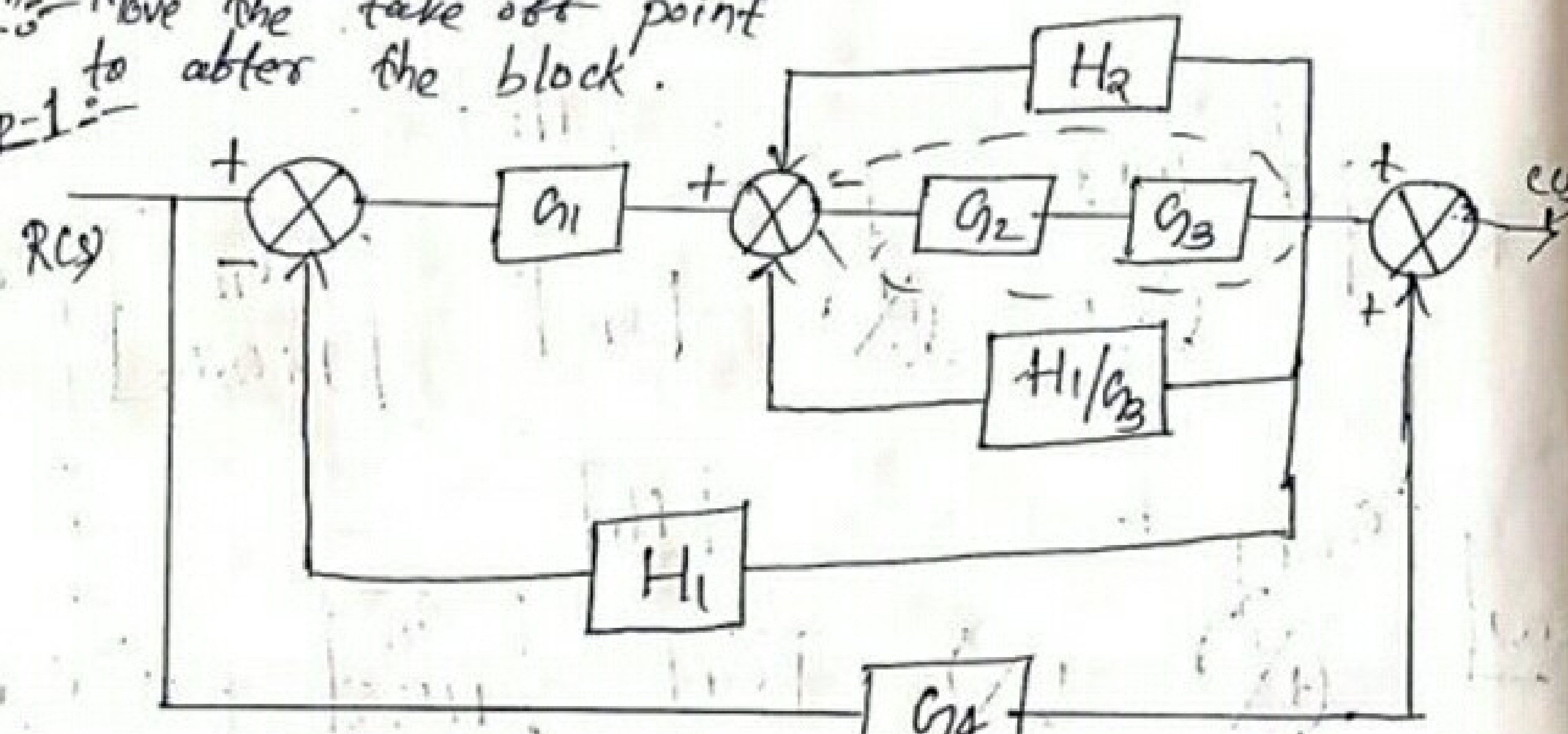
Q.

Dt- 07/01/2020

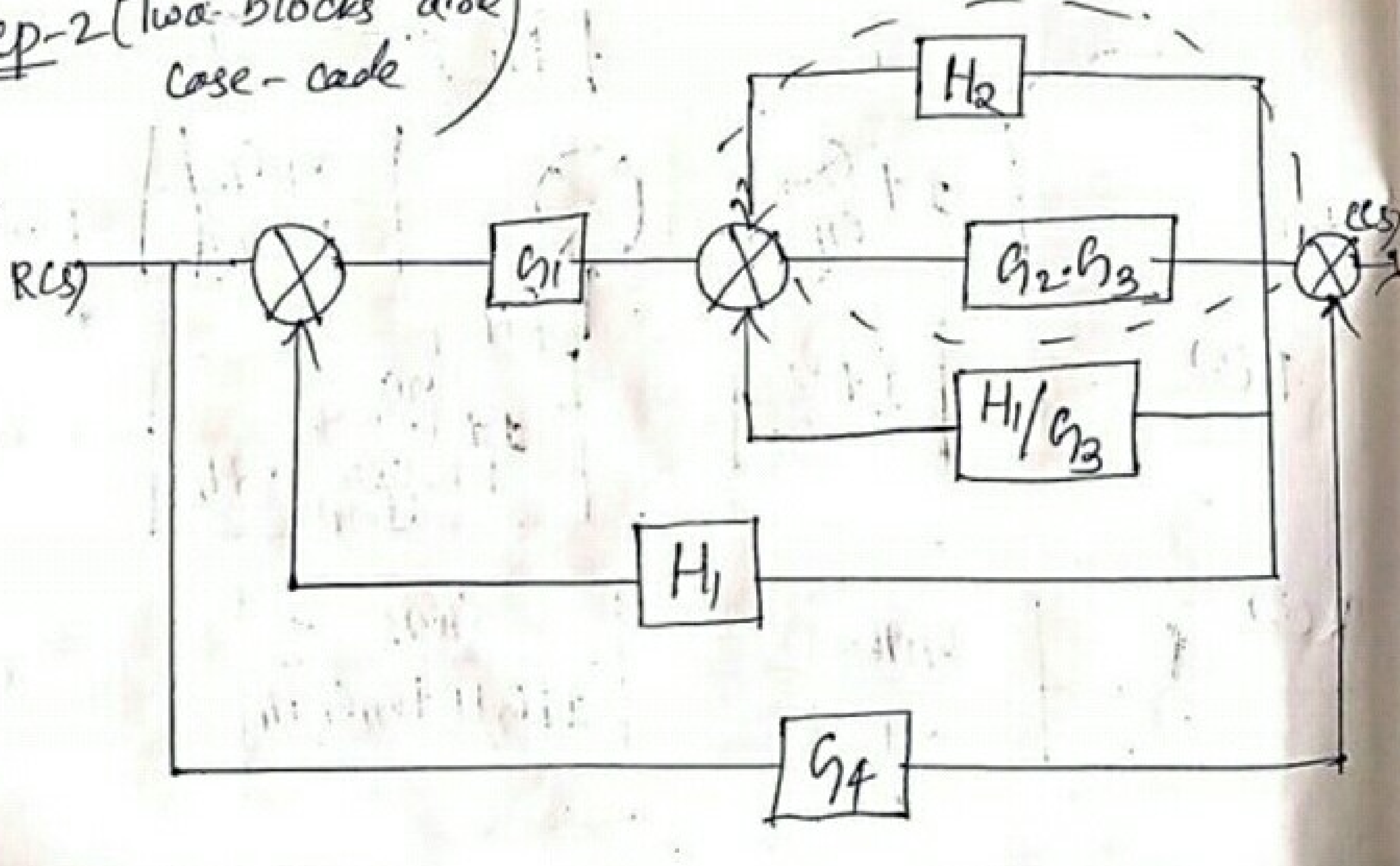


Soln:- Move the take off point to after the block.

Step-1:-



Step-2 (Two blocks are case-cade)



* **Signal flow graph :-**

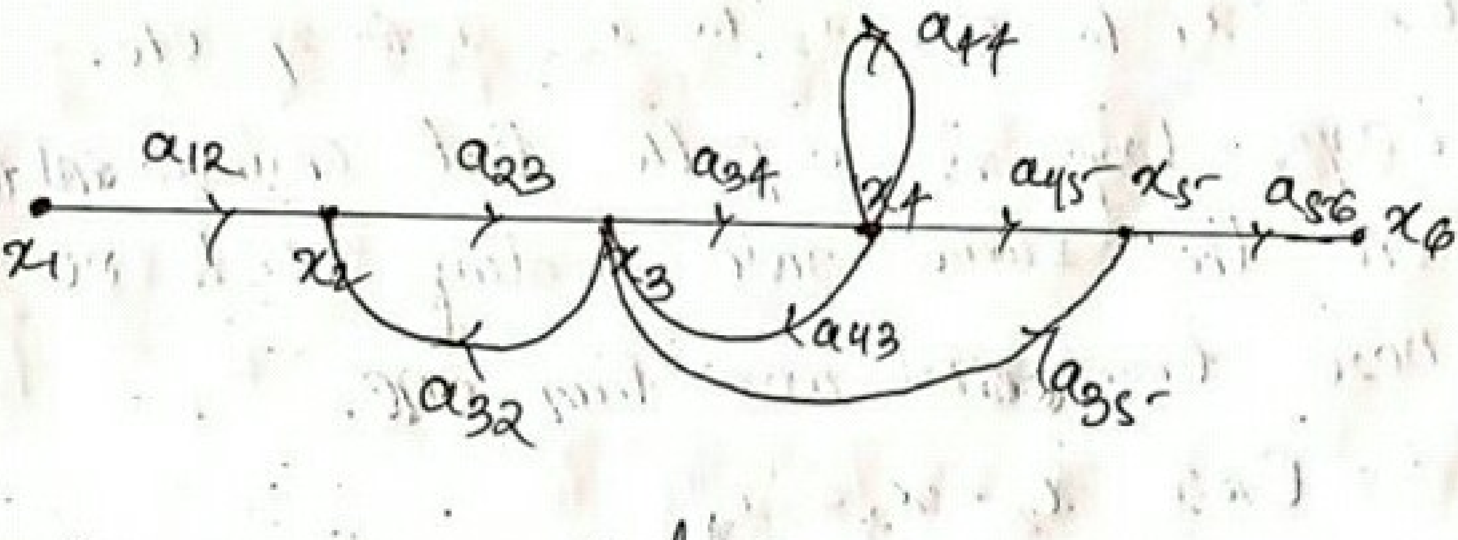
→ The process of block diagram reduction technic is time consuming because of every stage modify block diagram is to be redrawn.

→ A simple method was developed by S.J. Mason which is known as single & does not required any reduction technic.

→ Signal flow graph is applicable to linear system.

→ A signal flow graph is a diagram which is represent set of eqn & signal flow consists of nodes & these nodes are connected by a direct line called branches.

→ Every branch of signal flow graph having an error which represent the flow of signal.



1) **I/p node or source node :-**

An i/p node is a node which has a only out going branches.

Ex :- x_1 is the i/p node.

2) **O/p node or sink node :-**

An o/p node is a node that has only one or more incoming branches.

Ex :- x_6 is the o/p node.

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3) Mixed Node :- A node having incoming & outgoing branch is known as mixed node.

Ex:- x_2 to x_5.

4) Transmittance :- It is also known as transfer function, which is normally written on the branch near the arrow.

Ex:- $a_{12}, a_{23}, a_{34}, a_{44}, a_{43}, a_{45}, a_{56}$ etc

5) Forward path :- It is a path which originate to the i/p node & terminate at the output node & along which no node transerved more than once.

Ex:- x_1 to x_2; x_2 to x_3, x_3 to x_4 etc.

6) Loop :- loop is a path that orginate and terminate on the same node & along which no other node transerved more than one.

Ex:- $x_2 - x_3 - x_2$

7) Self-loop :- It is a path which originate or terminate at the same node.

Ex:- x_4 to x_4.

8) Path gain :- The product of branch gain along the path is called path gain.

Ex:- x_1 to x_2 is a_{12}.

9) Loop gain :- The gain of the loop is known as loop gain.

Ex:- a_{32}, a_{43}

1d) **Non-touching loop** :- In this loop having no common node branch & path.

Ex:- x_2 to x_3 to x_2 & x_4 to x_4.

* **Construction** :-

→ construction of signal flow graph from equation:-

Q1//
$$Y_2 = t_{21} Y_1 + t_{23} Y_3$$
$$Y_3 = t_{32} Y_2 + t_{33} Y_3 + t_{31} Y_1$$ [Not applicable]
$$Y_4 = t_{43} Y_3 + t_{42} \cdot Y_2$$
$$Y_5 = t_{54} Y_4$$
$$Y_6 = t_{65} Y_5 + t_{64} Y_4$$

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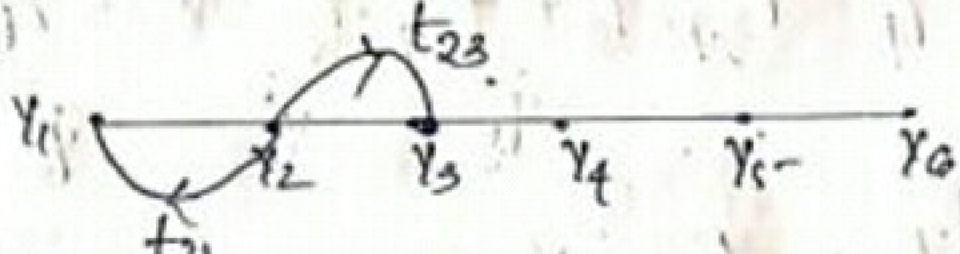
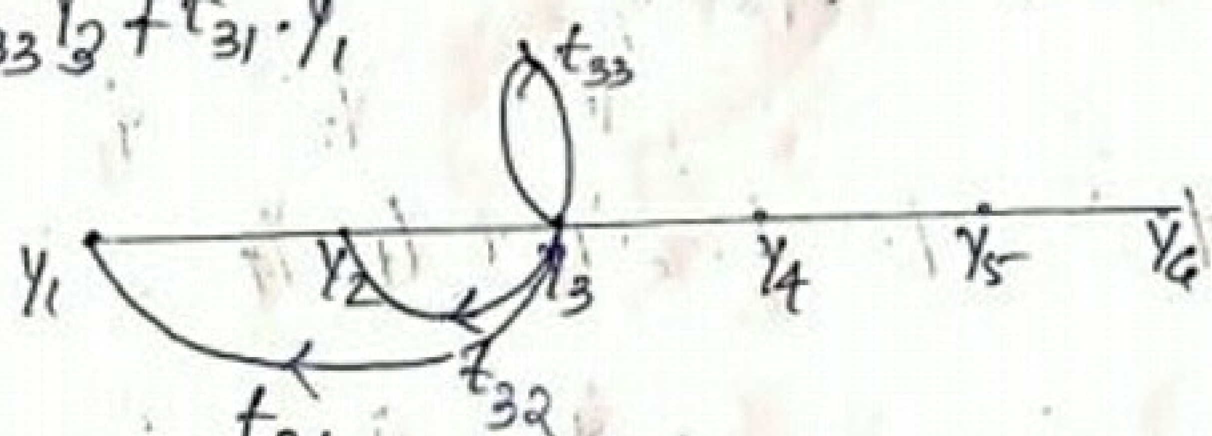
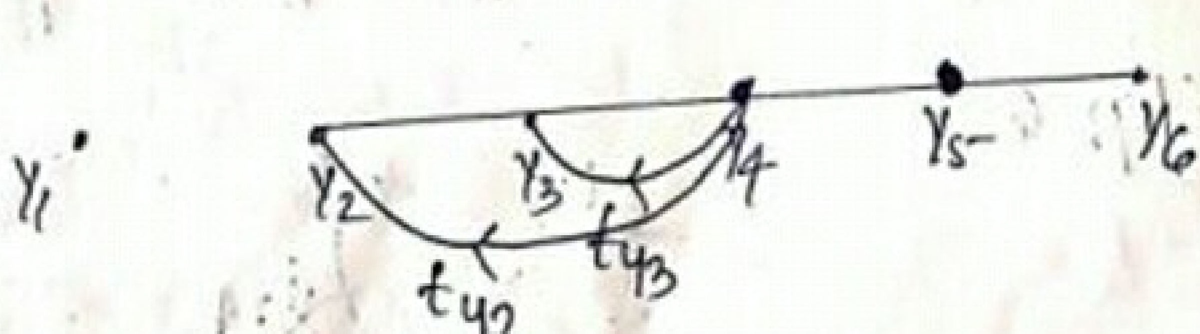
step-1:-
$$Y_2 = t_{21} Y_1 + t_{23} Y_3$$



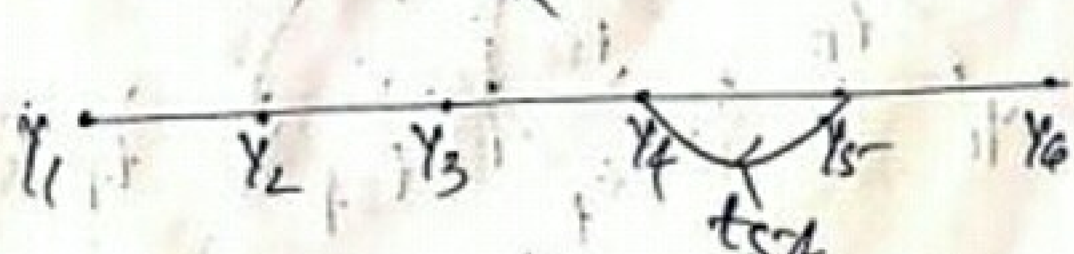
step-2:-
$$Y_3 = t_{32} Y_2 + t_{33} Y_3 + t_{31} \cdot Y_1$$



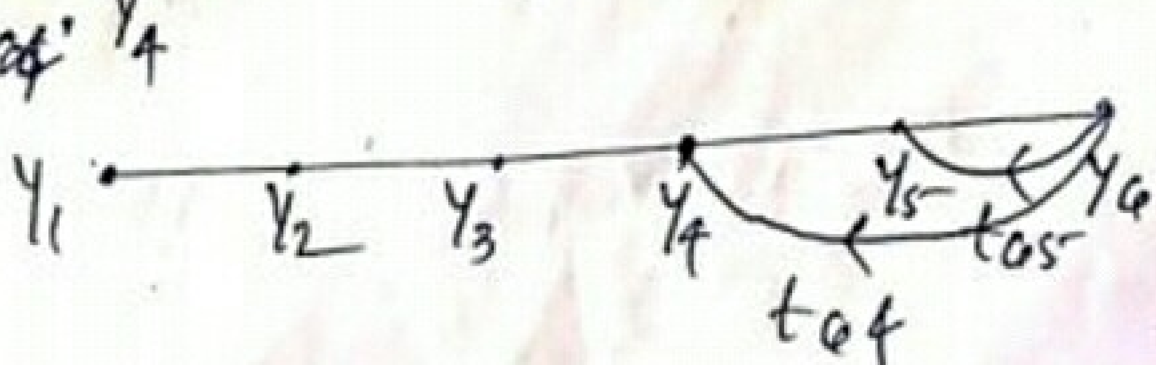
step-3
$$Y_4 = t_{43} \cdot Y_3 + t_{42} Y_2$$



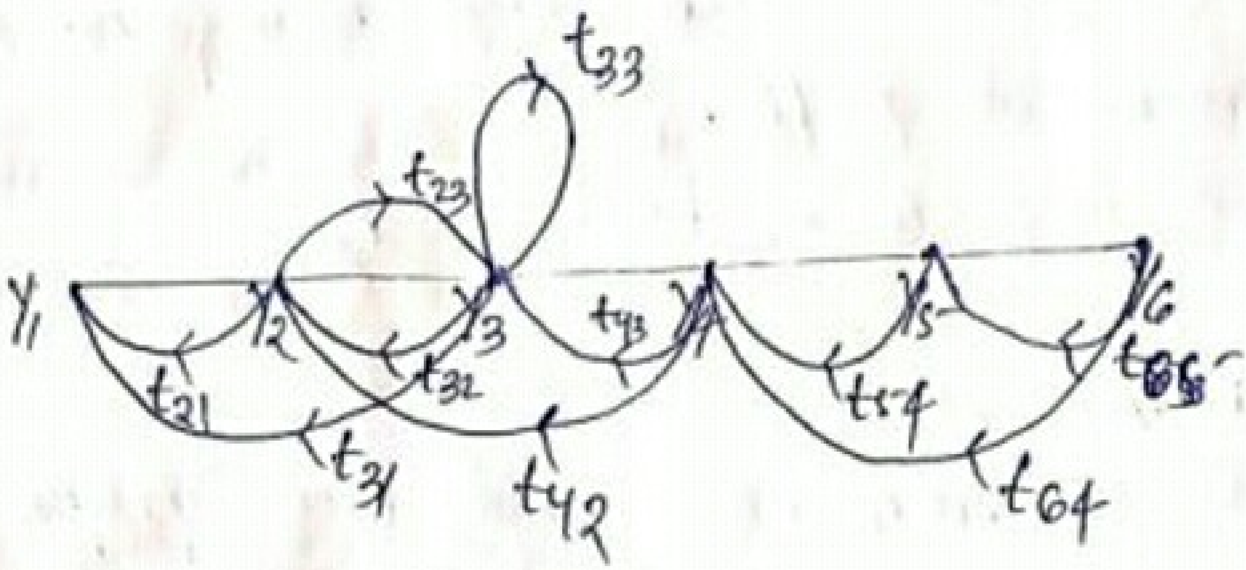
step-4
$$Y_5 = t_{54} \cdot Y_4$$



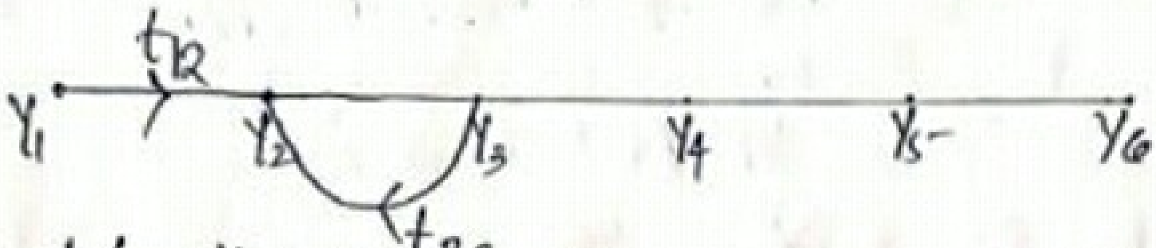
step-5
$$Y_6 = t_{65} \cdot Y_5 + t_{64} \cdot Y_4$$



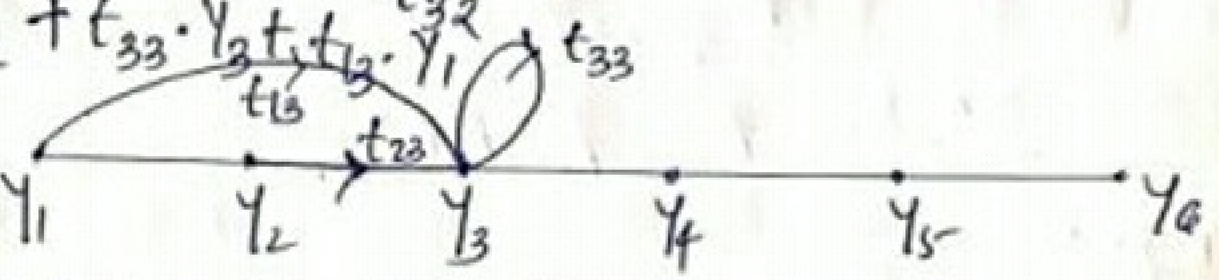
Step-6



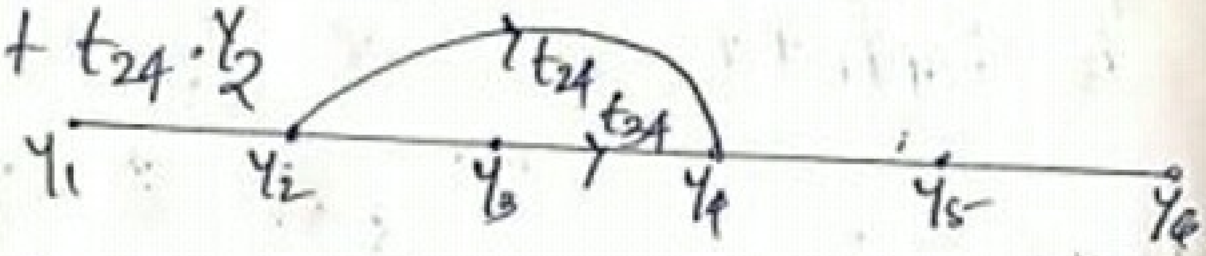
Q2/ Step-1 $y_2 = t_{12}\cdot y_1 + t_{32}\cdot y_3$ [Applicable]



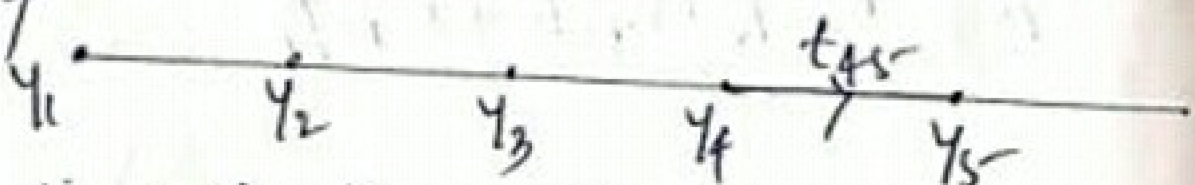
Step-2 $y_3 = t_{23}\cdot y_2 + t_{33}\cdot y_3 + t_{13}\cdot y_1$



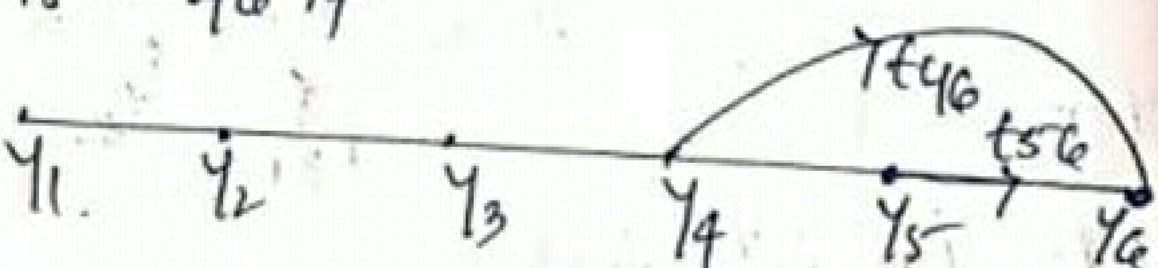
Step-3 $y_4 = t_{34}\cdot y_3 + t_{24}\cdot y_2$



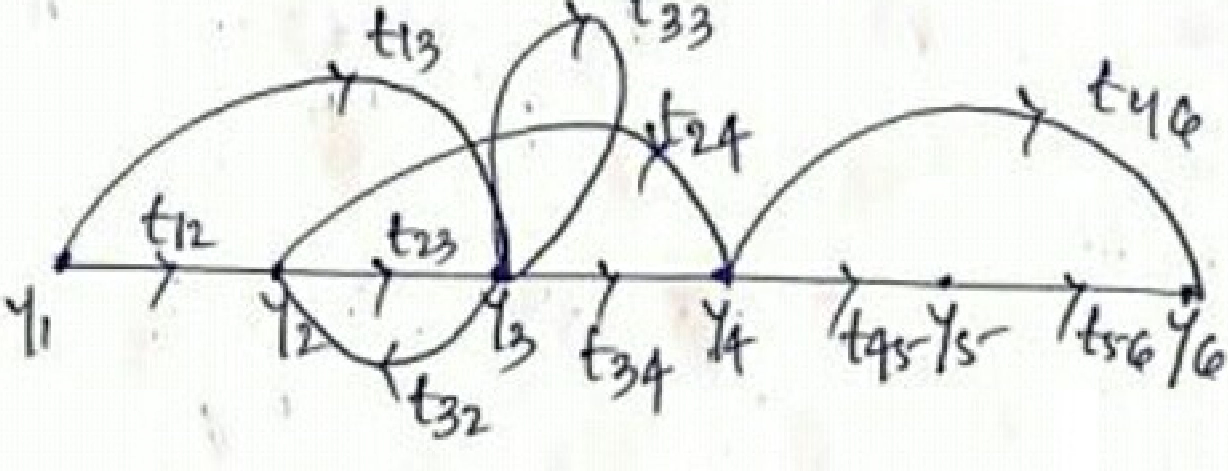
Step-4 $y_5 = t_{45}\cdot y_4$



Step-5 $y_6 = t_{56}\cdot y_5 + t_{46}\cdot y_4$



Step-6



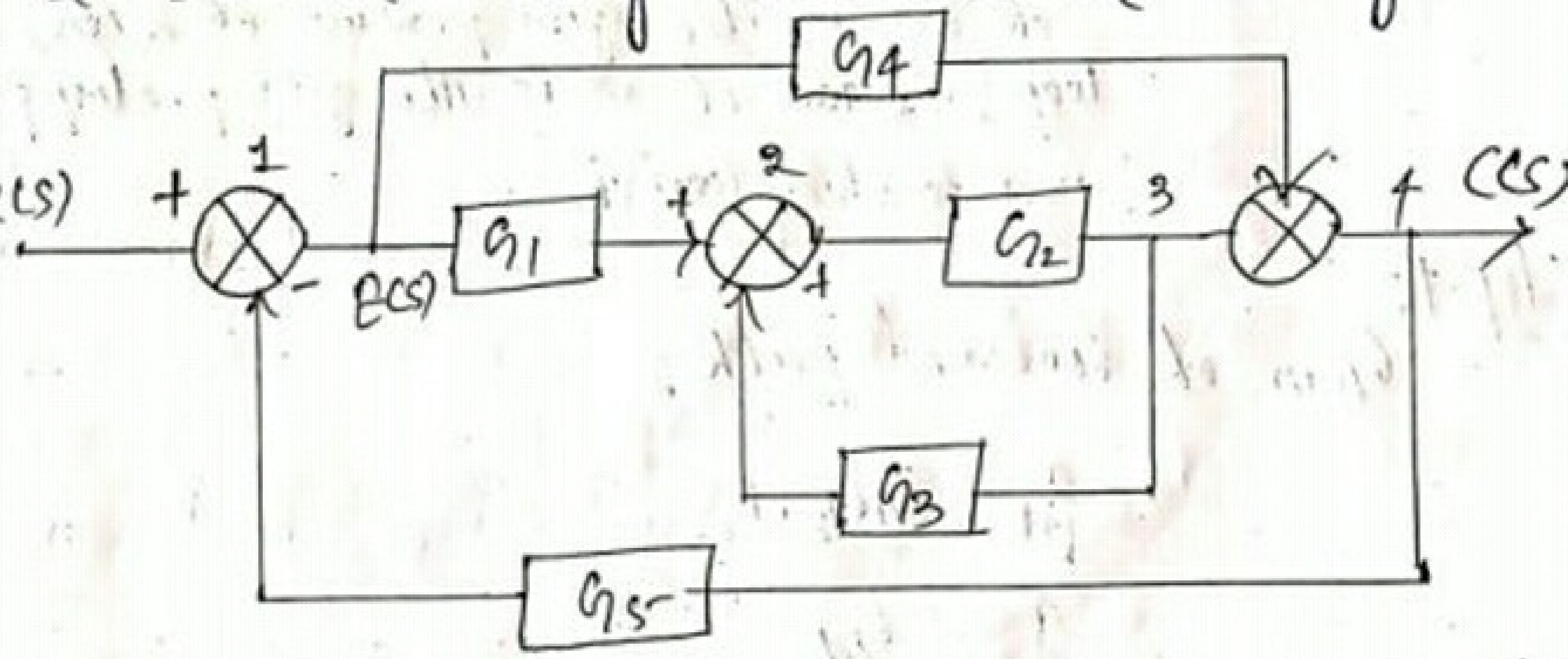
Construction of signal flow graph from block diagram

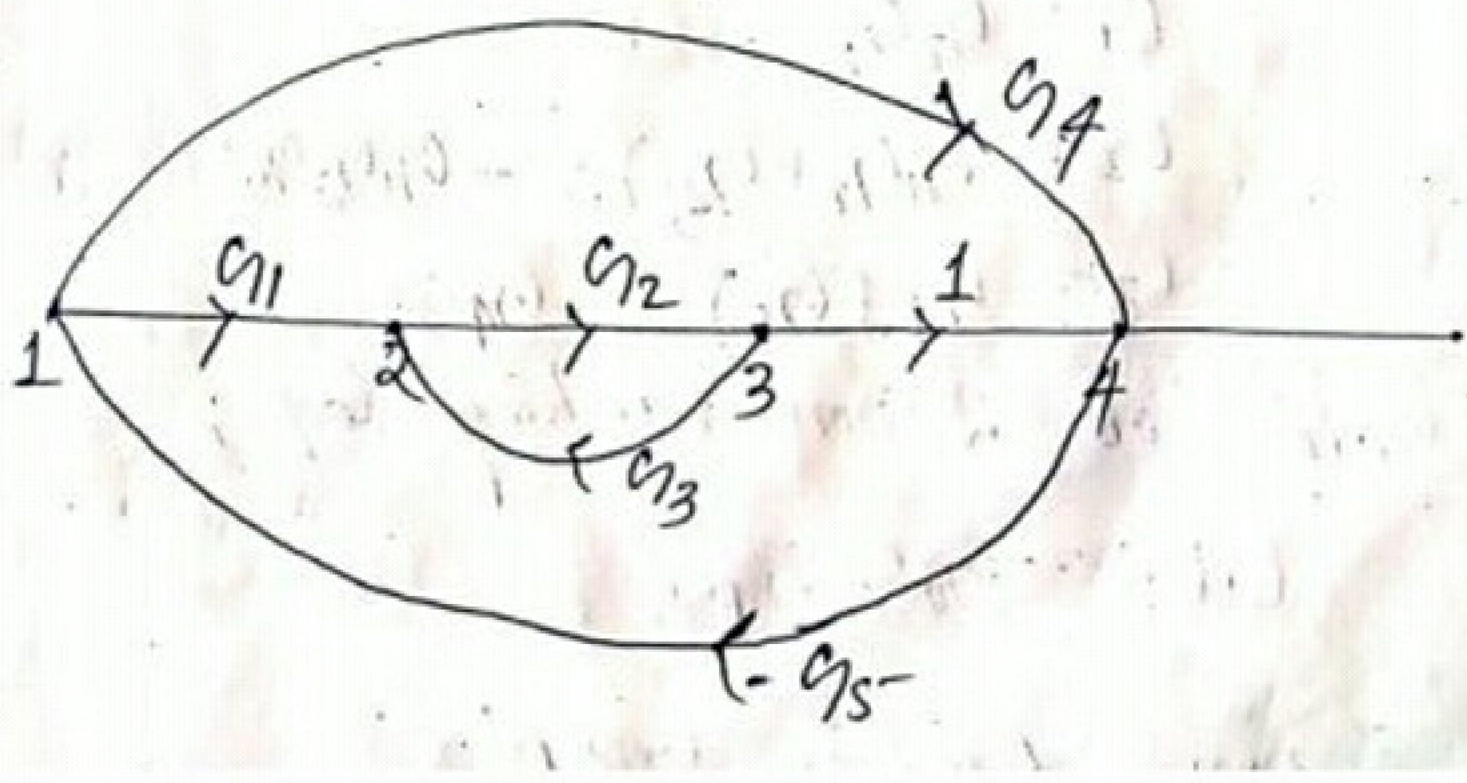
Rule-1:- All variable, summing point & take off point are represented by node.

Rule-2:- If summing point is placed before a take off point at the dirc of signal flow in such case the represent the summing point & take off by a single node.

R-3:- If a summing point is placed after a take off point are dirc of signal flow in such case represent the summing point & take off point by separate node connected by a branch having transmittance unity.

Q/1)





* MASON's gain formula :-

The overal transmitance or graph transmitance betn source node or sink node is given by Mason's gain formula.

$$\boxed{T.F. = \frac{\sum g_K \Delta_K}{\Delta}}$$

g = gain of the kth forward path (P)

Δ_1 = The part of Δ not touching the kth forward path.

Δ = 01 − [sum of all indivisual loop gain] + [sum of all possible gain product of 2 non-touching loop] − [sum of all possilbe gain product of 3 non touching loop] + ...

Step-1:- Gain of forward path;

$$g_1 = G_1 G_2 \cdot 1$$

$$g_2 = G_4$$

Step-2:- Gain of indivisual loop;

$$L_1 = G_2 G_3$$

$$L_2 = G_1 G_2 \cdot (-G_5) = -G_1 G_2 G_5$$

$$L_3 = G_4 \cdot (-G_5) = -G_4 G_5$$

Step-3:- Gain of 2 non-touching loop;

$$L_1 L_3 = -G_2 G_3 G_4 G_5$$

Step-4:- 3 non - touching loop = 0 (X)

Step-5:- $\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - G_2 G_3$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_3] - 0 + \ldots$$

$$\Rightarrow \Delta = 1 - [(G_2 G_3) + (-G_1 G_2 G_5) + (G_4 G_5)] + (-G_2 G_3 G_4 G_5)$$

$$\Rightarrow \Delta = 1 - G_2 G_3 + G_1 G_2 G_5 + G_4 G_5 - G_2 G_3 G_4 G_5$$

$$T.F = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}$$

$$\Rightarrow T.F. = \frac{G_1 G_2 \cdot 1 + G_4 \cdot (1 - G_2 G_3)}{1 - G_2 G_3 + G_1 G_2 G_5 + G_4 G_5 - G_2 G_3 G_4 G_5}$$

$$\Rightarrow T.F. = \frac{G_1 G_2 + G_4 - G_2 G_3 G_4}{1 - G_2 G_3 + G_1 G_2 G_5 + G_4 G_5 - G_2 G_3 G_4 G_5}$$

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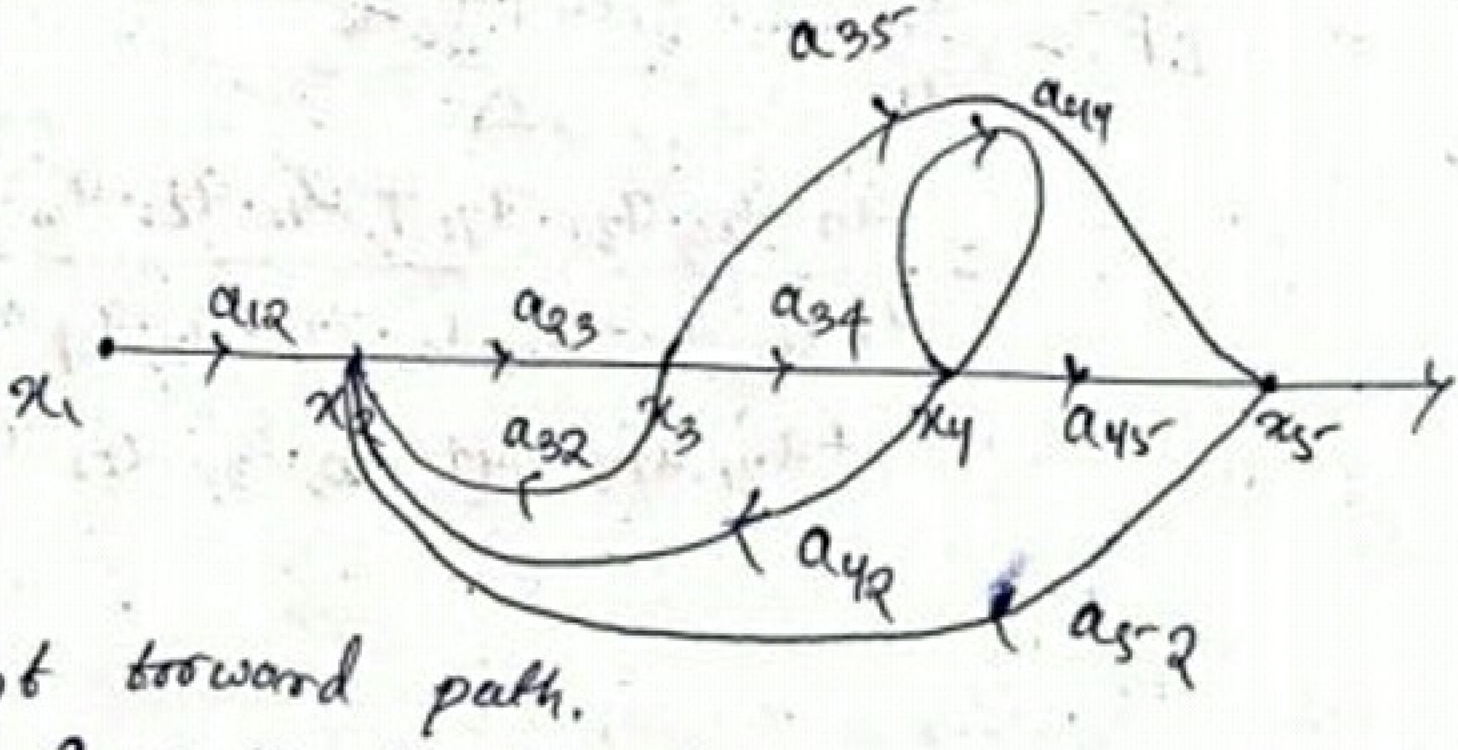
Q2) For the system represented by the given eqn, find the T.F. x_5 by x_1 (x_5/x_1) by the help of signal flow graph (SFG) technique.

$$x_2 = a_{12} x_1 + a_{32} x_3 + a_{42} x_4 + a_{52} \cdot x_5$$

$$x_3 = a_{23} x_2$$

$$x_4 = a_{34} x_3 + a_{44} \cdot x_4$$

$$x_5 = a_{35} \cdot x_3 + a_{45} \cdot x_4$$



Step-1:- Gain of forward path.

$$g_1 = a_{12} \cdot a_{23} \cdot a_{34} \cdot a_{45}$$

$$g_2 = a_{12} \cdot a_{23} \cdot a_{35}$$

Step-2 gain of individual loop.

$$L_1 = a_{23} \cdot a_{32}$$
$$L_2 = a_{23} \cdot a_{34} \cdot a_{42}$$
$$L_3 = a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{52}$$
$$L_4 = a_{23} \cdot a_{35} \cdot a_{52}$$
$$L_5 = a_{44}$$

Step-3

Non-touching loop.

$$L_1 L_5 = a_{23} \cdot a_{32} \cdot a_{44}$$
$$L_4 L_5 = a_{23} \cdot a_{35} \cdot a_{52} \cdot a_{44}$$

Step-4

Non

Step-5

$$\Delta_1 = 1 - 0 = 1$$
$$\Delta_2 = 1 - a_{44}$$

Step-6:-

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5] + [L_1 L_5 + L_4 L_5]$$

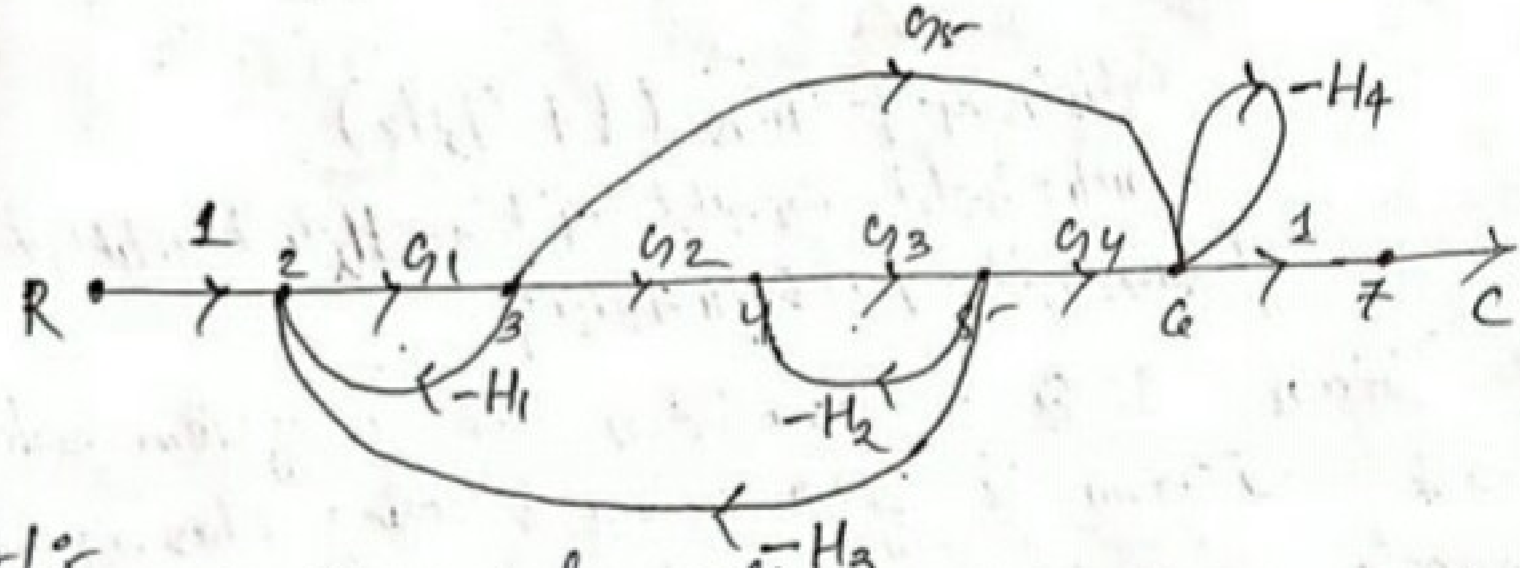
$$\Rightarrow \Delta = 1 - [a_{23} \cdot a_{32} + a_{23} \cdot a_{35} \cdot a_{42} + a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{52} + a_{44}] + [a_{32} \cdot a_{23} \cdot a_{44} + a_{23} \cdot a_{35} \cdot a_{52} \cdot a_{44}]$$

$$= 1 - a_{23} \cdot a_{32} - a_{23} \cdot a_{35} \cdot a_{42} - a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{52} - a_{44} + a_{32} \cdot a_{23} \cdot a_{44} + a_{23} \cdot a_{35} \cdot a_{52} \cdot a_{44}$$

$$T.F. = \frac{x_5}{x_1} = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}$$

$$= \frac{a_{12} \cdot a_{23} \cdot a_{34} \cdot a_{45} + a_{12} \cdot a_{23} \cdot a_{35} (1 - a_{44})}{1 - a_{23} \cdot a_{32} - a_{23} \cdot a_{35} \cdot a_{42} - a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{52} - a_{44} + a_{32} \cdot a_{23} \cdot a_{44} + a_{23} \cdot a_{35} \cdot a_{52} \cdot a_{44}}$$

3) From the given SFG, find the ratio of C/R.



Step-1:- step of forward path.

$$g_1 = G_1 G_2 G_3 G_4$$

$$g_2 = G_1 \cdot G_5$$

Step-2 gain of individual loop.

$$L_1 = -G_1 H_1$$

$$L_2 = -G_3 H_2$$

$$L_3 = -G_1 G_2 \cdot G_3 \cdot H_3$$

$$L_4 = -H_4$$

Step-3 2 Non-touching loop.

$$L_1 L_2 = G_1 H_1 G_3 H_2$$

$$L_1 L_4 = G_1 H_1 H_4$$

$$L_2 L_4 = G_3 H_2 H_4$$

$$L_3 L_4 = G_1 G_2 G_3 H_3 H_4$$

Step-4 3 non-touching loop.

$$L_1 L_2 L_4 = -G_1 H_1 G_3 H_2 H_4$$

Step-5:- $\Delta_1 = 1 - 0 = 0$

$$\Delta_2 = 1 - L_2 = 1 + G_3 H_2$$

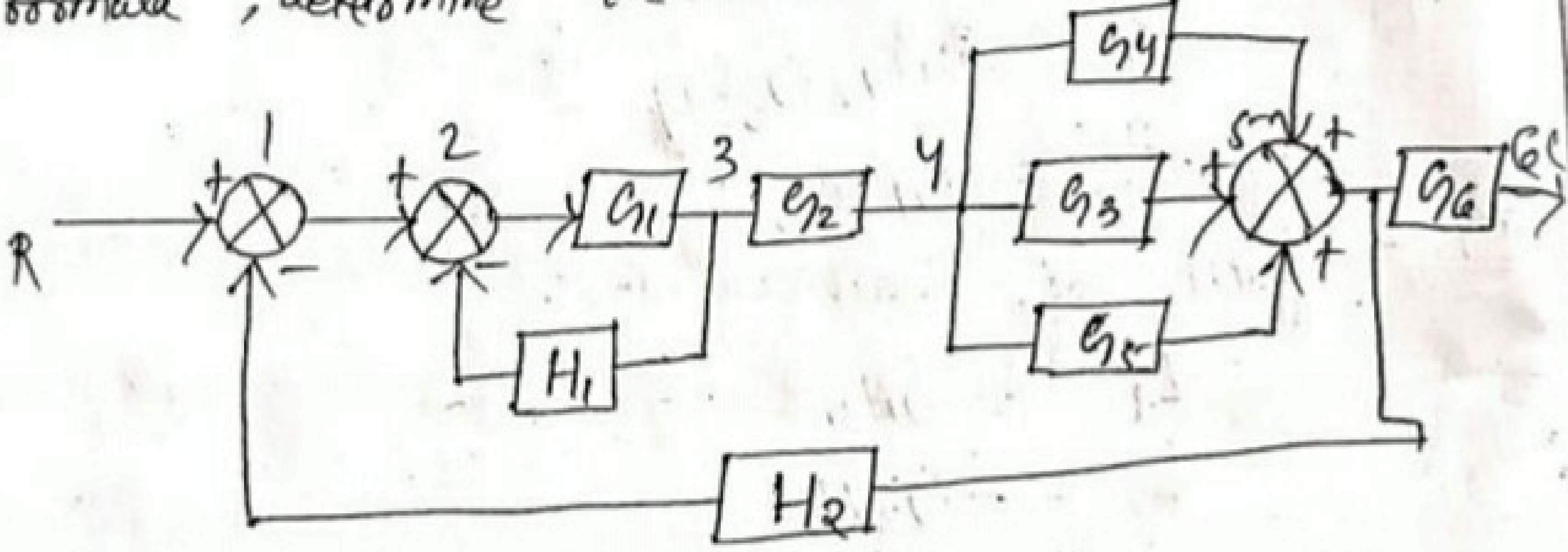
Step-6:- $\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_2 + L_1 L_4 + L_2 L_4 + L_3 L_4]$

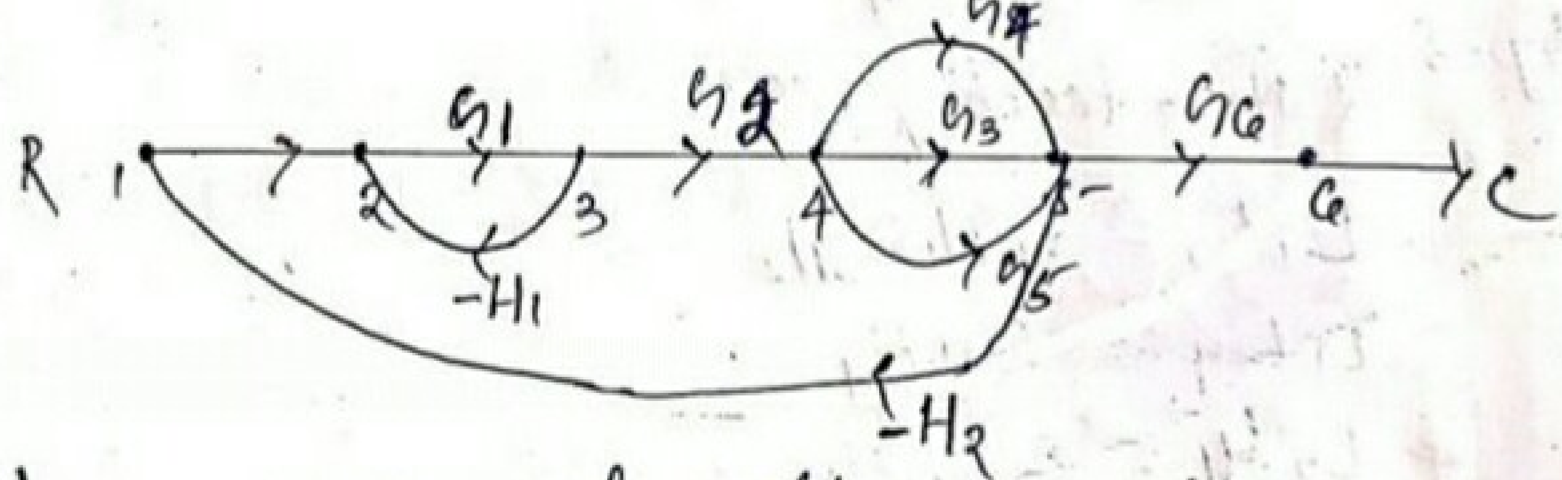
$$[-G_1 H_1 G_3 H_2 H_4]$$

$$T.F. = \frac{C}{R} = \frac{g_1\Delta_1 + g_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4 + G_1G_5 - (1 + G_3H_2)}{1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_1G_2H_2H_1 + G_1H_1H_4 + G_3H_2H_4 + G_1G_2G_3H_3H_4 + G_1H_1G_3H_2H_4}$$

Q. Obtain SFG representation for a system whose block diagram is given below & using Mason's gain formula, determine the ratio of C/R.





Step-1 Step of forward path;

$$g_1 = G_1G_2G_3G_6$$
$$g_2 = G_1G_2G_4G_6$$
$$g_3 = G_1G_2G_5G_6$$

Step-2 Gain of individual loop.

$$L_1 = -G_1H_1$$
$$L_2 = -G_1G_2G_4H_2$$
$$L_3 = -G_1G_2G_5H_2$$
$$L_4 = -G_1G_2G_3H_2$$

step-3 2 non touching loop. (X)

step-4 3 non-touching loop. (X)

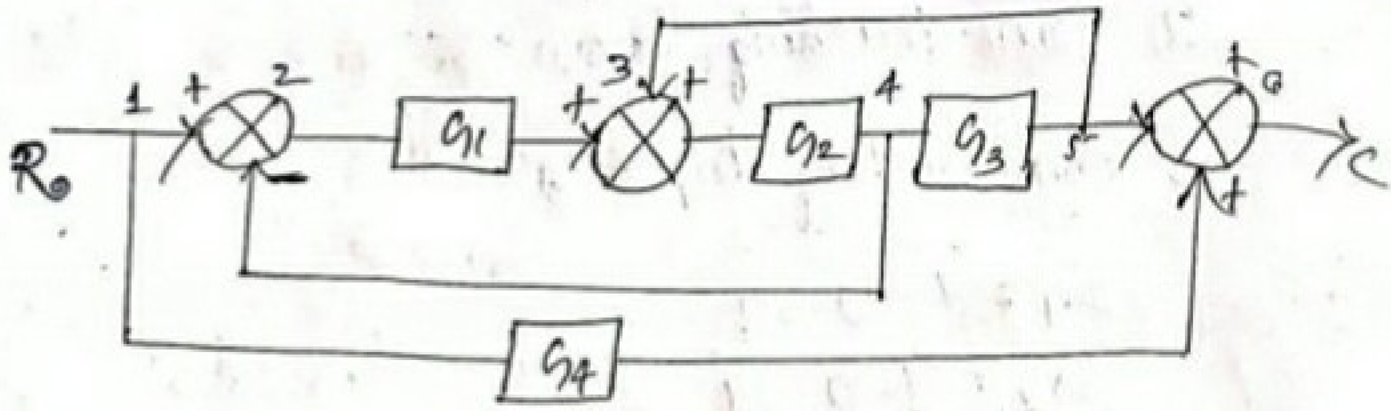
step-5
$$\Delta_1 = 1 - 0 = 1$$
$$\Delta_2 = 1 - 0 = 1$$
$$\Delta_3 = 1 - 0 = 1$$

step-6
$$\Delta = 1 - [L_1 + L_2 + L_3] + 0 - 0$$
$$= 1 + G_1H_1 + G_1G_2G_4H_2 + G_1G_2G_5H_2 + G_1G_3G_3H_2$$

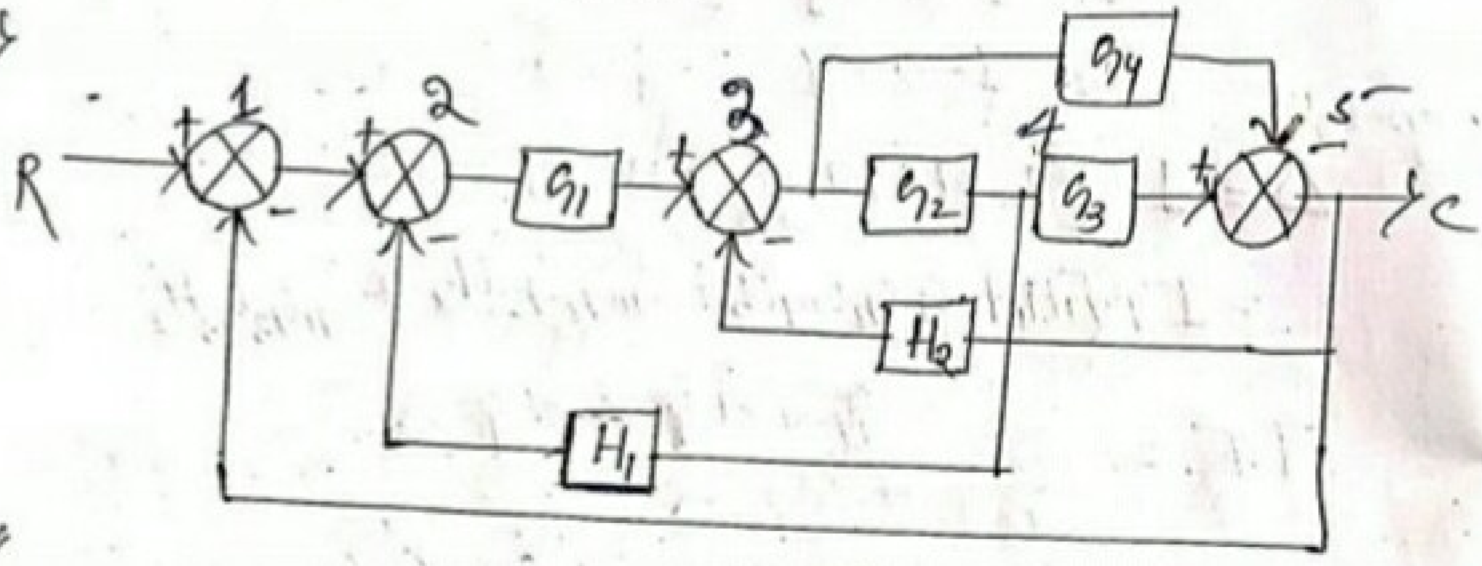
$$T.F. = \frac{C}{R} = \frac{g_1\Delta_1 + g_2\Delta_2 + g_3\Delta_3}{\Delta}$$

$$= \frac{G_1G_2G_3G_6 + G_1G_2G_4G_6 + G_1G_2G_5G_6}{1 + G_1H_1 + G_1G_2G_4H_2 + G_1G_2G_5H_2 + G_1G_2G_3H_2}$$

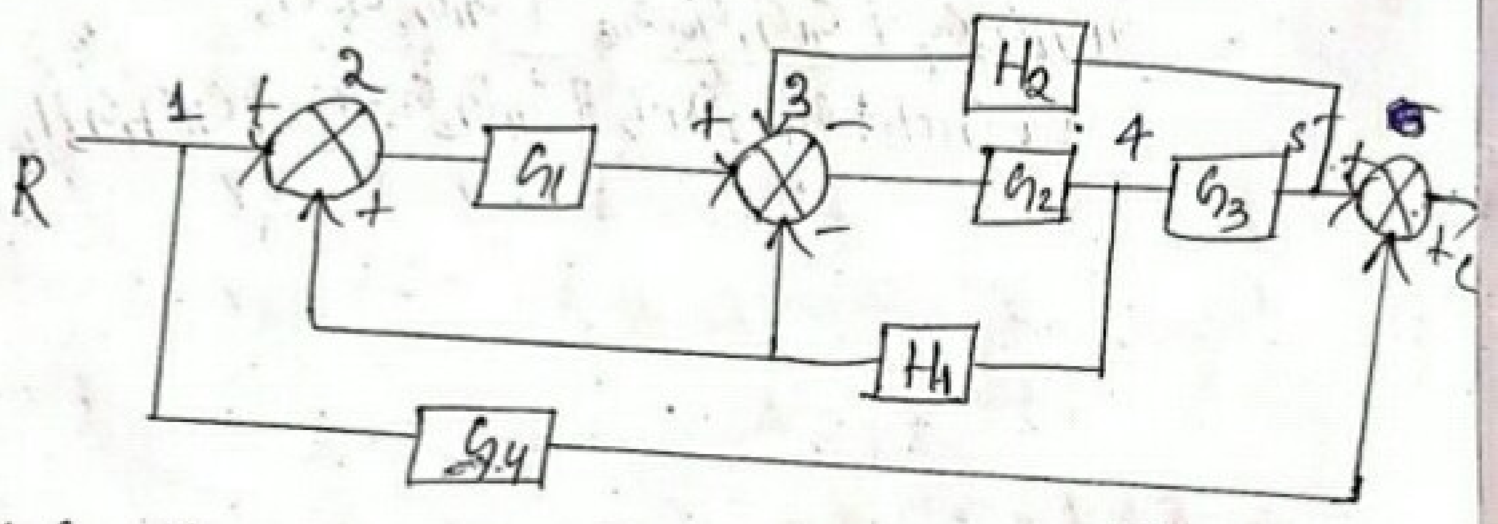
Q/5)



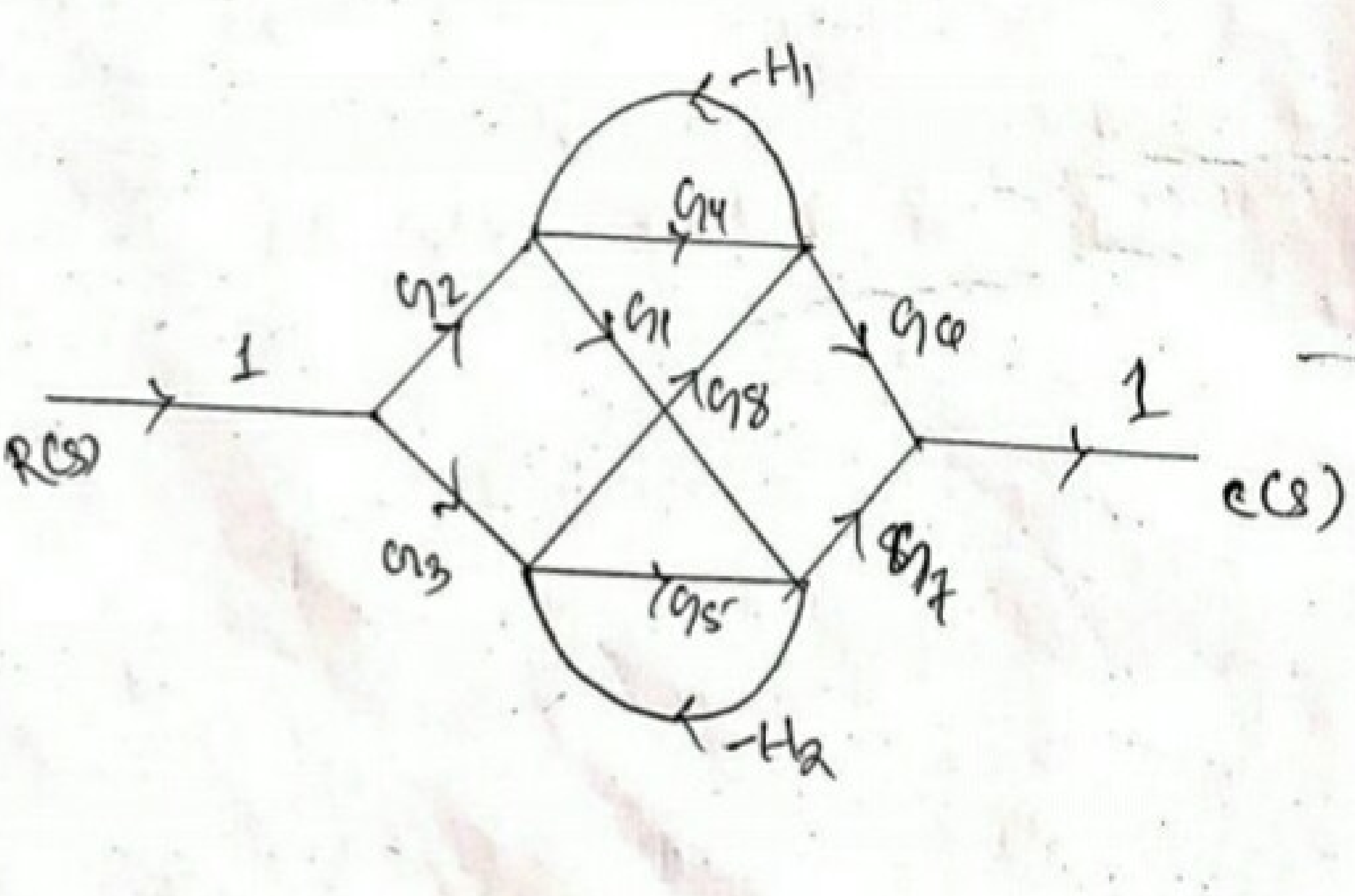
Q/6)



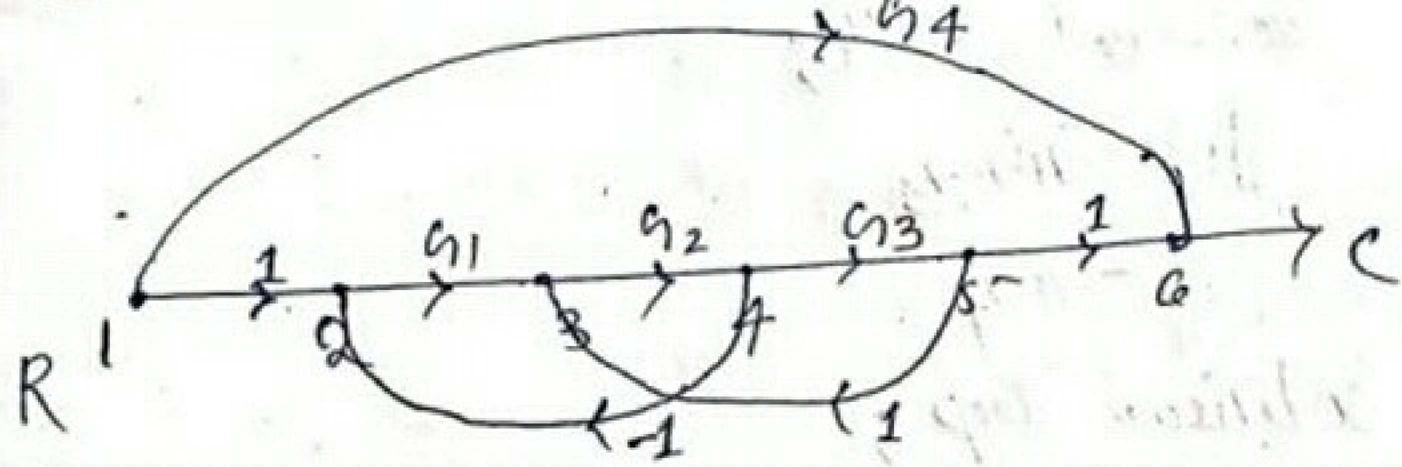
Q/4)



Q/ Obtain the T.F. C/R from the above SFG.
Dt-13/01/2020



Q5//



step-1 Forward path;

$$g_1 = G_1 G_2 G_3$$
$$g_2 = G_4$$

step-2 Indivisual loop;

$$L_1 = -G_1 G_2$$
$$L_2 = G_2 G_3$$

step-3 2 non-touching loop; (X) Non

step-4 3 non touching loop ⇒ Non

step-5 $\Delta_1 = 1 - 0 = 1$

$$\Delta_2 = 1 - (-G_1G_2 + G_2G_3) = 1 + G_1G_2 - G_2G_3$$

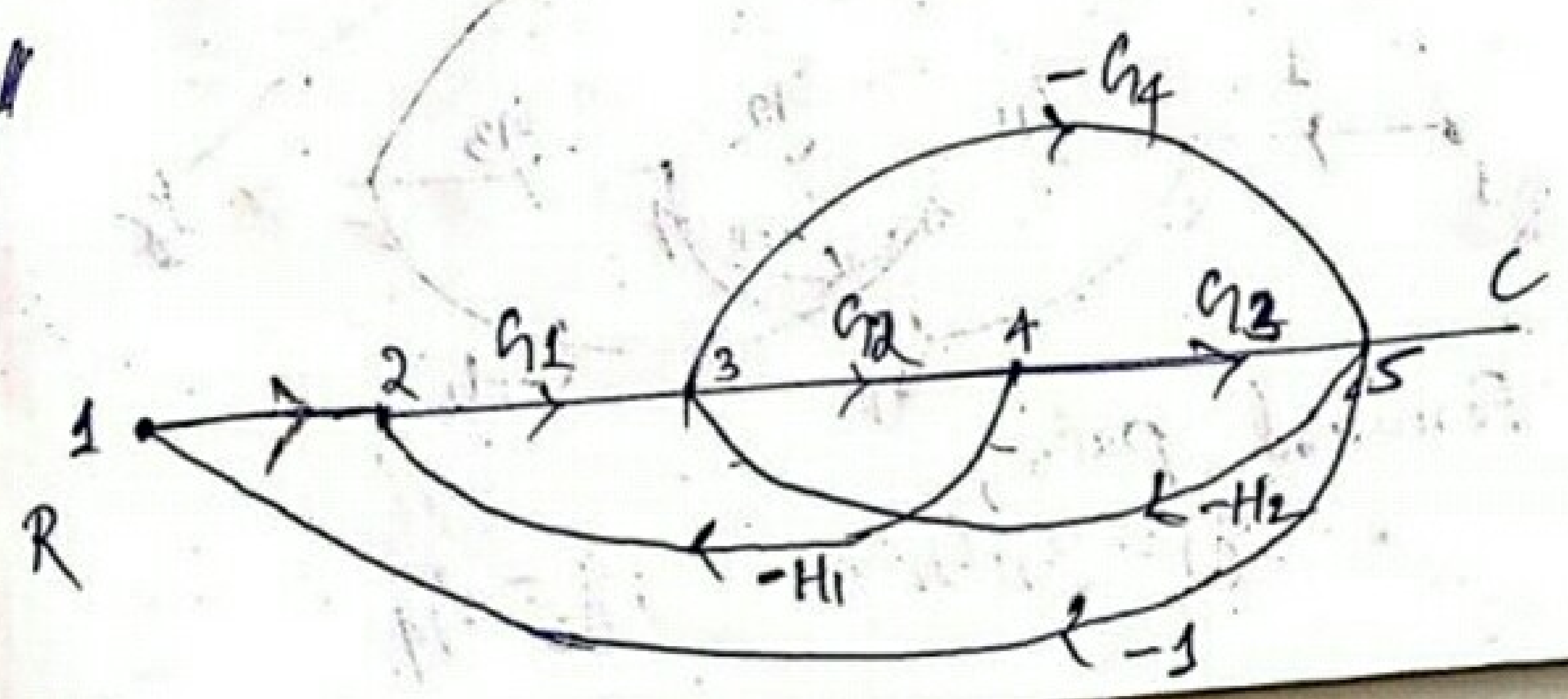
step-6 :- $\Delta = 1 - [L_1 + L_2] + 0 - 0$

$$\Rightarrow \Delta = 1 - [-G_1G_2 + G_2G_3] = 1 + G_1G_2 - G_2G_3$$

$$T.F. = \frac{C}{R} = \frac{g_1\Delta_1 + g_2\Delta_2}{\Delta}$$

$$\Rightarrow T.F = \frac{G_1G_2G_3 + G_4(1 + G_1G_2 - G_2G_3)}{1 + G_1G_2 - G_2G_3}$$

Q6//



Step-1 Forward path;

$g_1 = G_1G_2G_3$

$g_2 = -G_1G_4$

Step-2 Individual loop;

$L_1 = -G_1G_2H_1$

$L_2 = -G_2G_3H_2$

$L_3 = -G_1G_2G_3$

$L_4 = +G_4H_2$

$L_5 = +G_1G_4$

Step-3 2 non-touching loop; ⇒ Non

Step-4 3 non-touching loop ⇒ Non

Step-5 $\Delta_1 = 1-0 = 01$

$\Delta_2 = 1-0 = 1$

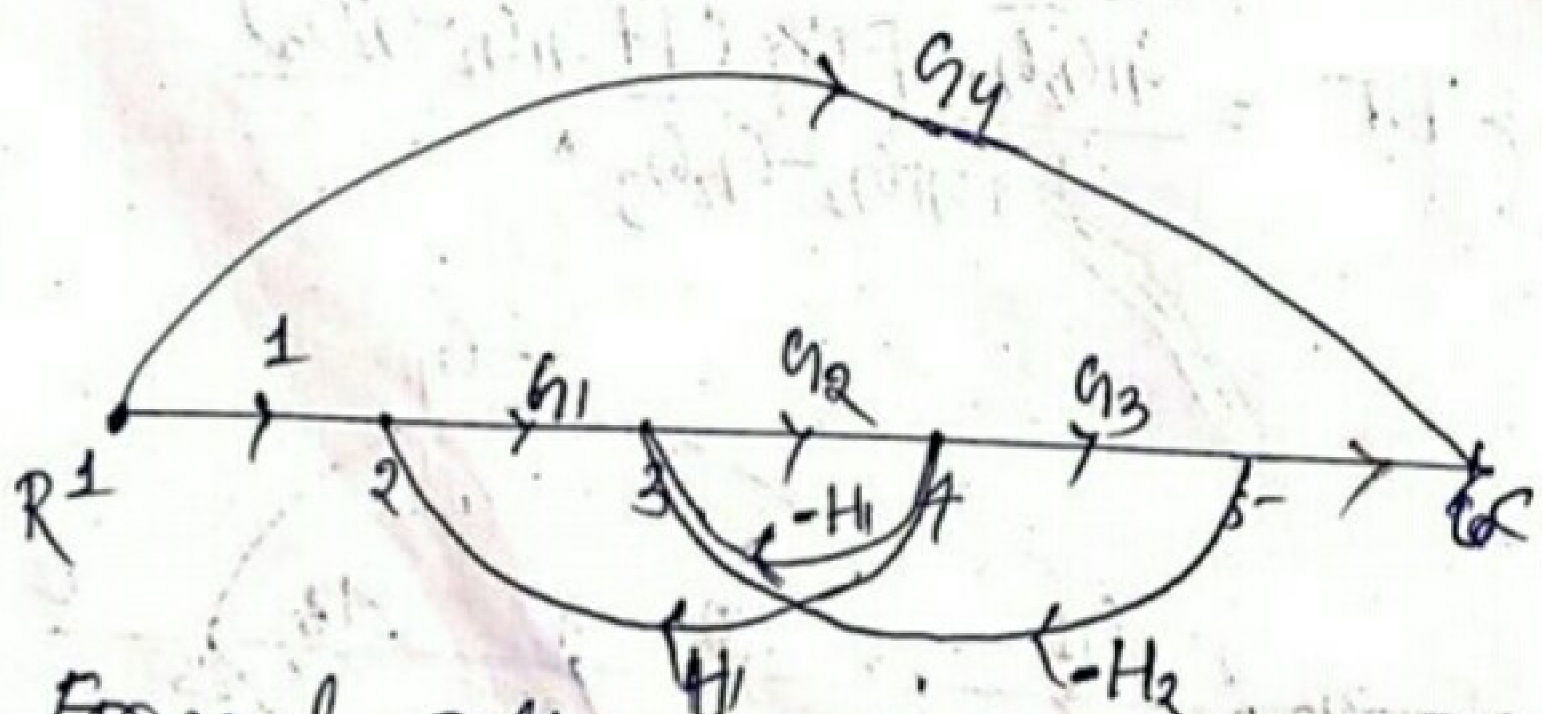
Step-6 $\Delta = 1-[L_1+L_2+L_3+L_4+L_5]+0-0$

$= 1+G_1G_2H_1+G_2G_3H_2+G_1G_2G_3-G_4H_2-G_1G_4$

$$T.F. = \frac{C}{R} = \frac{g_1\Delta_1+g_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3-G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_1G_2G_3-G_4H_2-G_1G_4}$$

Q7)



Step-1 Forward path;

$g_1 = G_1G_2G_3$, $g_2 = G_4$

Step-2:- Individual loop;

$L_1 = G_1G_2H_1$

$L_2 = -G_2H_1$, $L_3 = -G_2G_3H_2$

Step-3 2 non-touching loop; ⇒ Non

Step-4 3 non-touching loop, ⇒ Non

Step-5:

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - [G_1G_2H_1 - G_2H_1 - G_2G_3H_2] = 1 - G_1G_2H_1 + G_2H_1 + G_2G_3H_2$

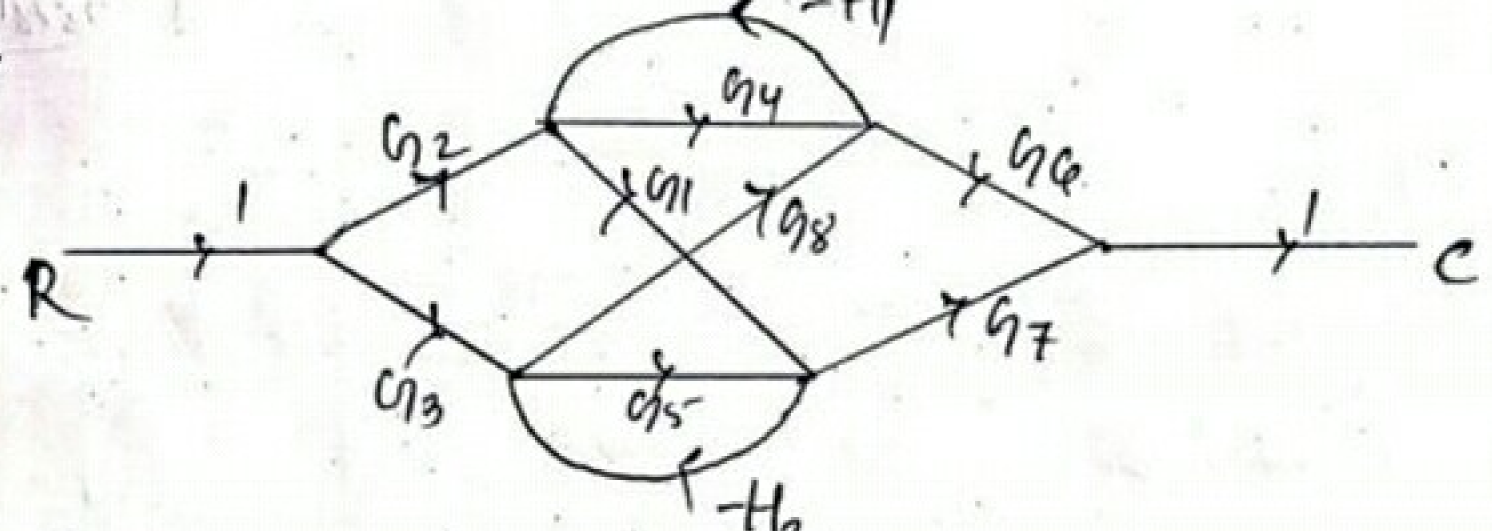
Step-6 :- $\Delta = 1 - [L_1 + L_2 + L_3]$

$= 1 - G_1G_2H_1 + G_2H_1 + G_2G_3H_2$

$$T.F. = \frac{C}{R} = \frac{g_1\Delta_1 + g_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_4(1 - G_1G_2H_1 + G_2H_1 + G_2G_3H_2)}{1 - G_1G_2H_1 + G_2H_1 + G_2G_3H_2}$$

Q8



Step-1 Forward path;

$g_1 = G_2G_4G_6$ $\qquad$ $g_5 = -G_2G_1H_2G_8G_6$

$g_2 = G_3G_5G_7$ $\qquad$ $g_6 = -G_3G_8H_1G_1G_7$

$g_3 = G_2G_1G_7$

$g_4 = G_3G_8G_6$

Step-2 Individual loop;

$L_1 = -G_4H_1$

$L_2 = -G_5H_2$

$L_3 = G_1H_2G_8H_1$

Step-3:- 2 non-touching loop.

$$L_1 L_2 = G_4 G_5 H_1 H_2$$

step-4 :-

Non => 3 non-touching loop.

step-5 :-

$$\Delta_1 = 1 + G_5 H_2$$
$$\Delta_2 = 1 + G_4 H_1$$
$$\Delta_3 = 1 - 0 = 1$$
$$\Delta_4 = 1 - 0 = 1$$

Step-6

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_2] - 0$$
$$= 1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 G_5 H_1 H_2$$

$$T.F. = \frac{C}{R} = \frac{g_1\Delta_1 + g_2\Delta_2 + g_3\Delta_3 + g_4\Delta_4}{\Delta}$$

$$= \frac{G_2 G_4 G_6 (1 + G_5 H_2) + G_3 G_5 G_7 (1 + G_4 H_1) + G_2 G_1 G_7 + G_3 G_8 G_6 - G_1 G_2 H_2 G_6 G_8 - G_3 G_8 H_1 G_5 H_1}{1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2}$$

2nd chapter — Time Domain Analysis (Response) — Dt-16/01/20

→ It refers to the analysis of system performances in time. That means the study of evolution (specifically o/p) of the system variables with time.

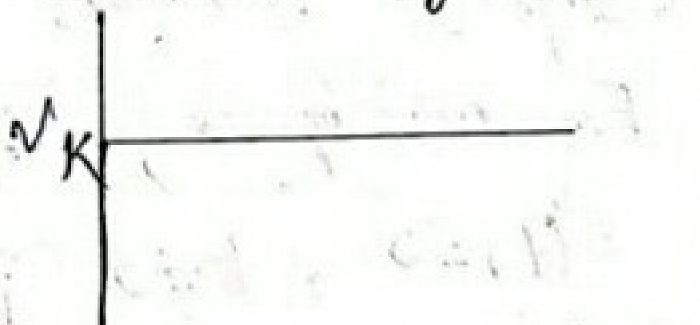
→ There are 2 common ways of analysis the responce of system;

1) Natural Responce & forced responce
2) Transient responce & steady responce.

In both cases the complete responce of the system is given by the combination of both responces, i.e. natural & forced responce or transient & steady state respone

⇛ Test i/p signal for transient analysis :-

1. Step signal:-



→ For unit step function in Laplace romain;

$$L[f(t)] = \int_{-\infty}^{\infty} f(t).e^{-st}.dt$$

$r(t) = 0 \quad -\infty < t < 0$

$r(t) = K \quad +\infty > t > 0$

→ step function is also called displacement funf, if test signal as a step signal then,

$R(s) = 1/s$

$$= \int_{0}^{\infty} 1.\ e^{-st}.dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_{0}^{\infty}$$

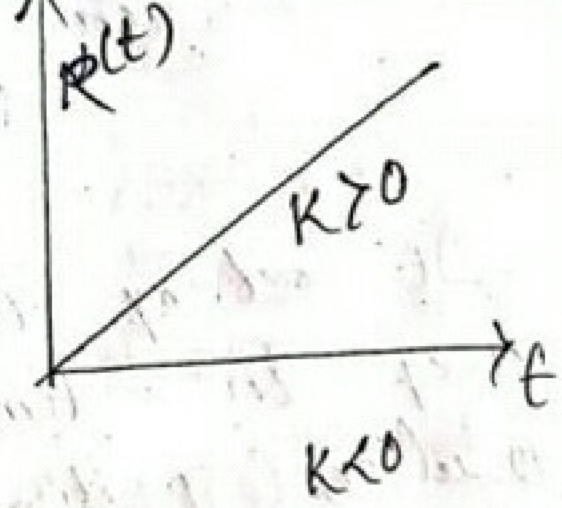
$$\boxed{\begin{matrix} e^{0} = 1 \\ e^{-\infty} = 0 \end{matrix}}$$

$$= \left[\frac{e^{-\infty s} - e^{0}}{-s}\right] = \frac{e^{-\infty} - 1}{-s} = \frac{+1}{+s} = \frac{1}{s}$$

2. Ramp signal:-

$R(t) = 0 \quad -\infty < t < 0$

$R(t) = Kt \quad 0 < t < \infty$



→ For unit ramp signal;

$R(t) = t \quad t > 0$

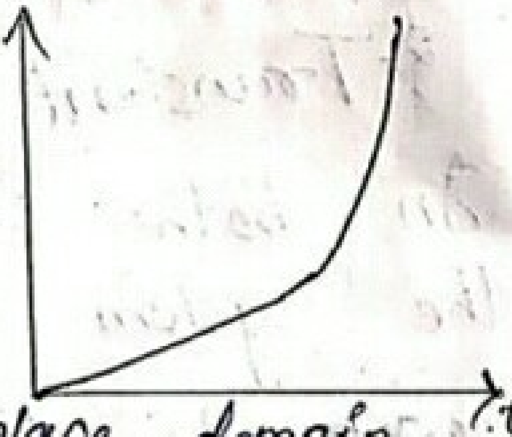
→ Unit Ramp test signal in laplace domain,

$$R(s) = \frac{1}{s^2}$$

3. Parabolic signal:-

$P(t) = 0 \quad -\infty < t < 0$

$P(t) = \frac{Kt^2}{2} \quad 0 < t < \infty$



→ For unit parabolic signal in laplace domain.

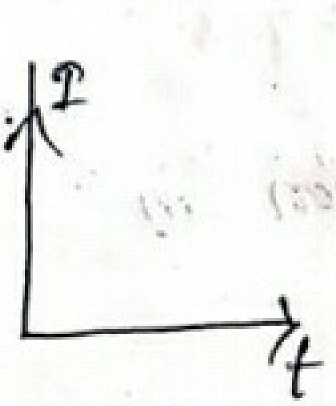
$$P(s) = 1/s^3$$

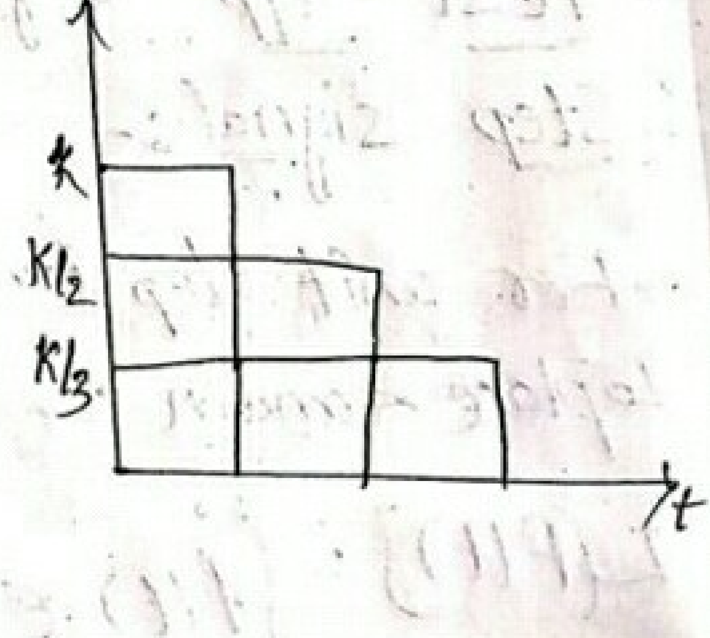
4. Impulse function:-

→ For unit impulse;

$\delta(t) = 0 \ , \ t \neq 0$

$\delta(t) = 1 \ , \ t = 0$

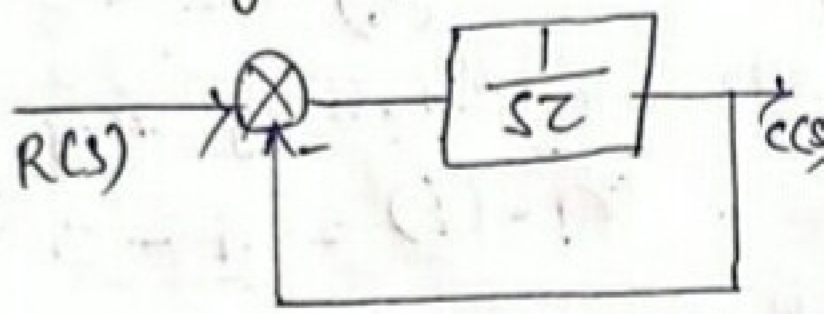




Time Response of 1st order system :-

Unit step i/p :-

→ $\frac{C(s)}{R(s)} = \frac{\frac{1}{s\tau}}{1+\frac{1}{s\tau}\times 1}$



$G(s) = \frac{1}{s\tau}$

$H(s) = 1$

$$T.F. = \frac{\frac{1}{s\tau}}{\frac{s\tau+1}{s\tau}} = \frac{1}{1+s\tau}$$

→ Response of the 1st order system with unit step i/p. $(R(s) = 1/s)$

$$\frac{C(s)}{R(s)} = \frac{1}{1+s\tau}$$

$$\Rightarrow C(s) = \frac{1}{1+s\tau}\times\frac{1}{s} = \frac{1}{s(s\tau+1)}$$

→ By partial Fraction;

$$\frac{A}{s} + \frac{B}{s\tau+1} = \frac{1}{s(s\tau+1)} \quad \text{——— (A)}$$

$$\Rightarrow A(s\tau+1) + Bs = 1$$

→ $s \to 0$, $\Rightarrow A(0+1) + 0 = 1$

$$\Rightarrow A = 1$$

→ $s \to 1$ $\Rightarrow A(1.\tau+0) + B.1 = 0$

$$\Rightarrow A\tau + B = 0 \qquad [\because A = 1]$$

$$\Rightarrow B + \tau = 0$$

$$\Rightarrow B = -\tau$$

→ Put the value of A & B in eqn (A)

$$\Rightarrow \frac{1}{s} + \frac{-\tau}{s\tau+1} = \frac{1}{s(s\tau+1)}$$

$$\Rightarrow \frac{1}{s} - \frac{\tau}{s\tau+1} = \frac{1}{s(s\tau+1)}$$

$\Rightarrow C(S) = \frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}$

→ In time domain by using inverse laplace, we get

$\Rightarrow \boxed{C(t) = 1 - e^{-t/\tau}}$ $\boxed{\frac{1}{s+a} = e^{-ta}}$

→ When $t = \tau$;

$\Rightarrow c(t) = 1 - e^{-1} = 1 - \frac{1}{e} = 0.632 = 63.2\%$

→ Where τ is known as time constant.

→ It is defined as the time required for the signal to attain 63.2% of final or steady state value

*Note:-

Time constant indicates how fast the system reaches the final value.

Smaller the τ → Faster the system

larger τ → sluggish the system

*Unit Impulse:-

→ Responce of the 1st order with unit ramp i/p. $(R(s) = 1)$ for unit impulse

$$T.F. = \frac{C(S)}{R(S)} = \frac{1}{1+s\tau}$$

$$\Rightarrow C(S) = \frac{1}{1+s\tau} \times 1$$

$$\Rightarrow C(S) = \frac{\frac{1}{\tau}}{\frac{1}{\tau}+S}$$ $\frac{1}{s+a} = e^{-at}$

$$\Rightarrow C(s) = \frac{1}{\tau} \times \frac{1}{S+\frac{1}{\tau}}$$

$$\Rightarrow C(t) = \frac{1}{\tau} \times e^{-\frac{t}{\tau}}$$

* **Unit ramp** :- $R(s) = \frac{1}{s^2} = t.u(t)$

$$\frac{C(s)}{R(s)} = \frac{1}{1+s\tau}$$

$$\Rightarrow C(s) = \frac{1}{1+s\tau} \times \frac{1}{s^2}$$

$$\Rightarrow C(s) = \frac{1}{s^2(s\tau+1)}$$

$\Rightarrow$ By partial fraction;

$$\Rightarrow \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s\tau+1} = \frac{1}{s^2(s\tau+1)} \quad \text{—— (B)}$$

$$\Rightarrow As(s\tau+1) + B(s\tau+1) + Cs^2 = 1$$

$$\Rightarrow As(s\tau+1) + B(s\tau+1) + Cs^2 = 1$$

$s \to 0$ $\quad A0(0+1) + B\times(0+1) + 0 = 1$

$$\Rightarrow B = 1$$

$s \to 1$ $\quad As(s\tau+1) + B(s\tau+1) + Cs = 0$

$$\Rightarrow A\cancel{s^2}\tau + As + Bs\tau + \cancel{B} + \cancel{C}s^2 = 0$$

$$\Rightarrow A + B\tau = 0 \qquad [\because B = 1]$$

$$\Rightarrow A + \tau = 0$$

$$\Rightarrow A = -\tau$$

$s^2 \to 1$ $\quad As^2\tau + \cancel{A}s + B\cancel{s}\tau + \cancel{B} + Cs^2 = 0$

$$\Rightarrow As^2\tau + Cs^2 = 0 \qquad [\because A \to -\tau]$$

$$\Rightarrow -\tau.\tau + C = 0$$

$$\Rightarrow -\tau^2 + C = 0 \Rightarrow C = \tau^2$$

Put the value of A, B & C in eqn (B), we get;

$$\Rightarrow \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{s\tau+1} = \frac{1}{s^2(s\tau+1)}$$

$$\Rightarrow C(s) = \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{s\tau+1}$$

→ Apply inverse laplace on both the side;

$$c(t) = t - \tau + \tau \cdot \frac{\tau}{s\tau+1}$$

$$\Rightarrow c(t) = t - \tau + \tau \cdot \frac{1}{s+\frac{1}{\tau}}$$

$$\Rightarrow c(t) = t - \tau + \tau \cdot e^{-t/\tau}$$

Error signal or steady state error:- (Ramp)

$$e(t) = r(t) - c(t)$$

$$\Rightarrow e(t) = t - (t - \tau + \tau \cdot e^{-t/\tau})$$

$$\Rightarrow e(t) = \tau - \tau \cdot e^{-t/\tau}$$

$$\Rightarrow e(t) = \tau(1 - e^{-t/\tau})$$

Error signal for step i/p:-

$$e(t) = r(t) - c(t)$$

$$\Rightarrow e(t) = 1 - (1 - e^{-t/\tau})$$

$$\Rightarrow e(t) = \cancel{1} - \cancel{1} + e^{-t/\tau}$$

$$\Rightarrow e(t) = e^{-t/\tau}$$

Error signal for impulse:-

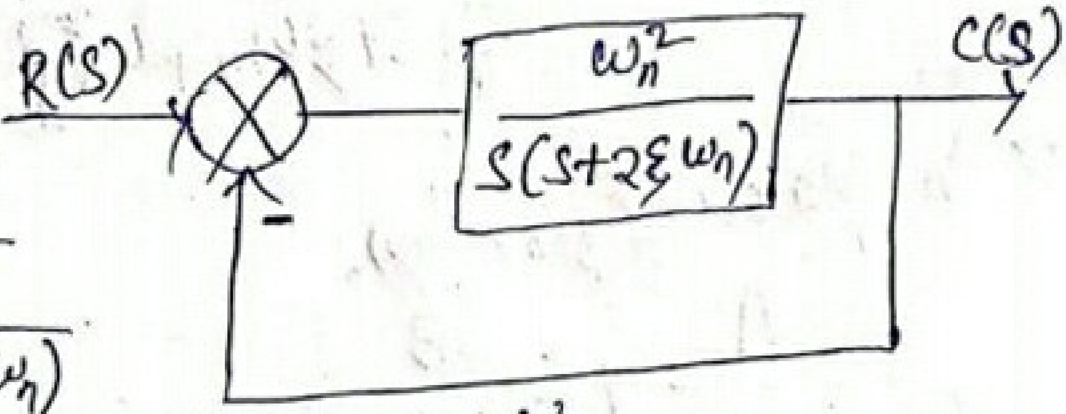
$$e(t) = r(t) - c(t)$$

$$\Rightarrow e(t) = \delta - \left(\frac{1}{\tau} \times e^{-t/\tau}\right)$$

$$\Rightarrow e(t) = \delta - \frac{1}{\tau} \cdot e^{-t/\tau}$$

* Time response of 2nd order system :-

$$\frac{C(s)}{R(s)} = \frac{\frac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}}$$



→ Where,

$$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$H(s) = 1$$

ω_n → Natural frequency of oscillation / undamped natural frequency $[\xi = 0]$

$\xi\omega_n$ → Damping factor / actual damping

ω_d → Damped frequency of oscillation.

ξ → Damping ration

* Damping Ratio :- $\xi = 0-1$ → Under damped

The ratio of actual damping to the critical damping is known as damping ratio.

Critical damping → $\xi = 1$

→ Time response of 2nd order system with unit step i/p.

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{\frac{\omega_n^2}{s^2+2\xi\omega_n s}}{\frac{s^2+2\xi\omega_n s+\omega_n^2}{s^2+2\xi\omega_n s}} \times \frac{1}{s}$$

$$\Rightarrow C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \times \frac{1}{s}$$

$(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)$
$= s^2+\cancel{\xi^2\omega_n^2}+2s\xi\omega_n$
$+\omega_n^2-\cancel{\omega_n^2\xi^2}$
$= s^2+2\xi\omega_n s+\omega_n^2$

→ Replace $s^2+2\xi\omega_n s+\omega_n^2 \to (s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)$

$$\Rightarrow C(s) = \frac{\omega_n^2}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi)^2} \times \frac{1}{s}$$

$$\boxed{\omega_n\sqrt{(1-\xi^2)} = \omega_d}$$

$$\Rightarrow C(s) = \frac{\omega_n^2}{s\{(s+\xi\omega_n)^2 + \omega_d^2\}} \quad \boxed{\omega_d^2 = \omega_n^2(1-\xi^2)}$$

→ In partial fraction;

$$\Rightarrow \frac{A}{s} + \frac{B}{(s+\xi\omega_n)^2 + \omega_d^2} = \frac{\omega_n^2}{s\{(s+\xi\omega_n)^2 + \omega_d^2\}}$$

$$\Rightarrow A\{(s+\xi\omega_n)^2 + \omega_d^2\} + Bs = \omega_n^2$$

$$(s+\xi\omega_n)^2+\omega_d^2=0$$
$$(s+\xi\omega_n)^2=\sqrt{-\omega_d^2}$$
$$=\sqrt{-1}\times\omega_d$$
$$=j\omega_d-\xi\omega_n$$

$$\omega_n^2=\frac{A}{s}+\frac{B\ \cancel{\omega_n^2}\ s}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\omega_n^2=\frac{A((s+\xi\omega_n)^2+\omega_d^2)}{(\ )}+\frac{Bs}{s}$$

$$\omega_n^2=A((\xi\omega_n)^2+\omega_d^2)$$

$$A=\frac{\omega_n^2}{\xi^2\omega_n^2+\omega_d^2}$$

$$A=\frac{\omega_n^2}{\xi^2\omega_n^2+\omega_n^2(1-\xi^2)}$$

$$=\frac{\cancel{\omega_n^2}\ 1}{\cancel{\omega_n^2}[\cancel{\xi^2}+(1-\cancel{\xi^2})]}$$

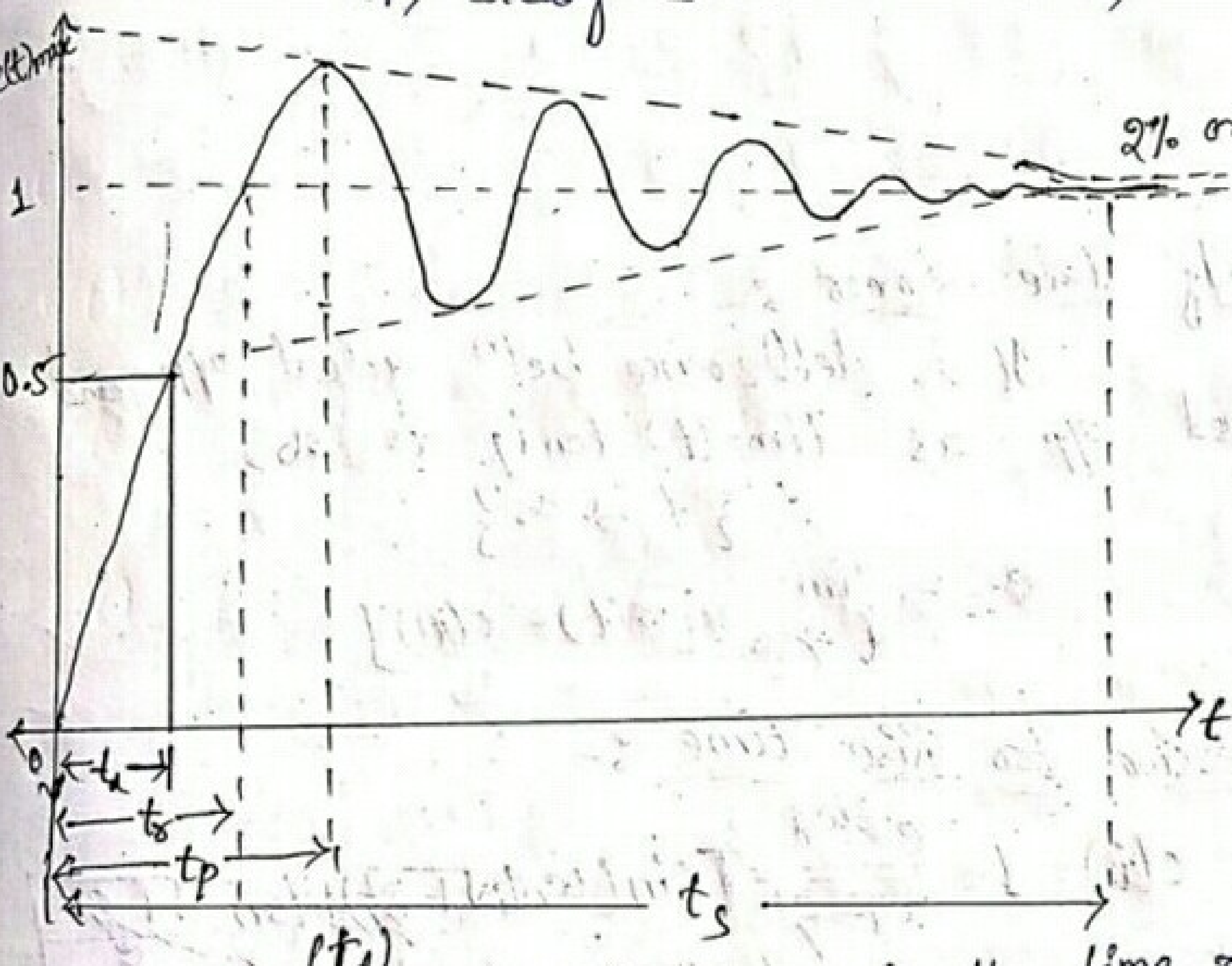
$$=$$

$$\omega_n^2=B(j\omega_d-\xi\omega_n)$$

$$B=\frac{\omega_n^2\ (-j\omega_d-\xi\omega_n)}{(j\omega_d-\xi\omega_n)(-j\omega_d-\xi\omega_n)}$$

→ The followings are the common transient responce characteristic ;

(a) Delay time (t_d)
(b) Rise time (t_r)
(c) Peak time (t_p)
(d) Max^n over shoot (M_p)
(e) Settling time (t_s)
(f) Steady state error (e_{ss})



* <u>Delay</u> <u>time</u> (t_d) :- The delay time is the time required for the response to reach 50% of the final value in 1st time.
* <u>Rise</u> <u>time</u> (t_r) :- It is the time required for the response to rise from 10% to 90% of its final value for over damped system and 0 to 100% for under damped system.
* <u>Peak</u> <u>time</u> (t_p) :- The peak time is the time required for the response to reach the 1st peak of the time response or 1st peak over shoot.
* <u>Settling</u> <u>time</u> (t_s) :- It is the difference bet^n the peak of the time responce & steady o/p.

Steady state error :-

It is difference betn actual o/p and desired o/p as time(t) tends to ∞.

$$\{t \to \infty\}$$

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

* **Expression for Rise time :-**

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\left(\omega_n t\sqrt{1-\xi^2}\right) + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

→ Let the responce reach 100% of desired value so put $c(t) = 1$

$$\Rightarrow 1 = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\left(\omega_n t\sqrt{1-\xi^2}\right) + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

$$\Rightarrow \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\left(\omega_n t\sqrt{1-\xi^2} + \phi\right)\right] = 0 \qquad \left[\because \tan\phi = \frac{\sqrt{1-\xi^2}}{\xi} \Rightarrow \phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

→ Let,

$$\left[\sin\left(\omega_n t\sqrt{1-\xi^2} + \phi\right)\right] = 0$$

$$\Rightarrow \sin\left(\omega_n\sqrt{1-\xi^2}\cdot t_r + \phi\right) = \sin n\pi$$

$$\Rightarrow \omega_n t\sqrt{1-\xi^2} + \phi = n\pi$$

$t \to$ delay time

$$\Rightarrow t_r = \frac{n\pi - \phi}{\omega_n\sqrt{1-\xi^2}} \Rightarrow \boxed{t_r = \frac{n\pi - \phi}{\omega_d}}$$

$\Rightarrow \xi \cdot \sin(\omega_n\sqrt{1-\xi^2}\cdot t+\phi) = \cos(t\omega_n\sqrt{1-\xi^2}+\phi)\cdot\sqrt{1-\xi^2}$

→ let, $\cos\phi = \xi$

then, $\sin\phi = \sqrt{1-\xi^2}$

$\Rightarrow \cos\phi \cdot \sin(t\omega_n\sqrt{1-\xi^2}+\phi) - \cos(t\omega_n\sqrt{1-\xi^2}+\phi)\cdot\sin\phi = 0$

$\Rightarrow \sin(\omega_n t\sqrt{1-\xi^2}+\phi-\phi) = 0$ $\quad \because [\sin(A+B)]$

$\Rightarrow \sin(\omega_n t\cdot\sqrt{1-\xi^2}) = 0$

* Expression of max^m over-shoot :-

→ Max^m over shoot identify by putting $n=1$, therefore the expression is $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$ $\quad |\because n=1.$

→ Max^m under shoot occurs at $n=2$. therefore the expression of t_p is :-

$$t_p = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} \quad |\because n=2$$

* Max^m Peak over shoot :-

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}[\sin(\omega_n t\cdot\sqrt{1-\xi^2}+\phi)]$$

→ For max^m over shoot put $t = t_p$; we get

$$c(t)_{Max} = 1 - \frac{e^{-\xi\omega_n\pi/\omega_n\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\cdot\sin\left[\omega_n\cdot\frac{\pi\cancel{\sqrt{1-\xi^2}}}{\omega_n\cancel{\sqrt{1-\xi^2}}}+\phi\right]$$

$$\Rightarrow c(t)_{Max} = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\cdot\sin(\pi+\phi)$$

$\boxed{\sin(\pi+\phi) = -\sin\phi}$

$$\Rightarrow c(t)_{Max} = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\cdot(-\sin\phi)$$

$$\Rightarrow c(t)_{Max} = 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\cdot\sin\phi \quad [\because \sin\phi = \sqrt{1-\xi^2}]$$

$$\Rightarrow c(t)_{max} = 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\cancel{\sqrt{1-\xi^2}}}\cdot\cancel{\sqrt{1-\xi^2}}$$

$\Rightarrow c(t)_{max} = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$

$\Rightarrow c(t)_{max} - 1 = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$\Rightarrow \boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$

→ % of max^n over shoot $= e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

* **Settling time:-** Settling time for a 2nd order system is 4 time of the time constant because the speed of the decay depends upon the time constant.

$$\tau = \frac{1}{\xi\omega_n}$$

→ 2% of settling time $(4\tau) = \frac{4}{\xi\omega_n} = t_s$

→ 5% of setting time $(3\tau) = \frac{3}{\xi\omega_n}$

Q. When a 2nd order C.S. is subjected to unit step i/p. The value of $\xi = 0.5$ & $\omega_n = 6$ radian/sec determine the rise time, peak time & settling time and max^n over-shoot.

G.D. 2nd order of C.S. (unit step i/p)

$\xi = 0.5$

$\omega_n = 6$ radian/sec

Sol^n Rise time $(t_r) = \frac{n\pi - \phi}{\omega_n\sqrt{1-\xi^2}}$

i) $n \to 1$, $\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 1.04$ radian

$$t_r = \frac{1\times\pi - 1.04}{6\sqrt{1-0.5^2}} = 0.404 \text{ sec.}$$

ii) $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\Rightarrow t_p = \dfrac{\pi}{6\sqrt{1-0.5^2}} = 0.604$ sec

(iii) Settling time :-

$t_s(4\%) = \dfrac{4}{\xi\omega_n}$

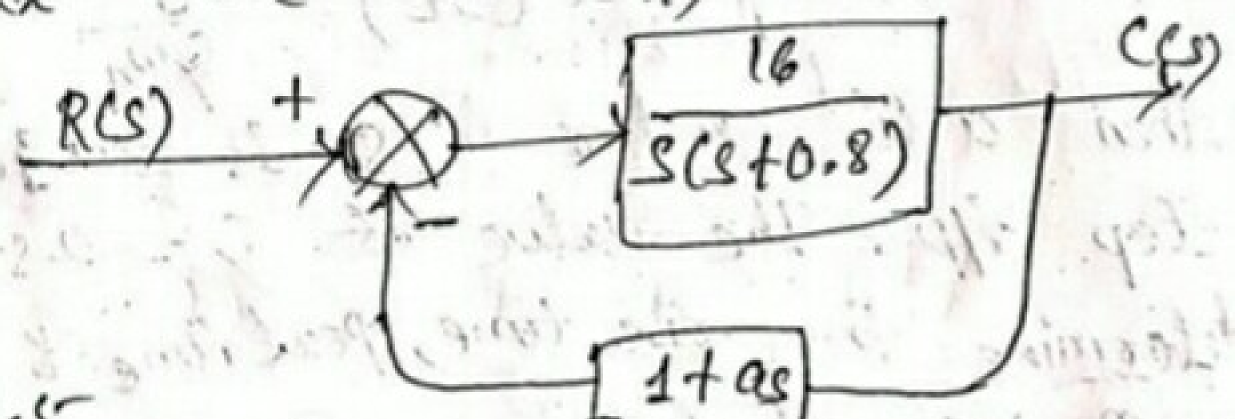
$\Rightarrow t_s = \dfrac{4}{0.5\times 6} = 1.33$ sec

(iv) Max^m over shoot $(M_p) = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$\Rightarrow M_p = e^{-0.5\pi/\sqrt{1-0.5^2}} \times 100$

$\Rightarrow M_p = 16.3$

Q) Consider this system as shown in fig. below. Determine the value of such that the damping ratio is 0.5. & also obtain the value of rise time & max^m over-shoot (M_p) in its step response.



Sol^n Given $\xi = 0.5$

$$T.F. = \frac{C(S)}{R(S)} = \frac{\dfrac{16}{S(S+0.8)}}{1+\dfrac{16}{S(S+0.8)}\cdot(1+aS)}$$

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{\dfrac{16}{S(S+0.8)}}{\dfrac{S(S+0.8)+16.(1+Sa)}{S(S+0.8)}}$$

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{16}{S(S+0.8)+16(1+aS)}$$

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{16}{S^2+0.8S+16+16aS} = \frac{16}{S^2+(0.8+16a

* For 2nd order system;

$$T.F. = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

→ Characteristics eq^n = $s^2+2\xi\omega_n s+\omega_n^2$ —— (ii)

→ But we have the data is,

$= s^2+(0.8+16a)s+16$ —— (i)

→ equating the both eq^n,

$$s^2+2\xi\omega_n s+\omega_n^2 = s^2+(0.8+16a)s+16$$

$$\Rightarrow 2\xi\omega_n s = (0.8+16a)s$$

$$\Rightarrow 2\xi\omega_n = 0.8+16a$$

→ Put the value of ξ & ω_n. $\xi = 0.5$ & $\omega_n = 4$

$$\Rightarrow 2\times 0.5\times 4 = 0.8+16a$$

$$\Rightarrow 16a = 4-0.8$$

$$\Rightarrow a = 3.2/16 = 0.2$$

→ $\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-0.5^2}}{0.5} = 1.04$ redian

→ $t_r = \frac{n\pi-\phi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi-1.04}{4\sqrt{1-0.5^2}} = 0.606$

→ $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}\times 100$

$$= e^{-0.5\pi/\sqrt{1-0.5^2}}\times 100$$

$$= 16.3$$

Q) A open loop t.f. of server system with unit feed-back is given that,

$G(s) = \frac{10}{(s+2)(s+5)}$, determine the damping ratio, undamped natural frequency of oscillation. What is the % over shoot of the respone to a unit step i/p.

* **Error Analysis :-**

Consider an open loop t.F. $G(s).H(s)$ =

$$G(s).H(s) = \frac{K(s+Z_1)(s+Z_2)(s+Z_3)\text{-------} \rightarrow \text{Zeros}}{s^m(s+P_1)(s+P_2)(s+P_3)\text{-------} \rightarrow \text{Poles}}$$

→ In this above eqn $s = -Z_1, -Z_2, -Z_3$ ------- are zeros and $s = -P_1, -P_2, -P_3$ ---- are poles, and the above eqn having a term of s^m in denominator, that means m is the no. of pole at the origin.

→ A system having no pole at origin of the s plane is said to be type - 0 system. i.e. $m = 0$.

→ If $m = 1$ it means the system have a pole at origin of the s plane. It is said to be a type-1 system.

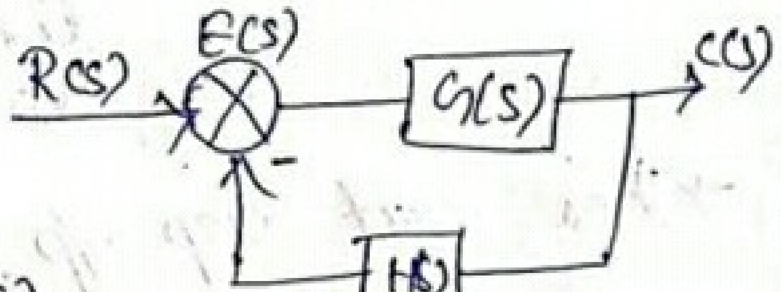
→ A system is called type-2 system, if $m = 2$ as show m.

* **Steady state error :-**

Steady state error :-

For closed loop $\quad C.S.\ (T.F.) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$C(s) = E(s).G(s)$



$\Rightarrow T.F. = \frac{E(s).G(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$

$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+G(s).H(s)}$

$\Rightarrow E(s) = \frac{R(s)}{1+G(s).H(s)}$

→ For unit feed back system ; $H(s)=1$

$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$

→ The steady state error of the system (e_{ss}) is obtained by appling final value theorem.

$e_{ss} = \lim_{t\to\infty} e(t)$

$= \lim_{s\to 0} s.E(s)$

$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{s \times \frac{1}{s}}{1+G(s)} = \frac{1}{1+G(s)}$

$\Rightarrow e_{ss} = \lim_{s\to 0} s.E(s)$

$\Rightarrow \boxed{e_{ss} = \lim_{s\to 0} s.\frac{R(s)}{1+G(s)}}$

* **Static error co-efficient:-**

→ Static position error (K_p):-

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s) \cdot H(s)}$$

→ For unit step i/p,

$$R(s) = 1/s$$

→ The e_{ss} is;

$$e_{ss} = \lim_{s \to 0} \frac{s \times \frac{1}{s}}{1 + G(s) \cdot H(s)}$$

$$\Rightarrow e_{ss} = \frac{1}{1 + \lim_{s \to 0} G(s)}$$ [for unit feedback system, $H(s) = 1$]

$$\boxed{K_p = \lim_{s \to 0} G(s)}$$

$$\boxed{\Rightarrow e_{ss} = \frac{1}{1 + K_p} \quad \text{(finite value)}}$$

* → **Static velocity error (K_v):-**

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)}$$

→ For unit ramp system, $R(s) = \frac{1}{s^2}$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{s \times \frac{1}{s^2}}{1 + G(s)}$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{1}{\cancel{s} + sG(s)}$$

$$\boxed{K_v = \lim_{s \to 0} sG(s)}$$

$$\boxed{\Rightarrow e_{ss} = \lim_{s \to 0} \frac{1}{sG(s)} = \frac{1}{K_v}}$$

Static acceleration error (Ka)

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1+G(s)} \qquad [H(s)=1]$$

For unit parabolic system, $1/s^3 = R(s)$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \times \frac{1}{s^{\cancel{3}}}}{1+G(s)}$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

$$\Rightarrow e_{ss} = \frac{1}{0 + \lim_{s \to 0} s^2 G(s)}$$

$$\boxed{K_a = \lim_{s \to 0} s^2 G(s)}$$

$$\Rightarrow \boxed{e_{ss} = \frac{1}{K_a}}$$

Steady state error for diff^t types of system :-

Case-I :-

Type '0' system with unit step i/p,

$$G(s) = \frac{K(s+Z_1)(s+Z_2) \cdots (s+Z_n)}{s^0 (s+P_1)(s+P_2) \cdots (s+P_n)}$$

$R(s) = 1/s$, so

$$K_p = \lim_{s \to 0} G(s)$$

$$\Rightarrow K_p = \lim_{s \to 0} \frac{K(s+Z_1)(s+Z_2) \cdots (s+Z_n)}{(s+P_1)(s+P_2) \cdots (s+P_n)}$$

$$\Rightarrow K_p = \frac{K Z_1 Z_2 \cdots}{P_1 P_2 \cdots}$$

$\Rightarrow K_p =$ finit value $= K$

$$\Rightarrow e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} = \text{constant}$$

*

Case-II

→ Type '0' system with unit ramp system,

$$R(s) = 1/s^2 \; ; \text{so}$$

$$K_v = \lim_{s \to 0} sG(s)$$

$$\Rightarrow K_v = \lim_{s \to 0} \frac{sK(s+Z_1)(s+Z_2)\cdots(s+Z_n)}{s^0(s+P_1)(s+P_2)\cdots(s+P_n)}$$

$$\Rightarrow \boxed{K_v = 0}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

Case-III

→ Type '0' system with unit parabolic system,

$$R(s) = 1/s^3 \; , \text{so}$$

$$K_a = \lim_{s \to 0} s^2 G(s)$$

$$\Rightarrow K_a = \lim_{s \to 0} \frac{s^2 K(s+Z_1)(s+Z_2)\cdots(s+Z_n)}{s^0(s+P_1)(s+P_2)\cdots(s+P_n)}$$

$$\Rightarrow K_a = 0$$

$$\Rightarrow e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Hence ramp & parabolic i/p are not acceptable in type zero system.

Type-'1' system:-

Type '1' system for unit step i/p:-

Case-I $R(S) = 1/s$

$m = 1$

$H(S) = 1$

$$G(S) = \frac{K(S+Z_1)(S+Z_2)\text{-------}(S+Z_n)}{S(S+P_1)(S+P_2)\text{-------}(S+P_n)}$$

$$K_p = \lim_{S\to 0} G(S)$$

$$\Rightarrow K_p = \lim_{S\to 0} \frac{K(S+Z_1)(S+Z_2)\text{-----}(S+Z_n)}{S(S+P_1)(S+P_2)\text{-----}(S+P_n)}$$

$$\Rightarrow K_p = \infty$$

$$\Rightarrow e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

Type '1' system for unit Ramp i/p:-

Case-II $R(S) = 1/S^2$

$m = 1$

$$G(S) = \frac{K(S+Z_1)(S+Z_2)\text{-------}(S+Z_n)}{S(S+P_1)(S+P_2)\text{-------}(S+P_n)}$$

$$K_v = \lim_{S\to 0} SG(S)$$

$$\Rightarrow K_v = \lim_{S\to 0} \frac{\cancel{S}K(S+Z_1)(S+Z_2)\text{-------}(S+Z_n)}{\cancel{S}(S+P_1)(S+P_2)\text{-------}(S+P_n)}$$

$$\Rightarrow K_v = K \text{ (constant)}$$

$$\Rightarrow e_{ss} = \frac{1}{K_v} = \text{finite error}$$

Case-III

Type '1' system for unit Parabolic system:-

$R(S) = 1/S^3$

$$\Rightarrow K_a = \lim_{S\to 0} S^2 G(S)$$

$$K_a = \lim_{s \to 0} \frac{\cancel{s^2} K(s+Z_1)(s+Z_2) \cdots (s+Z_n)}{\cancel{s}(s+P_1)(s+P_2) \cdots (s+P_n)}$$

$\Rightarrow K_a = 0$

$\Rightarrow e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$

→ From the above eqn it is clear that for type unit system step i/p & ramp i/p is acceptable & parabolic is not acceptable

* Type-2:- case-I

→ Unit step i/p :-

$R(s) = 1/s \quad , m = 2$

$$\Rightarrow G(s) = \frac{K(s+Z_1)(s+Z_2) \cdots (s+Z_n)}{s^2(s+P_1)(s+P_2) \cdots (s+P_n)}$$

$\Rightarrow K_p = \lim_{s \to 0} G(s)$

$$\Rightarrow K_p = \lim_{s \to 0} \frac{K(s+Z_1)(s+Z_2) \cdots (s+Z_n)}{s^2(s+P_1)(s+P_2) \cdots (s+P_n)}$$

$\Rightarrow K_p = \infty$

$\Rightarrow e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$

Case-II :- Unit ramp system :-

$R(s) = 1/s^2 \quad , m = 2$

$K_v = \lim_{s \to 0} sG(s)$

$$\Rightarrow K_v = \lim_{s \to 0} \frac{\cancel{s} K(s+Z_1)(s+Z_2) \cdots (s+Z_n)}{s^{\cancel{2}}(s+P_1)(s+P_2) \cdots (s+P_n)}$$

$\Rightarrow K_v = \infty$

$\Rightarrow e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$

Case-III Unit parabolic system :-

$R(s) = 1/s^3 \quad , m = 2$

$K_a = s^2 G(s)$

$$\Rightarrow K_a = \lim_{s\to 0} \frac{\cancel{s^2} K (s+z_1)(s+z_2)\ldots\ldots(s+z_m)}{\cancel{s^2}(s+P_1)(s+P_2)\ldots\ldots(s+P_n)}$$

$\Rightarrow K_a = K$ (constant)

$\Rightarrow e_{ss} = \frac{1}{K_a} = \frac{1}{K}$ = finite error.

→ Hence for type '2' system all the 3 i/p are acceptable.

System	type 0	type-1	type-2
1) Unit step i/p (K_p) $\Rightarrow e_{ss} = \frac{1}{1+K_p}$	finit value(K) constant	$K_p = \infty$ $\frac{1}{1+\infty} = 0$	$K_p = \infty$ 0
2) Unit ramp (K_v) $\Rightarrow e_{ss} = 1/K_v$	$0 = K_v$ $\frac{1}{0} = \infty$	K_v = constant finite error	$K_p = \infty$ 0
3) Unit Parabolic (K_a) $\Rightarrow e_{ss} = 1/K_a$	$K_a = 0$ $\frac{1}{0} = \infty$	$K_a = 0$ $e_{ss} = \frac{1}{0} = \infty$	K_a = constant e_{ss} = finite error

Q) The open loop T.F. for unit feed back system is given by $G(s) = \frac{50}{(1+0.1s)(s+10)}$. Determine the static error coefficient K_p, K_v & K_a.

→ $K_p = \lim_{s\to 0} G(s)$

$$\Rightarrow K_p = \lim_{s\to 0} \frac{50}{(1+0.1s)(s+10)}$$

$$\Rightarrow K_p = \frac{50}{(1+0)(0+10)} = \frac{50}{10} = 5$$

→ $K_v = \lim_{s\to 0} s.G(s)$

$$= \lim_{s\to 0} \frac{s \times 50}{(1+0.1s)(s+10)}$$

$$= 0 \times \frac{50}{10} = 0$$

$\rightarrow K_a = \lim_{s\to 0} s^2 G(s)$

$= \lim_{s\to 0} \frac{s^2 \cdot 50}{(1+0.1s)(s+10)}$

$= 0 \times \frac{50}{10} = 0$

Q) The forward path T.F. of a unit feed back C.S. is given by $G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$. Determine the step, ramp, parabolic error co-efficient & also determine types of system.

i) $K_p = \lim_{s\to 0} G(s)$

$= \lim_{s\to 0} \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$

$= \frac{5(0+0+100)}{0(0+5)(0+0+10)} = \frac{500}{0}$

$\Rightarrow K_p = \infty$

ii) $K_v = \lim_{s\to 0} s \times G(s)$

$= \lim_{s\to 0} \frac{\cancel{s} \cdot 5(s^2+2s+100)}{s^{\cancel{2}}(s+5)(s^2+3s+100)}$

$= \frac{5(0+0+100)}{0}$

$= \infty$

iii) $K_a = \lim_{s\to 0} s^2 G(s)$

$= \lim_{s\to 0} \frac{\cancel{s^2} \cdot 5(s^2+2s+100)}{\cancel{s^2}(s+5)(s^2+3s+10)}$

$= \frac{5(0+0+100)}{(0+5)(0+0+10)}$

$= \frac{\cancel{500}^{10}}{\cancel{50}_{1}} = 10$

iii) $R(t) = 3t^2/2$

$\Rightarrow R(s) = \frac{3}{2} \times \frac{2}{s^3} = \frac{3\times 2}{2s^3} = \frac{3}{s^3}$

$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{3}{s^3}}{1+\frac{20}{s(s+2)(s^2+2s+20)}}$

$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{\frac{3}{s^2}}{\frac{s(s+2)(s^2+2s+20)+20}{s(s+2)(s^2+2s+20)}}$

$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{\frac{3}{s^2} \times s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)}$

$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{3(s+2)(s^2+2s+20)}{s\{(s+2)(s^2+2s+20)+20\}}$

$\Rightarrow e_{ss} = \infty$

Q) The open loop T.F. of unit feed back system is given by $G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$. Find the static error coefficient & steady state error of the system when subjected to an i/p is given by

$R(t) = 2+5t+2t^2$

$R(t) = 2+5t+2t^2$

$\Rightarrow R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{2\times 2}{s^3}$

$R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$

$\Rightarrow K_p = \lim_{s\to 0} G(s)$

$\Rightarrow K_p = \lim_{s\to 0} \frac{108}{s^2(s+4)(s^2+3s+12)}$

$\Rightarrow K_p = \infty$

Q. The open loop T.F. of unit feed back system is given by $G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$. Find the static error coefficient & steady state error of the system when subjected to an i/p is given by

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$$R(t) = 2+5t+2t^2$$

$$\Rightarrow R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{2\times 2}{s^3}$$

$$R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$$

$$\Rightarrow K_p = \lim_{s\to 0} G(s)$$

$$\Rightarrow K_p = \lim_{s\to 0} \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$\Rightarrow K_p = \infty$$

$$K_v = \lim_{s\to 0} sG(s)$$

$$= \lim_{s\to 0} \frac{\cancel{s}\cdot 108}{s^{\cancel{2}}(s+4)(s^2+3s+12)}$$

$\Rightarrow K_v = \infty$

$\rightarrow$ $$K_a = \lim_{s\to 0} s^2 G(s)$$

$$= \lim_{s\to 0} \frac{\cancel{s^2}\cdot 108}{\cancel{s^2}(s+4)(s^2+3s+12)}$$

$$= \frac{108}{4\times 12} = \frac{108}{48}$$

$$\Rightarrow K_a = \frac{108}{48}$$

$\rightarrow$ The given i/p is $r(t) = 2+5t+2t^2$

$$R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$$

$\rightarrow$ $\boxed{e_{ss} = \frac{1}{1+K_p} + \frac{1}{K_v} + \frac{1}{K_a}}$ (for $\frac{1}{s}, \frac{1}{s^2}, \frac{1}{s^3}$)

$\rightarrow$ As per given i/p ;

$$\cancel{e_{ss} = \frac{1}{1+\frac{2}{s}} + \frac{1}{\frac{5}{s^2}} + \frac{1}{\frac{4}{s^3}}}$$

$=$

$$e_{ss} = \frac{R_1}{1+K_p} + \frac{R_2}{K_v} + \frac{R_3}{K_a}.$$

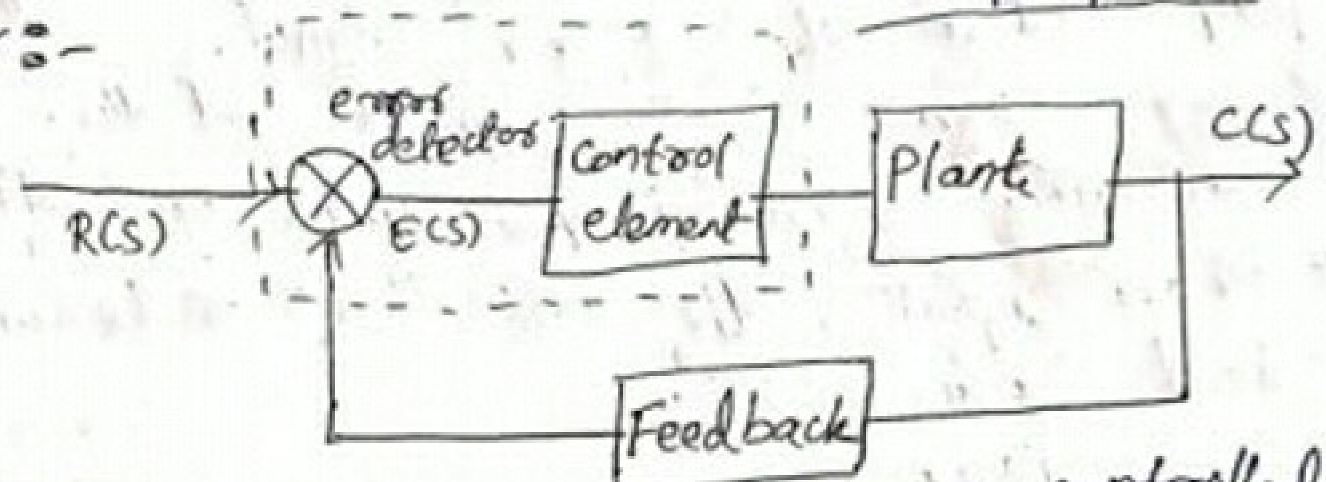
$$= \frac{2}{1+\infty} + \frac{5}{\infty} + \frac{4}{\frac{108}{48}}$$

$$= \frac{2}{\infty} + 0 + \frac{48\times 4}{108}$$

$\Rightarrow e_{ss} = 1.77$

Dt- 11/02/2020

* Controller:-



Defn:-
A controller is one which compares controlled value with desired value & has a function to correct the deviation produced.

* Use of controller:-
i) It improve steady state accuracy by reducing steady state error.
ii) As the steady state accuracy improves, the stability also improve.
iii) They also help in reducing the offset in the system.
iv) Max^m over shoot of the system can be controlled by using controllers.
v) It also reducing the noise signal.
vi) slow response can be made faster by using controlling element.

* Classification:- It classified depending upon the type of controlling action used. They are:-
i) Proportional controllers (P)
ii) Integral " (I)
iii) Derivative " (D)
iv) PI (PI) "
v) PD "
vi) PID ".

i) Proportional controller (P):–

In this controller the actuating signal is proportional to the error signal.

→ Error signal is the i/p of controlling element & actuating signal is the o/p of controlling element.

→ Error signal is the diff^n bet^n reference i/p & feed back signal.

$$e_a(t) \propto e(t)$$

$$e_a(t) = K_p e(t) \qquad k_p = \text{proportionality constant}$$

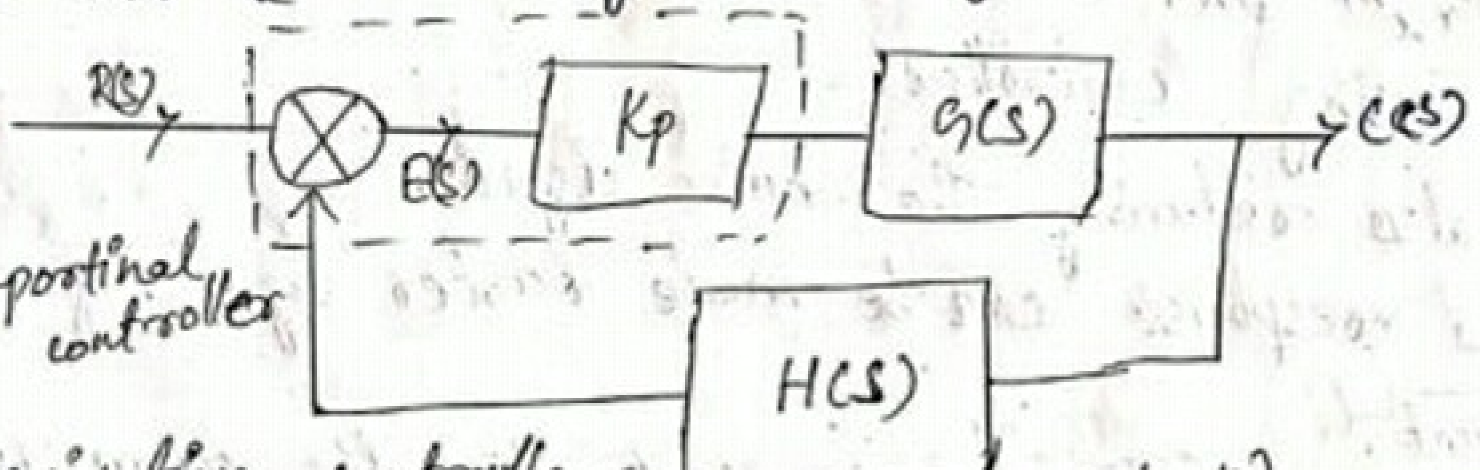
→ In laplace transfer domain;

$$E_a(s) \propto E(t)$$

$$E_a(s) = K_p E(s) \qquad K_p = \text{proportinal gain}$$

$$\Rightarrow K_p = \frac{E_a(s)}{E(s)} \quad \text{———} (i)$$

→ In controller with proportinal controlation there is a contineous linear relation bet^n o/p of the controller & actuating error signal.



ii) Derivative controller:–(D) (Rate controller)

In A controller with derivative controlation the o/p of the controller depend on the rate of change of actuating error signal.

$$e_a(t) \propto \frac{d}{dt}.e(t)$$

$$\Rightarrow e_a(t) = K_d \frac{de(t)}{dt}$$

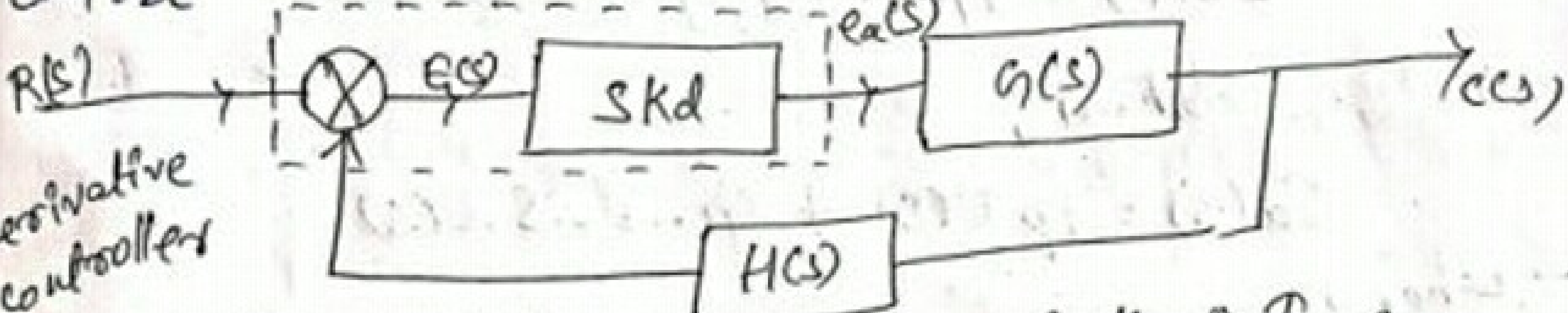
→ In LT domain,

$$\Rightarrow E_a(S) = K_d \cdot S \cdot E(s)$$

$$\Rightarrow sK_d = \frac{E_a(s)}{E(s)} \quad \{\text{Transfer func}\} \quad \text{—— (1)}$$

where, $K_d \rightarrow$ derivative gain constant.

* In the above eqn, it is clear that where error is zero or constant, the o/p of the controller will be zero. Therefore these type of controller can't be use alone.



derivative controller

(iii) Integral controller :— (Reset controller) - I :—

→ In a controller with an integral controlaction the o/p of the controller is changed at a rate which is proportinal to the actuating error signal.

$$\Rightarrow \frac{d}{dt} e_a(t) \propto e(t) \qquad \Rightarrow e_a(t) = K_i \int e(t) + e(0)^{\to 0}$$

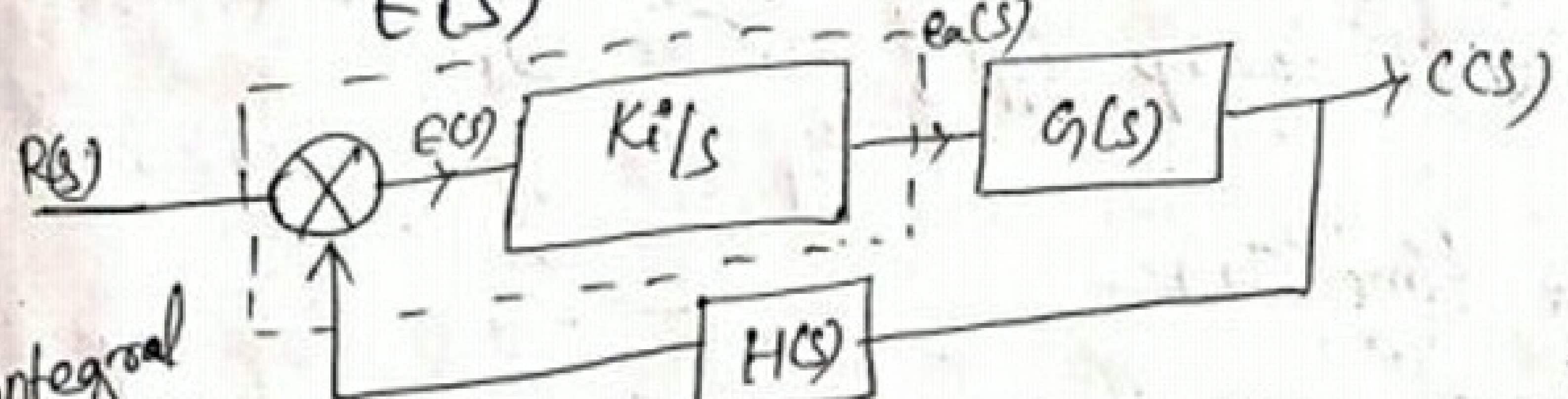
all initial condn is '0'.

$$\Rightarrow \frac{d}{dt} e_a(t) = K_i\, e(t) \qquad \Rightarrow e_a(t) = K_i \int e(t) \text{ —— (1)}$$

→ In LT domain ;

$$\Rightarrow S \cdot E_a(S) = K_i E(S)$$

$$\Rightarrow \frac{E_a(S)}{E(S)} = K_i/S$$



Integral controller

$K_i \rightarrow$ Integral gain constant

(iv) Proportional plus derivative controller :- (P-D)

→ When a derivative controller is added in series to proportional controlation, then this combination is turned as P-D controller.

→ In matematically,

$$e_a(t) = K_p\, e(t) + K_p \cdot K_d \frac{d}{dt} e(t)$$

→ In L.T. domain;

$$E_a(S) = K_p\, E(S) + K_p \cdot K_d \cdot S \cdot E(S)$$

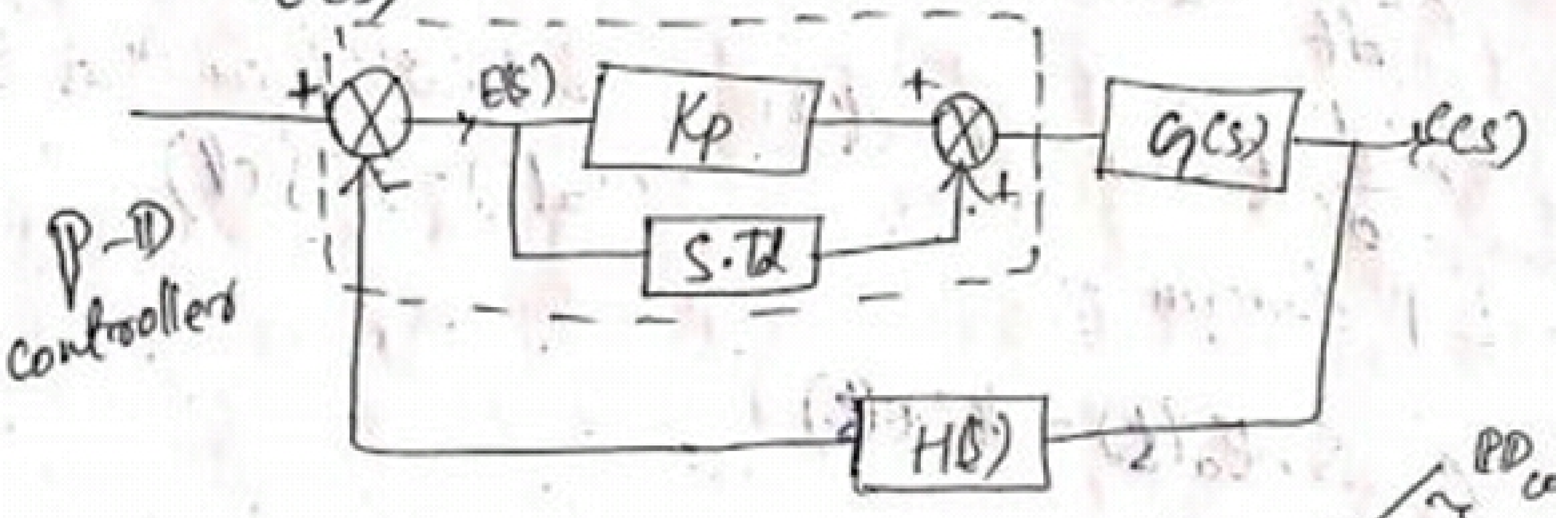
→ where,

$[K_d \propto T_d]$ ——* ((Kd → Derivative Time)) → Td

$$\Rightarrow E_a(S) = K_p\, E(S) + K_p \cdot T_d \cdot S \cdot E(S)$$

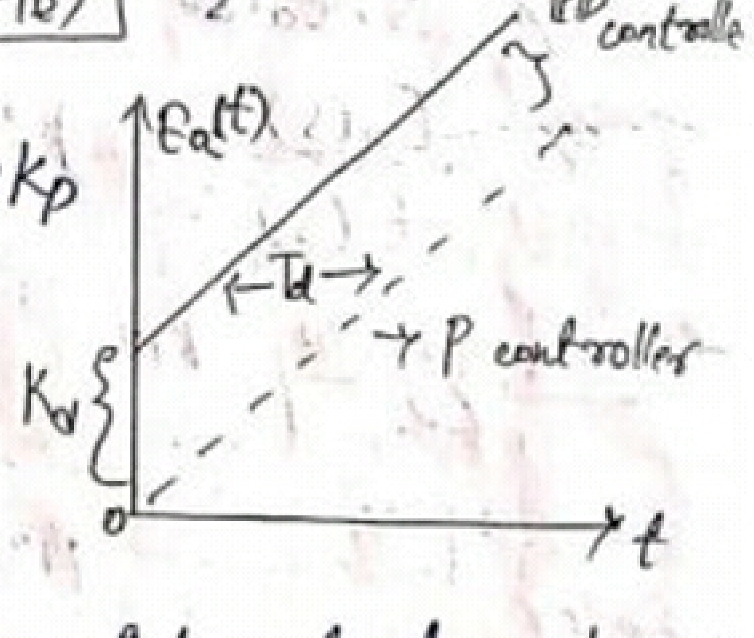
$$\Rightarrow E_a(S) = E(S)\{K_p(1 + S T_d)\}$$

$$\Rightarrow \frac{E_a(S)}{E(S)} = K_p(1 + S T_d)$$



$$E_a(S) = E(S) K_p + E(S) \cdot S T_d K_p$$

⇒ For unit ramp i/p,



*Derivative time (Td):-

It is defn as the time interval by which the rate action advanced the effect of the proportional controller or as the amount of lead.
→ It expressed in Td & unit is second.

→ Proportional plus integral controller (P-I):-

It is the combination of proportional controller & integral controller. action is called P-I controller.

→ In mathematically.

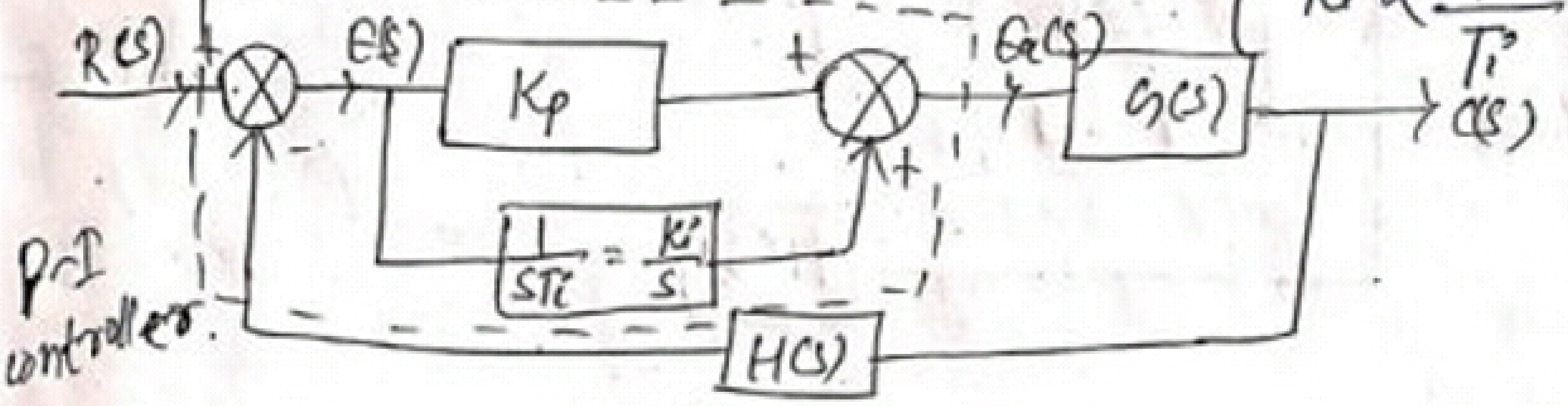
$$e_a(t) = K_p e(t) + K_p \cdot K_i \int e(t)$$

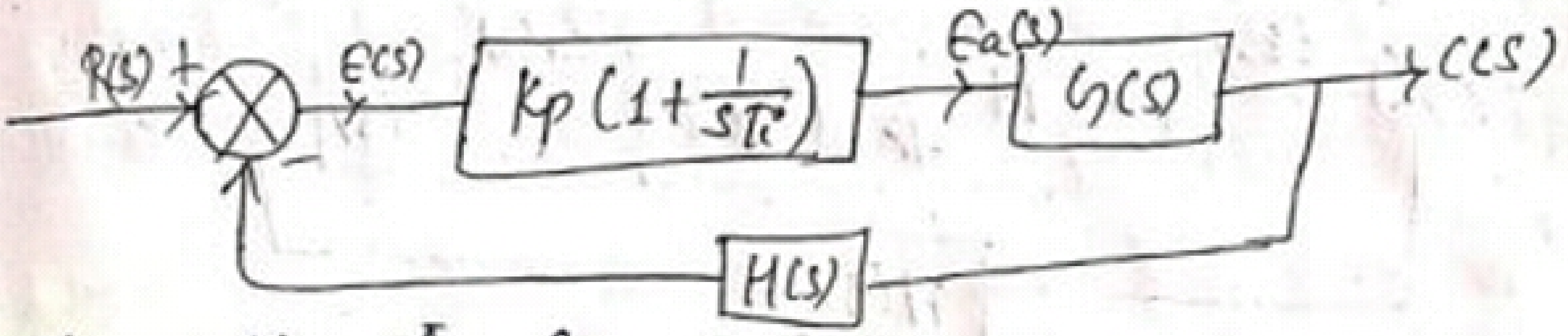
→ In L.T. domain;

$$\Rightarrow E_a(s) = K_p \cdot E(s) + K_p \cdot K_i \cdot \frac{1}{s} \cdot E(s)$$

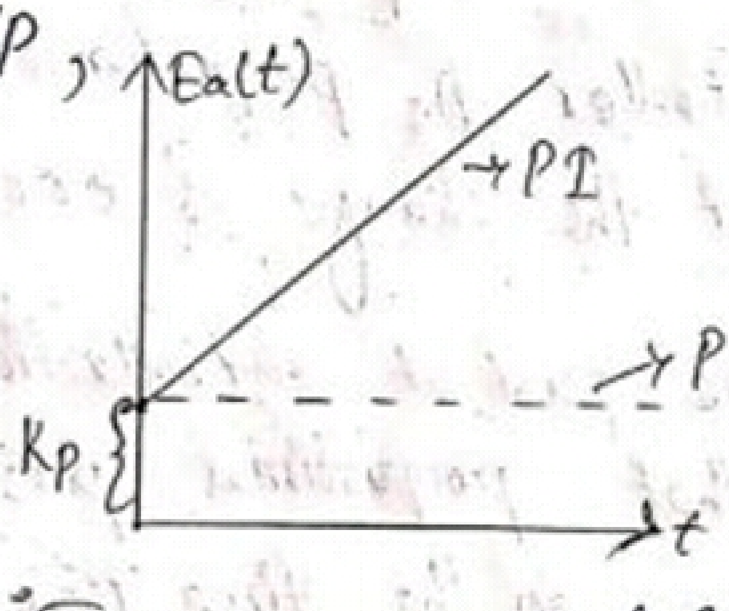
$$\Rightarrow E_a(s) = E(s)\left\{K_p\left(1 + K_i \cdot \frac{1}{s}\right)\right\}$$

$$\text{T.F.} \Rightarrow \frac{E_a(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right) \quad \because \left\{ \begin{array}{l} T_i \propto \frac{1}{K_i} \\ K_i \propto \frac{1}{T_i} \end{array} \right.$$





→ For unit step i/p,



* P-I-D controller:-

The combination of proportional, integral & derivative controllers is called P-I-D controlaction. And the controllaction is called P-I-D controller.

→ In mathematically;

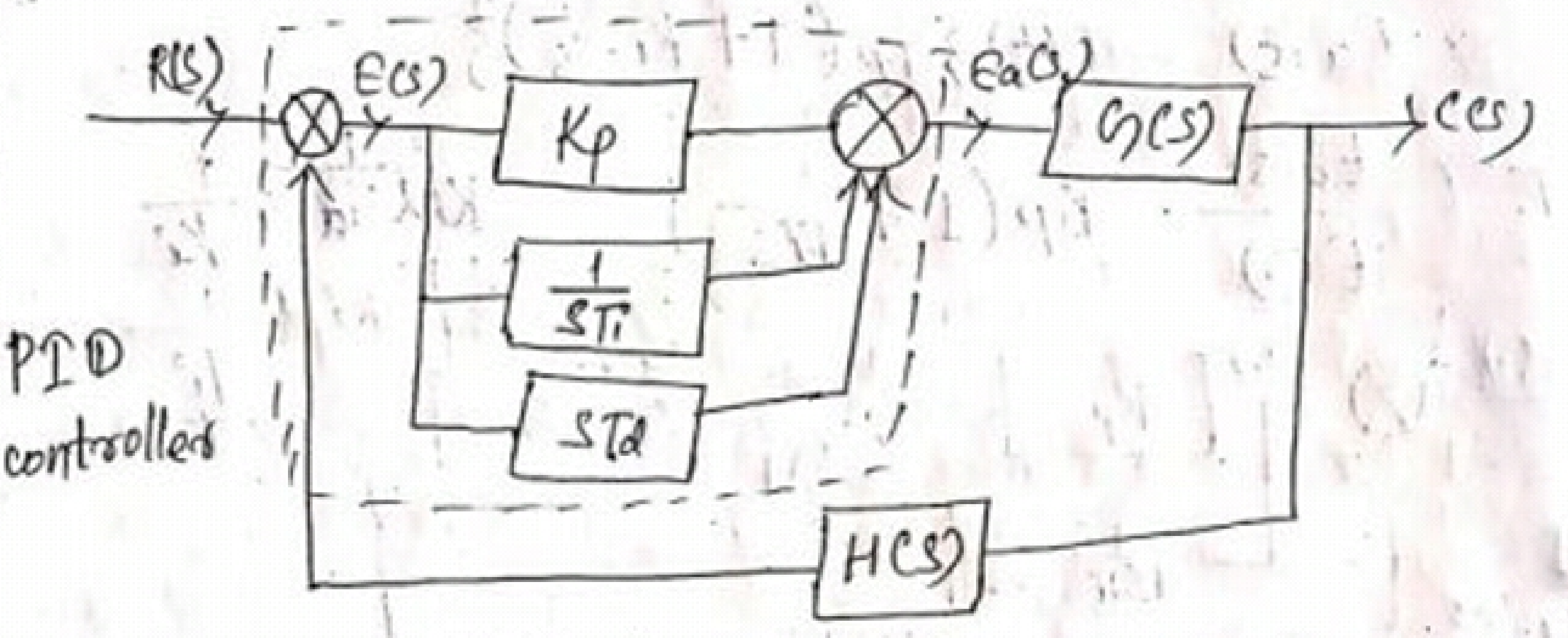
$$e_a(t) = K_p e(t) + K_p \cdot \frac{1}{T_i} \int e(t) + K_p T_d \frac{de(t)}{dt}$$

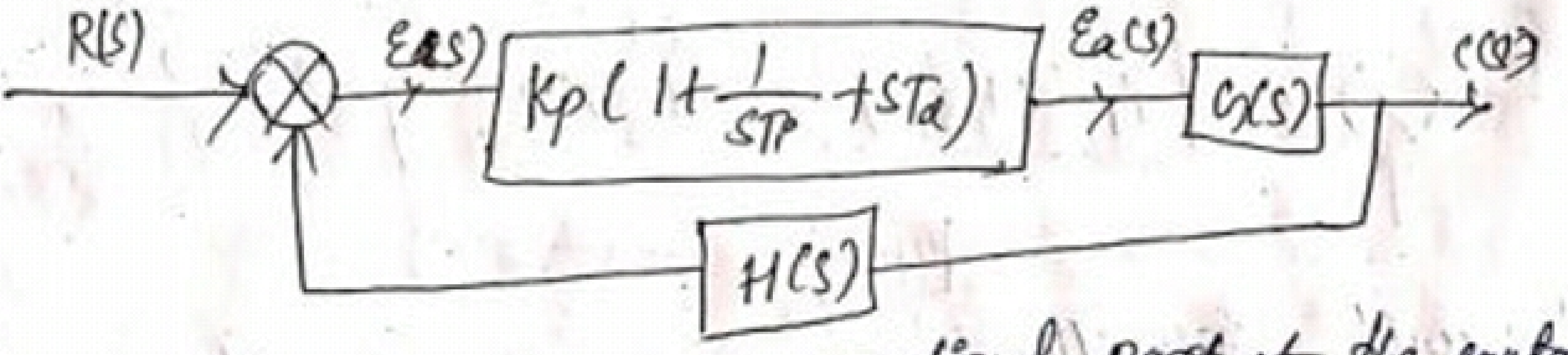
→ In L.T. domain;

$$e_a(s) = K_p \cdot e(s) + K_p \frac{1}{sT_i} \cdot e(s) + K_p \cdot T_d \cdot s \cdot e(s)$$

$$\Rightarrow e_a(s) = K_p \cdot e(s)\left(1 + \frac{1}{sT_i} + T_d \cdot s\right)$$

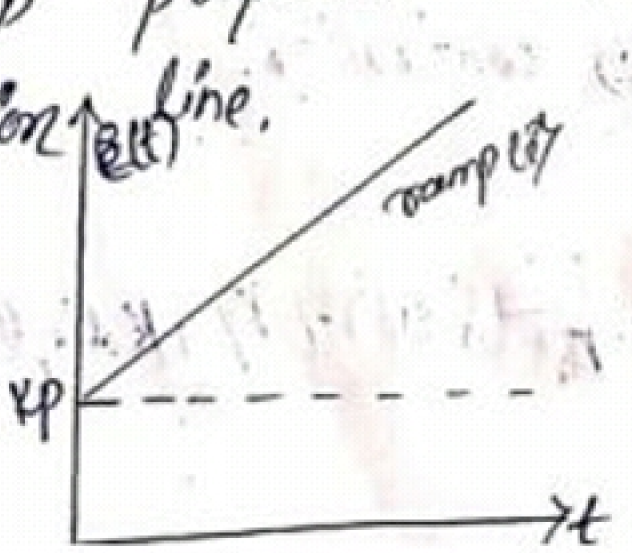
$$\Rightarrow \frac{e_a(s)}{e(s)} = K_p\left(1 + \frac{1}{sT_i} + sT_d\right)$$

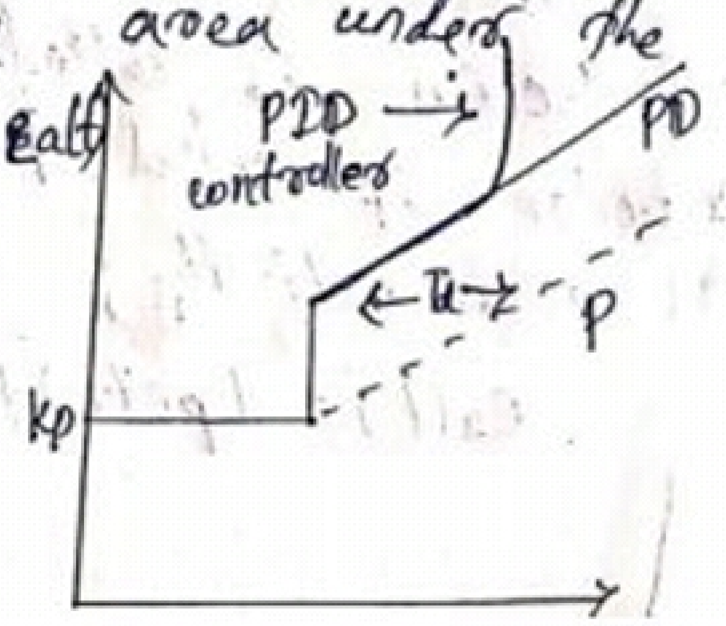




→ In this controller the proportional part of the control action repeat the change of error.

→ The derivative part of controllaction adds a increment of o/p so that proportional plus derivative action is shifted ahead in the time. Then the integral part of controllaction at a further increment of the o/p proportional to the area under the deviation line.





5th 04-15/10/2018 Analysis of stability of ROOT LOCUS TECHNIQUE

* Defn :-
It is a graphical method in which ploting the locus of the roots in the s-plane for the locus of the system parameter is varied different value of system parameter is varied over the complete range of value i.e. from '0' to '∞' is called "Root locus".

* Rules for construction of Root Locus :-

→ Rule-1 :- The root locus is symmetrical about real axis.

→ Rule-2 :- The roots starts from an open loop poles with K = 0 . (K → constant gain)

→ Rule-3 :- The root locus will terminate either on an open loop zeros or on infinite with K = infinite (∞)

→ Rule-4 :- No. of root locii or no. of root locus branches will be equal to the no. of poles if the poles are more than the no. of zeros i.e.

$$\left.\begin{array}{lll} N = P & \text{if} & P > Z \\ \text{or}\ N = Z & \text{if} & Z > P \\ \text{or}\ N = P = Z & \text{if} & Z = P \end{array}\right\}$$

where, N → No. of locii or branch
P → No. of poles
Z → No. of zeros

→ Rule-5 :- Locus of the roots on real axis.

→ Any point on the real axis is a part of the root locus if & only if the no. of poles & zeros to its right is odd.

→ Rule-6:- Asymptotes

→ The branches of root locus tends to ∞ (infinity) along a set of straight line called asymptotes.

→ These asymptotes making an angle with real axis & is given by $\theta = \frac{(2K+1) \times 180^\circ}{P-Z}$

where, $K \to 0, 1, 2, 3, \ldots\ldots\ldots (P-Z)$

→ Total no. of asymptotes (equal to) = No. of branches + 1 $(N+1)$

Ex:- Here, no. of asymptotes = 3 (∵ 0, 1, 2 = 3 no.)

$\theta_1 = \frac{(2\times0+1)\times180^\circ}{1} = 180^\circ$

$\theta_2 = \frac{(2\times1+1)\times180^\circ}{1} = 360^\circ$

$\theta_3 = \frac{(2\times2+1)\times180^\circ}{1} = 540^\circ$
$= 540 - 360 = 180^\circ$

$P = 2$
$Z = 1$
$N = P = 2$

→ Rule-7:- Centroid of asymptotes.

→ The point of intersection of asymptotes with real axis is called centroid of asymptotes.

$$\boxed{\sigma_A = \frac{\Sigma P - \Sigma Z}{P-Z}}$$

→ Centroid always has real axis.

→ Rule-8:- Angle of departure & angle of arrival at the root locus.

→ It always comes from complex conjugate poles.

Ex:- $s = -1 \pm j3$

→ Angle of departure $(\phi_d) = 180^\circ - \phi$

ϕ = sum of angles of vectors drawn to this pole from the other poles + sum of angle of vector drawn to this pole from the zeros.

→ Angle of arrival at a complex zero is given by $\phi_a = 180 + \phi$

ϕ = sum of vector drawn to this zeros from remaining zeros + sum of angle vector drawn to this zero from poles.

→ Rule-9 :- Break away point on real axis.

→ If the root locus lies two adjucent open loop poles on the real axis than, there will be at-least one break away point. Because the roots move towards each other as K (constant) is increased & meet at a point at this point K is maximum, If we increase the value of K betn 2 poles.

→ The root locus breaks into 2 parts.

→ Similarly if root locus lies betn 2 adjucent zero on real axis than, there will be at-least one break in point.

→ If the root locus lies betn open loop poles zeros than there will be no break away or break in point may be both occurs.

→ The break away or break in point can be determine from the roots of $\frac{dK}{ds} = 0$

$$\boxed{G(s).H(s) = \frac{K}{s(s^2+6s+10)}}$$

→ Characterstic eqn = 1 + GH

$$K + s(s^2+6s+10) = 0$$

$$\Rightarrow K = -(s^2+6s+10)s = -s^3 - 6s^2 - 10s$$

$$\Rightarrow \frac{dK}{ds} = 0$$

$$\Rightarrow -3s^2 - 12s - 10 = 0$$

$$s = -1.18, -2.81$$

Rule-10 Inter section of root locus branch with jω axis can be detemine through RH criteria.

→ Characteristic eqn =

$$s^3 + 6s^2 + 10s + K = 0$$

RH:-

s^3	1	10	0
s^2	6	K	0
s^1	$\frac{60-K}{6}$	0	
s^0	K	0	

$$\frac{60-K}{6} = 0$$

$$\Rightarrow 60-K=0 \Rightarrow K=60.$$

→ Auxiliary eqn (A.E) $\Rightarrow 6s^2+K=0$

$$\Rightarrow 6s^2+60=0$$

$$\Rightarrow s^2 = -\frac{60}{6}$$

$\Rightarrow$

$$s^2 = -10$$

$$s = \sqrt{-10} = \sqrt{-1} \times \sqrt{10}$$

$$= j \times \sqrt{10}$$

Q) The forward path transfer function of a unit feedback system is given by $G(s) = \frac{K}{s(s+4)(s+5)}$ sketch the root locus as K varies 0 to ∞.

Step 1:- Plot the poles & zeros symmetrical about real axis.

Pole → $s=0$, $s=-4$, $s=-5$

Zero → Non

→ No. of pole is 3

→ No. of zero is 0.

Step-2:- No. of branches;

$N=P$ $\quad P>Z$

$\Rightarrow P=N=3$ $\quad$ 3 branches.

3) The root locus exist betn $s=-0$ to $s=-4$ and to the left of -5 mark as the root locus on real axis.

~~Dr. Qts~~ of asymptotes
Centroid

$$\sigma_A = \frac{\Sigma P - \Sigma Z}{P - Z}$$

$$= \frac{(-5)+(-4)-0}{3-0} = \frac{-9}{3} = -3$$

5) Angle of Asymptotes:-

$$\theta = \frac{(2K+1)\times 180^\circ}{P-Z}$$

K = 0, 1, 2

$$\theta_1 = \frac{(2\times 0+1)\times 180^\circ}{3} = 60^\circ$$

$$\theta_2 = \frac{(2\times 1+1)180}{3} = 180^\circ$$

$$\theta_3 = \frac{(2\times 2+1)\times 180}{3} = 300^\circ$$

6) Calculation of break away point.

$$\frac{dK}{ds} = 0$$

→ Characteristics eqn = $1 + GH$

$$\Rightarrow 1 + \frac{K}{s(s+4)(s+5)} = 0$$

$$\Rightarrow s(s+4)(s+5) + K = 0$$

$$\Rightarrow K = -\{s(s+4)(s+5)\}$$

$$\Rightarrow K = -\{s^3 + s^2 4 + s^2 5 + 20s\}$$

$$\Rightarrow -s^3 - 9s^2 - 20s = K$$

$$\Rightarrow \frac{dK}{ds} = 0 \Rightarrow -3s^2 - 18s - 20 = 0$$

$$\Rightarrow +3s^2 + 18s + 20 = 0$$

$$s = -1.472, \; s = -4.527$$

→ Since −4 to −5 is not the branch of root locus ∴ we consider −1.472 as break away point.

4. Determine the point of intersection of root locus with imaginary axis by RH criteria.

characteristic eqn:—

$$s^3+9s^2+20s+K=0$$

RH table :—

$$\frac{9\times 20-1\times K}{9}$$

s^3 →	1	20	0	
s^2 →	9	K	0	A.E
s^1 →	$\frac{180-K}{9}$	0		ROZ
s^0 →	K	0		

$$\frac{180-K}{9}=0$$

$$\Rightarrow K=180$$

→ A.E. → $9s^2+K=0$

$$\Rightarrow 9s^2=-180$$

$$\Rightarrow s^2=\frac{-\cancel{180}^{20}}{\cancel{9}}=-20$$

$$\Rightarrow s=\sqrt{-20}=\sqrt{-1}\times\sqrt{20}=2\sqrt{5}j=\pm 4.47$$

Q2) For an unit feed back system the open loop T.F. is given by, $G(s)=\frac{K}{s(s+2)(s^2+6s+25)}$

a) Sketch the root locus for $0\leq K\leq\infty$

b) At what value of K the system becomes unstable.

c) At this pt. of instability determine the frequency of oscillation of the system.

1. Plot the poles & zeros symmetrical about real axis.

→ No. of pole → 4

→ No. of zeros → 0

$$\frac{K}{s(s+2)(s^2+6s+25)}$$

$s=0$, $s=-2$, $s=-3+4j$, $s=-3-4j$

2. No. of branch;

$$P > Z$$

$$\Rightarrow N = P = 4$$

3. The sagment on the real axis betn $s=0$ to $s=-2$ is the part of root locus.

4. Centroid of asymptos;

$$\sigma_A = \frac{\Sigma P - \Sigma Z}{P-Z}$$

$$= \frac{0+(-2)+(-3+4j)+(-3-4j)-0}{4-0}$$

$$= \frac{-2-3+\cancel{4j}-3-\cancel{4j}}{4} = \frac{-8}{4} = -2$$

5. Angle of asymptos;

$$\theta = \frac{(2K+1)\times 180^\circ}{P-Z}$$

$K = 0, 1, 2, 3$

$$\theta_1 = \frac{(2\times 0+1)\times 180^\circ}{4} = 45^\circ$$

$$\theta_2 = \frac{(2\times 1+1)\times 180^\circ}{4} = 135^\circ$$

$$\theta_3 = \frac{(2\times 2+1)\times 180^\circ}{4} = 225^\circ$$

$$\theta_4 = \frac{(2\times 3+1)\times 180^\circ}{4} = 315^\circ$$

6. Calculate the break away point.

$$\frac{dK}{ds} = 0$$

→ characteristic eqn = 1+GH

$$1 + \frac{K}{s(s+2)(s^2+6s+25)} = 0$$

$$\Rightarrow s(s+2)(s^2+6s+25) + K = 0$$

$$\Rightarrow K = -\{s(s+2)(s^2+6s+25)\}$$

$$\Rightarrow K = -(s^4+2s^3+6s^3+12s^2+25s^2+50s)$$

$$\Rightarrow K = -s^4 - 8s^3 - 37s^2 - 50s$$

$$\Rightarrow \frac{dK}{ds} = 0$$

$$\Rightarrow \frac{dK}{ds} = -4s^3 - 24s^2 - 74s - 50$$

$$\Rightarrow 4s^3 + 24s^2 + 74s + 50 = 0$$

$s_1 = -0.898$, $s_2 = -2.55 + 2.72j$

$s_3 = -2.55 - 2.72j$,

→ Since $-2.55+2.72j$ & $-2.55-2.72j$ not the branch of root locus ∴ we consider -0.898 as break away point.

7. RH criteria;

→ characteristics eqn → $s^4 + 8s^3 + 37s^2 + 50s + K = 0$

RH table:-

$s^4 \rightarrow 1 \quad 37 \quad K \quad 0$

$s^3 \rightarrow 8 \quad 50 \quad 0 \quad 0$

$s^2 \rightarrow \dfrac{37 \times 8 - 50 \times 1}{8} = 30.75 \quad \dfrac{8K - 37 \times 0}{8} = K \quad 0 \quad 0$

$s^1 \rightarrow \dfrac{30.75 \times 50 - 8K}{30.75} \quad 0$

$s^0 \rightarrow K \quad 0$

30.75

$\frac{50K}{80} = K$

$$\frac{30.75 \times 50 - 8K}{30.75} = 0$$

$\Rightarrow 1537.5 - 8K = 0$

$\Rightarrow 8K = 1537.5$

$\Rightarrow K = 192.18$

$\Rightarrow$ A.E $\Rightarrow$ $30.75 s^2 + K = 0$

$\Rightarrow s^2 \times 30.75 + 192.18 = 0$

$\Rightarrow 30.75 \times s^2 = -192.18$

$$\Rightarrow s^2 = \frac{-192.18}{30.75}$$

$$\Rightarrow s = \sqrt{\frac{-192.18}{30.75}} = \sqrt{-1} \times 2.49 = \pm 2.49j$$

8. Angle of depature from complex pole.

$\theta_d = 180° - \theta$

$= 180° - (27.104 + 90) = 180 - 321 = \cancel{141}°$

a) Graph

b) $K = 192.18$ (Marginary stable)

If $K > 192.18$ this has unstable.

c) $30.75 s^2 + K = 0$

$\Rightarrow 30.75 s^2 + 192.18 = 0$

$\Rightarrow 30.75 (j\omega)^2 + 192.18 = 0$ $\left\{ \begin{array}{l} j\omega = s \\ j^2 = -1 \end{array} \right.$

$\Rightarrow -30.75 \omega^2 + 192.18 = 0$

$$\Rightarrow \omega^2 = \frac{-192.18}{-30.75}$$

$\Rightarrow \omega = \sqrt{\dfrac{192.18}{30.75}}$

$\Rightarrow \omega = 2.499 = 2.5$ Radian/sec

Dt → 06/03/20

Q. Consider a unit feed back CS with the following feed forward T.F. $G(s) = \dfrac{K}{s(s^2+4s+8)}$ plot the root loci for the system.

1. Plot the no. of zeros & poles symmetrical about real axis.
   - → No. of pole – 3 $s = 0$, $s = -2+2j$, $s = -2-2j$
   - → No. of zero – 0
2. No. of branches;

$P > Z \qquad P = N = 3$

3. The segment on the real axis beth $s=0$ ~~to~~ & left ~~of~~ the ~~$s=-2$~~ is the part of root locus.
4. Centroid of symptos;

$$\sigma_A = \frac{\Sigma P - \Sigma Z}{P - Z}$$

$$= \frac{0 + (-2+2j) + (-2-2j) - 0}{3}$$

$$= \frac{-4}{3} = -1\frac{1}{3}$$

5. Angle of asymptoty :– $K = 0, 1, 2$

$$\theta_1 = \frac{(2\times 0+1)\times 180^\circ}{3} = 60^\circ$$

$$\theta_2 = \frac{(2\times 1+1)\times 180^\circ}{3} = 180^\circ$$

$$\theta_3 = \frac{(2\times 2+1)\times 180^\circ}{3} = 300^\circ$$

6. Calculate the break away point.

$$\frac{dK}{ds} = 0$$

→ Characteristics eq^n := $1+GH$

$$\Rightarrow 1 + \frac{K}{s(s^2+4s+8)} = 0$$

$$\Rightarrow s(s^2+4s+8) + K = 0$$

$$\Rightarrow K = -(s^3 - 4s^2 - 8s)$$

$$\rightarrow \quad \frac{dK}{ds} = 0$$

$$\Rightarrow 3s^2 + 8s + 8 = 0$$

$$s_1 = -\frac{4}{3} + \frac{2\sqrt{2}}{3}j = -1.33 + 0.94j$$

$$s_2 = -\frac{4}{3} - \frac{2\sqrt{2}}{3}j = -1.33 - 0.94j$$

→ Since there has the angle $cond^n$ is not satisfied ~~bcz~~ hence there has no break away point.

7. RH criateria;

→ characteristics $eq^n \Rightarrow s^3 + 4s^2 + 8s + K = 0$

RH Table:-

$s^3 \rightarrow$	1	8	0
$s^2 \rightarrow$	4	K	0
$s^1 \rightarrow$	$\frac{4\times 8 - 1\times K}{4}$	0	
$s^0 \rightarrow$	K	0	

$$\Rightarrow \frac{4\times 8 - 1\times K}{4} = 0$$

$$\Rightarrow 32 - K = 0$$

$$\Rightarrow K = 32$$

→ A.E. → $4s^2 + K = 0$

$\Rightarrow 4s^2 = -32$

$\Rightarrow s^2 = -\frac{\cancel{32}\,8}{\cancel{4}\,1} = -8$

$\Rightarrow s = \sqrt{-8} = \sqrt{-1} \times \sqrt{8} = \pm 2\sqrt{2}j = \pm 2.82j$

8. Angle of depature from complex pole.

$\theta_d = 180^\circ - (135 + 90) = -45^\circ$

—o—

Q4 Sketch the root locus & determine the stability of the system having open loop T.F. $G(s) = \frac{K}{s(s+6)(s^2+4s+13)}$

Step-1:- Plot the no. of poles & zeros.

Poles → 4 $\quad s_1 = 0,\ s_2 = -6,\ s_3 = -2+3j$
$s_4 = -2-3j$

Zeros → 0

2. No. of branch:-

$P > Z$. $\quad P = N = 4$

3. The sagment of the root locus on the red axis betn $s=0$ to $s=-6$.

4. centrooid of asymptos;

$\sigma_A = \frac{\Sigma P - \Sigma Z}{P - Z}$

$= \frac{0 + (-6) + (-2+3j) + (-2-3j) - 0}{4 - 0}$

$= \frac{-10}{4} = -2.5$

5. Angle of asymptos; $\quad K = 0, 1, 2, 3$

$\theta_1 = \frac{(2 \times 0 + 1) \times 180}{4} = 45^\circ$

$$\theta_2 = \frac{(2\times1+1)\times180^\circ}{4} = 135^\circ$$

$$\theta_3 = \frac{(2\times2+1)\times180^\circ}{4} = 225^\circ$$

$$\theta_4 = \frac{(2\times3+1)\times180^\circ}{4} = 315^\circ$$

6. Calculate the break away point.

$$\frac{dK}{ds} = 0$$

→ characteristics eq^n ⇒ $1+GH$

$$\Rightarrow 1 + \frac{K}{s(s+6)(s^2+4s+13)} = 0$$

$$\Rightarrow \{s(s+6)(s^2+4s+13)\} + K = 0$$

$$\Rightarrow K = -(s^4+4s^3+13s^2+6s^3+24s^2+78s)$$

$$\Rightarrow K = -(s^4+10s^3+37s^2+78s)$$

$$\Rightarrow \frac{dK}{ds} = 0$$

⇒ [illegible]

$$\Rightarrow 4s^3+30s^2+74s+78 = 0$$

$s_1 = -4.2$, $s_2 = -1.65+1.38j$

$s_3 = -1.65-1.38j$

→ Since root locus not be for $-1.65+1.38j$ & $-1.65-1.38j$.

∴ we consider -4.2 as break away point.

7) <u>RH</u> criteria:-

→ Characteristics eq^n :⇒ $s^4+10s^3+37s^2+78s+K=0$

<u>RH</u> table:-

s^4 →	1	37	K
s^3 →	10	78	0

$s^2 \rightarrow \dfrac{10\times 37 - 78\times 1}{10} = 29.2$ $\quad \dfrac{78\times K - 37\times 0}{78} = K$ $\quad 0 \quad 0$

$s^1 \rightarrow \dfrac{29.2\times 78 - 10K}{29.2}$ $\quad 0 \quad 0 \quad 0$

$s^0 \rightarrow K \quad 0$

$\Rightarrow$ A.E $\Rightarrow$

$29.2s^2 + K = 0$

$\Rightarrow 29.2s^2 + 227.76 = 0$

$\Rightarrow s^2 = -\dfrac{227.76}{29.2}$

$\Rightarrow s = \sqrt{\dfrac{-227.76}{29.2}} = \sqrt{-1} \times 2.79 = \pm 2.79j$

$= \pm 2.8j$

$\dfrac{29.2\times 78 - 10K}{10} = 0$

$\Rightarrow 2277.6 - 10K = 0$

$\Rightarrow 10K = 2277.6$

$\Rightarrow K = 227.76.$

POLAR PLOT

Defination :-

The Polar plot is a plot, which can be drawn between the magnitude and the phase angle of $G(j\omega) H(j\omega)$ by varying ω from zero to ∞.

Procedure to sketch the polar plot

Step 1 :- Determine the transfer function $G(s)$ of the system.

Step 2 :- Put $s = j\omega$ in the transfer function to obtain $G(j\omega)$

Step 3 :- At $\omega = 0$ and $\omega = \infty$ Calculate $|G(j\omega)|$, by $\lim_{\omega \to 0} |G(j\omega)|$ and $\lim_{\omega \to \infty} |G(j\omega)|$.

Step 4 :- Calculate the phase angle of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$. by $\lim_{\omega \to 0} \angle G(j\omega)$ and $\lim_{\omega \to \infty} \angle G(j\omega)$

Step 5 :- Rationalize the function $G(j\omega)$ and separate the real and imaginary parts.

Step 6 :- Equate the imaginary part $Im[G(j\omega)]$ to zero and determine the frequencies at which plot intersects the real axis and Calculate the value of $G(j\omega)$ at the point of intersection by substituting the determined value of frequency in the expression of $G(j\omega)$.

Step 7 :- Equate the real part $Re[G(j\omega)]$ to zero and determine the frequencies at which plots intersects the imaginary axis and Calculate the value of $G(j\omega)$ at the point of intersection by substituting the determined value of frequency in the rationalized expression of $G(j\omega)$.

Step 8 :- Sketch the polar plot with the help of above information.

Type 'zero' system :-

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$

Step 1 :- Put $s = j\omega$

$$G(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$G(j\omega) = \frac{K}{\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}}$$

Step 2 :- Taking the limit for the magnitude of $G(j\omega)$.

$$\lim_{\omega \to 0} |G(j\omega)| = \lim_{\omega \to 0} \frac{K}{\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} = K$$

$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{K}{\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} = 0$$

Step 3 :- Taking the limit for the phase angle of $G(j\omega)$.

$$\lim_{\omega \to 0} \angle G(j\omega) = \lim_{\omega \to 0} \angle -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = 0$$

$$\lim_{\omega \to \infty} \angle G(j\omega) = \lim_{\omega \to \infty} \angle -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = -180^\circ$$

Step 4 :- Separating the real and imaginary part of $G(j\omega)$.

$$G(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)} \cdot \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1-j\omega T_1)(1-j\omega T_2)}$$

$$G(j\omega) = \frac{K(1-\omega^2 T_1 T_2)}{1+\omega^2 T_1^2 + \omega^2 T_2^2 + \omega^4 T_1^2 T_2^2}$$

$$- j \frac{K\omega(T_1 + T_2)}{1+\omega^2 T_1^2 + \omega^2 T_2^2 + \omega^4 T_1^2 T_2^2}$$

Equating the real part to zero.

$$\frac{K(1-\omega^2 T_1 T_2)}{1+\omega^2 T_1^2+\omega^2 T_2^2+\omega^4 T_1 T_2}=0$$

$$\omega^2=\frac{1}{T_1 T_2} \text{ or } \omega=\frac{1}{\sqrt{T_1 T_2}} \text{ \& } \omega=\pm\infty.$$

The frequency at which plot intersects the imaginary axis is $\frac{1}{\sqrt{T_1 T_2}}$.

→ for positive value of frequencies the polar plot intersects the imaginary axis at $\omega=\infty$

$$\omega=\frac{1}{\sqrt{T_1 T_2}} \text{ and } \omega=\infty$$

Value of $G(j\omega)$ when

$$\omega=\frac{1}{\sqrt{T_1 T_2}}$$

$$G(j\omega)=0-j\frac{K\cdot\frac{1}{\sqrt{T_1 T_2}}(T_1+T_2)}{1+\frac{1}{T_1 T_2}\cdot T_2^2+\frac{1}{T_1 T_2}T_1^2+\frac{1}{T_1^2 T_2^2}\cdot T_1^2 T_2^2}$$

$$=-j\frac{K\frac{T_1+T_2}{\sqrt{T_1 T_2}}}{2+\frac{T_2}{T_1}+\frac{T_1}{T_2}}=-j\frac{K\cdot\frac{T_1+T_2}{\sqrt{T_1 T_2}}}{\frac{(T_1+T_2)^2}{T_1 T_2}}.$$

$$=-j\frac{K\sqrt{T_1 T_2}}{T_1+T_2}$$

$$|G(j\omega)|=\frac{K\sqrt{T_1 T_2}}{T_1+T_2} \text{ and } \angle G(j\omega)=-90^\circ$$

∴ when $\omega=\frac{1}{\sqrt{T_1 T_2}}$ $\quad G(j\omega)=\frac{K\sqrt{T_1 T_2}}{T_1+T_2}\angle -90^\circ$

$\omega=\infty$ $\quad G(j\omega)=0\angle -180^\circ$

Step 5 :- Equating the imaginary part to zero.

$$\frac{K\omega(T_1+T_2)}{1+\omega^2 T_1^2+\omega^2 T_2^2+\omega^4 T_1 T_2}=0$$

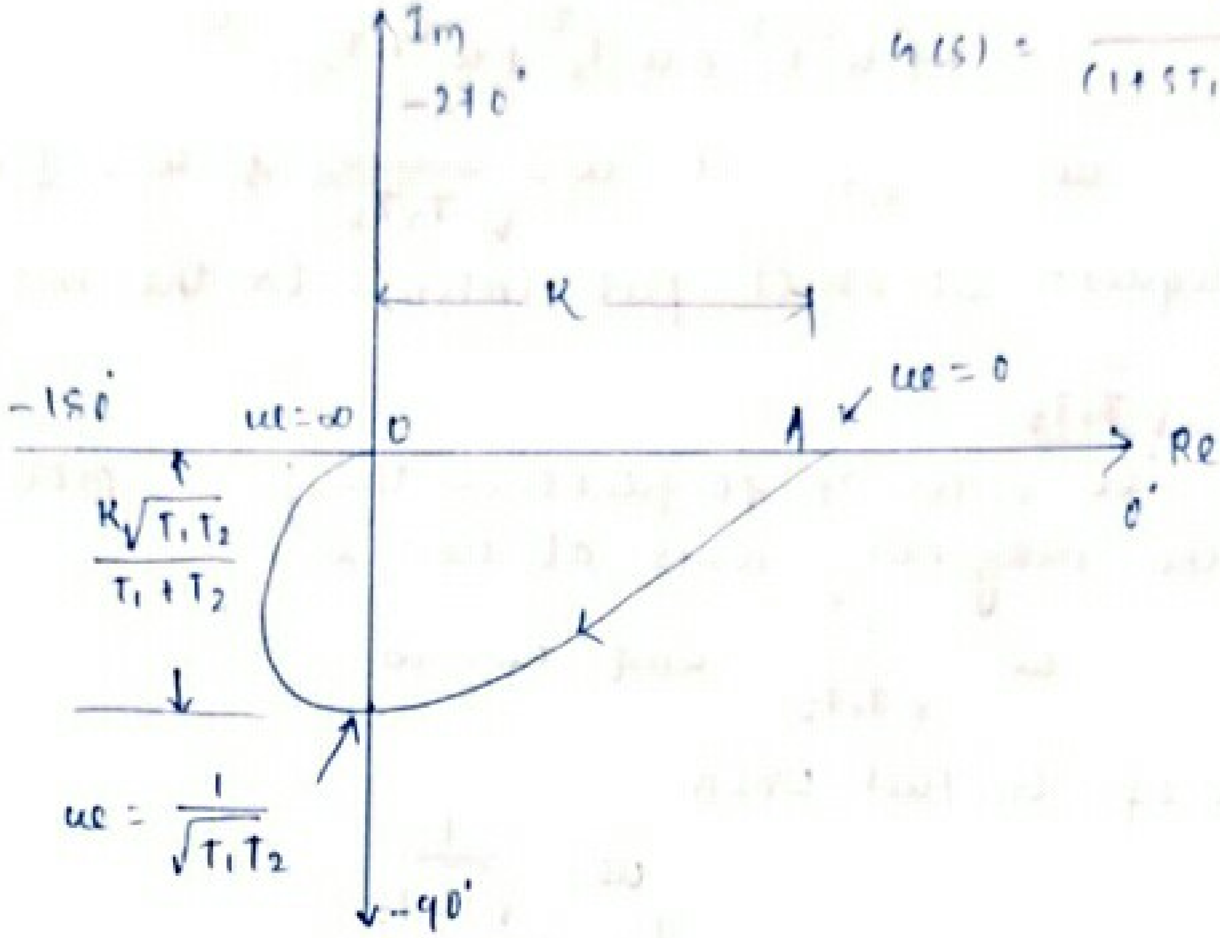
∴ when $\omega=0$ and $\omega=\pm\infty$

$\omega=0$ $\quad |G(j\omega)|=K$ $\quad \angle G(j\omega)=0^\circ$

$\omega=\infty$ $\quad |G(j\omega)|=0$ $\quad \angle G(j\omega)=0^\circ$

The coordinates of the intersection points 'A' & 'O'

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$



2. Type one system.

$$G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

Step 1 : Put $s = j\omega$

$$G(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$G(j\omega) = \frac{K}{\omega\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}}$$

$$G(j\omega) = \angle -90° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$$

Step 2: Taking the limit for the Magnitude of $G(j\omega)$

$$\lim_{\omega\to 0} |G(j\omega)| = \lim_{\omega\to 0} \frac{K}{\omega\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} = \infty$$

$$\lim_{\omega\to \infty} |G(j\omega)| = \lim_{\omega\to \infty} \frac{K}{\omega\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} = 0$$

Step 3 :- Taking the limit for the phase angle of $G(j\omega)$.

$$\lim_{\omega \to 0} \angle G(j\omega) = \lim_{\omega \to 0} \angle -90^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = -90^\circ$$

$$\lim_{\omega \to \infty} \angle G(j\omega) = \lim_{\omega \to \infty} \angle -90^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = -270^\circ$$

Step 4 :- Separating the real and imaginary parts.

$$G(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{-\omega K(T_1+T_2)}{\omega + \omega^3(T_1^2 + T_2^2 + \omega^2 T_1^2 T_2^2)}$$

$$+ \frac{j(K\omega^2 T_1 T_2 - K)}{\omega + \omega^3(T_1^2 + T_2^2 + \omega^2 T_1^2 T_2^2)} \quad \text{---}(A)$$

Equate the imaginary part equal to zero.

$$\frac{K\omega^2 T_1 T_2 - K}{\omega + \omega^3(T_1^2 + T_2^2 + \omega^2 T_1^2 T_2^2)} = 0$$

$$\therefore \omega = \frac{1}{\sqrt{T_1 T_2}} = \frac{1}{\sqrt{T_1 T_2}} \;\&\; \omega = \pm\infty$$

The frequency at which the point of intersection on real axis is $\frac{1}{\sqrt{T_1 T_2}}$. Now calculate the value of $G(j\omega)$ at this point.

Put $\omega = \frac{1}{\sqrt{T_1 T_2}}$ is equation (A)

$$G(j\omega) = -K\frac{T_1 T_2}{T_1 + T_2} \quad \angle G(j\omega) = 0^\circ$$

$$G(j\omega) = \infty \quad \angle G(j\omega) = 0^\circ$$

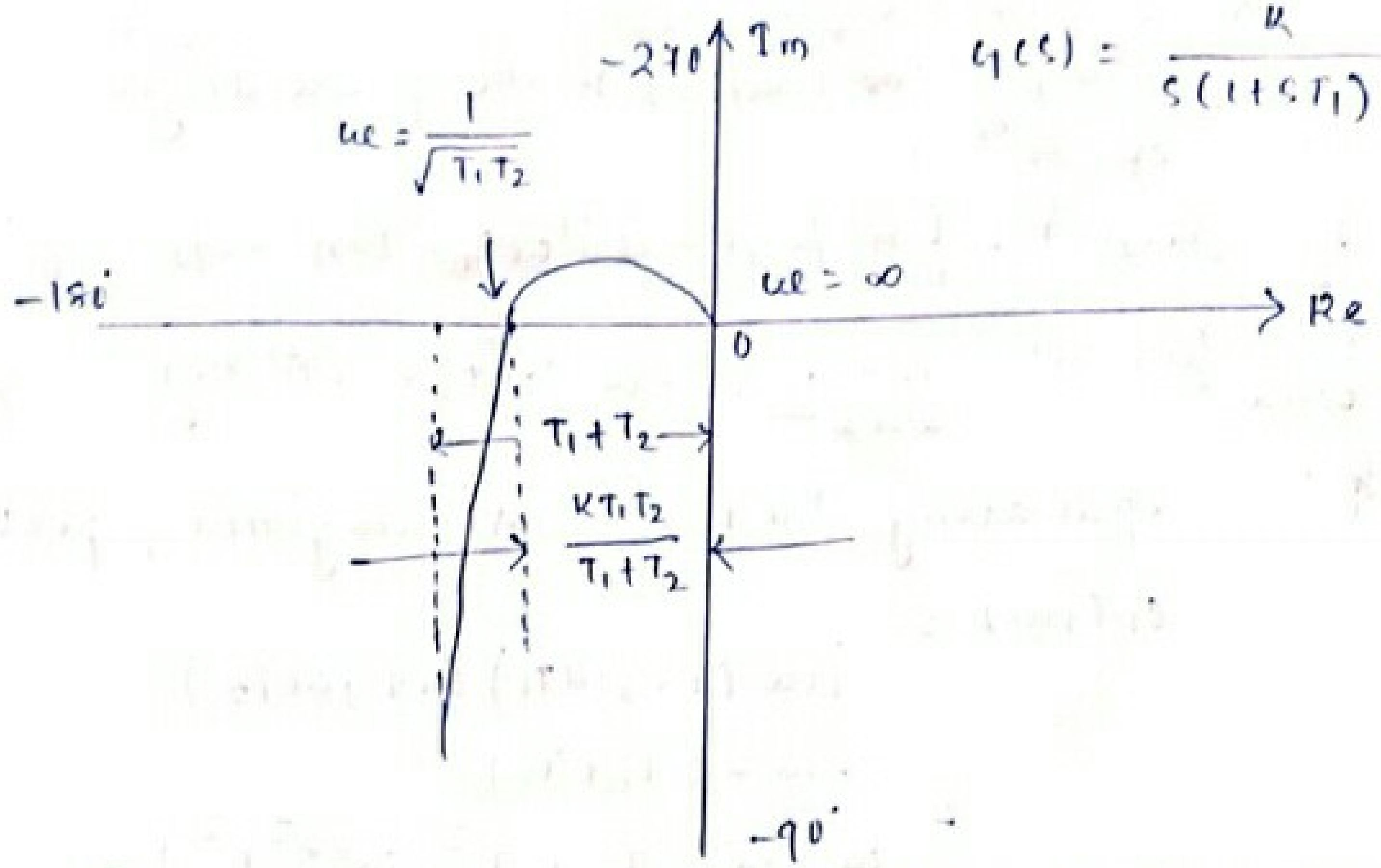
Step 5 :- Equate the real part to zero.

$$\frac{-\omega K(T_1+T_2)}{\omega + \omega^3(T_1^2 + T_2^2 + \omega^2 T_1 T_2)} = 0$$

$$\therefore \omega = \infty$$

For positive values of frequencies the polar plot intersects the imaginary axis at $\omega = \infty$.

$$\therefore G(j\omega) = 0 \quad \angle -270^\circ$$



$$G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

From the polar plot it is clear that in type one system the $j\omega$ term in denominator contributes $-90°$ to the total phase angle. At $\omega = 0$ the magnitude is infinity and phase angle $-90°$. At $\omega = \infty$ the magnitude becomes zero and curve converges to origin. At low frequency, the polar plot is asymptotic to a line parallel to negative imaginary axis.

Type TWO System

$$G(s) = \frac{K}{s^2 (1+sT_1)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{K}{(j\omega)^2 (1 + j\omega T_1)}$$

$$= \frac{K}{-\omega^2 \sqrt{1 + (\omega T_1)^2}}.$$

$$= \angle -180° - \tan^{-1} \omega T_1$$

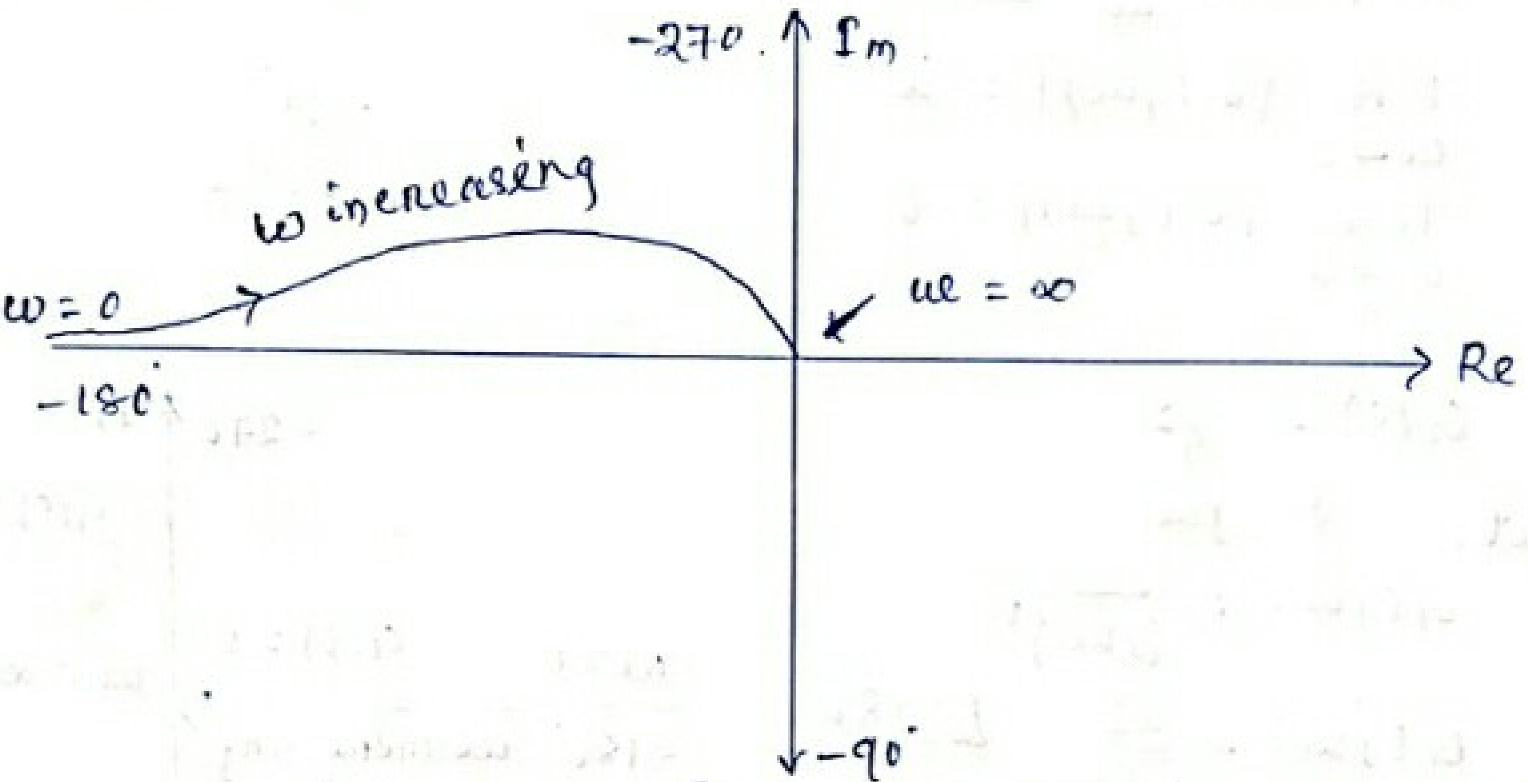
$$\lim_{\omega \to 0} |G(j\omega)| = \lim_{\omega \to 0} \frac{K}{-\omega^2 \sqrt{1 + (\omega T_1)^2}} = \infty$$

$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{K}{-\omega^2 \sqrt{1+(\omega T_1)^2}} = 0$$

$$\lim_{\omega \to 0} \angle -180^\circ - \tan^{-1} \omega T_1 = -180^\circ$$

$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{K}{-\omega^2 \sqrt{1+(\omega T_1)^2}}$$

$$= \lim_{\omega \to \infty} \angle -180^\circ - \tan^{-1} \omega T_1 = -270^\circ$$



The presence of s^2 in the denominator contributes a constant -180° to the angle of $G(j\omega)$ for all frequencies.

The polar plot is a smooth curve whose angle decreased continously from -180° to -270°.

From the polar plot it is clear that at $\omega = 0$, magnitude is infinity and phase angle -180°, at $\omega = \infty$ magnitude is zero and at low frequencies the polar is asymptotic to a line parallel to negative real axis.

Polar plots of some standard type function

1. $G(s) = \frac{1}{s}$

Put. $s = j\omega$

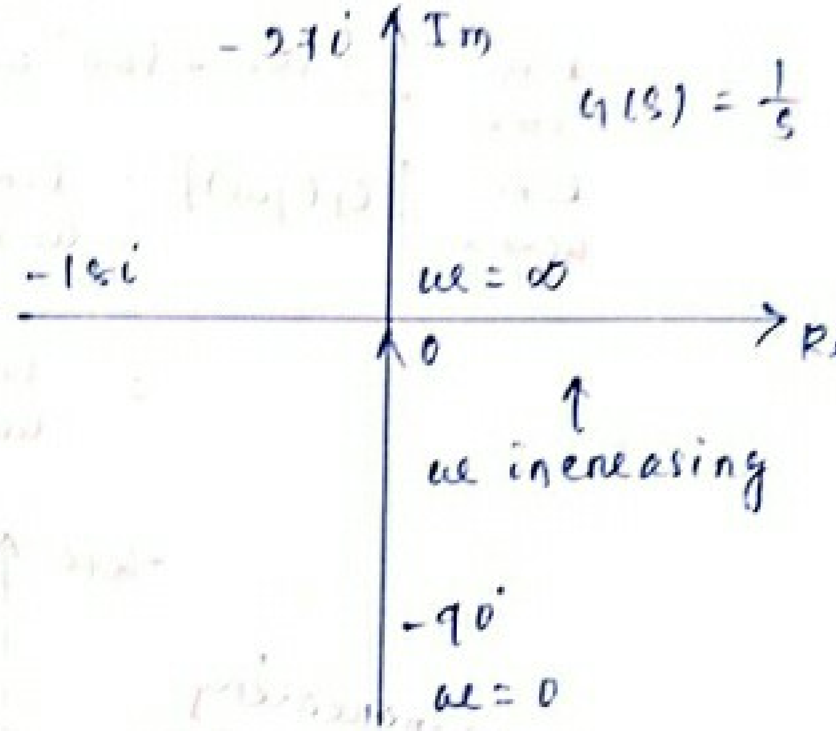
$G(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega}$

$= \angle -\tan^{-1}\infty$

$G(j\omega) = \frac{1}{\omega} \angle -90^\circ$

$\lim_{\omega \to 0} |G(j\omega)| = \infty$

$\lim_{\omega \to \infty} |G(j\omega)| = 0$



2. $G(s) = \frac{1}{s^2}$

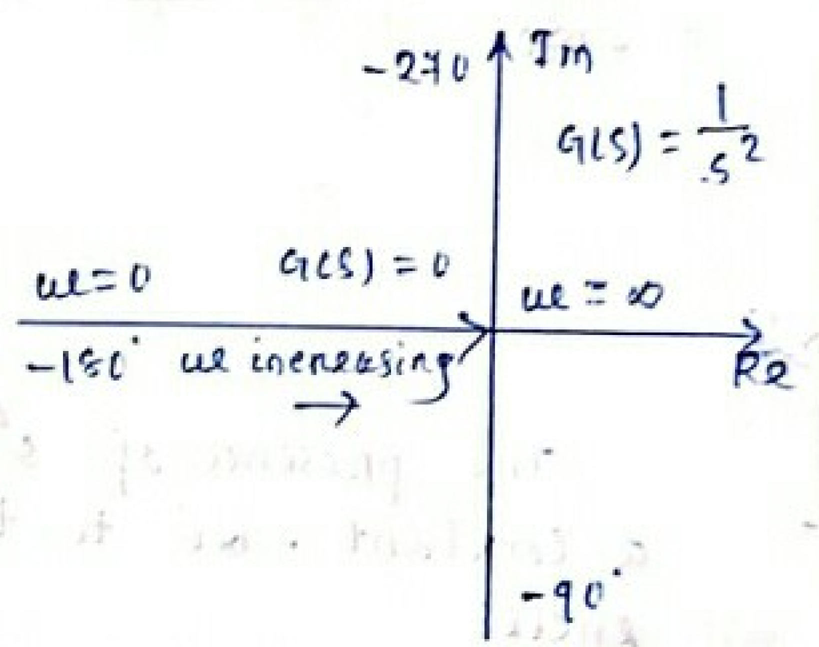
Put, $s = j\omega$

$G(j\omega) = \frac{1}{(j\omega)^2}$

$G(j\omega) = \frac{1}{-\omega^2} \angle -180^\circ$

$\omega = 0 \quad G(j\omega) = -\infty$

$\omega = \infty \quad G(j\omega) = 0$



3. $G(s) = \frac{1}{s+1}$

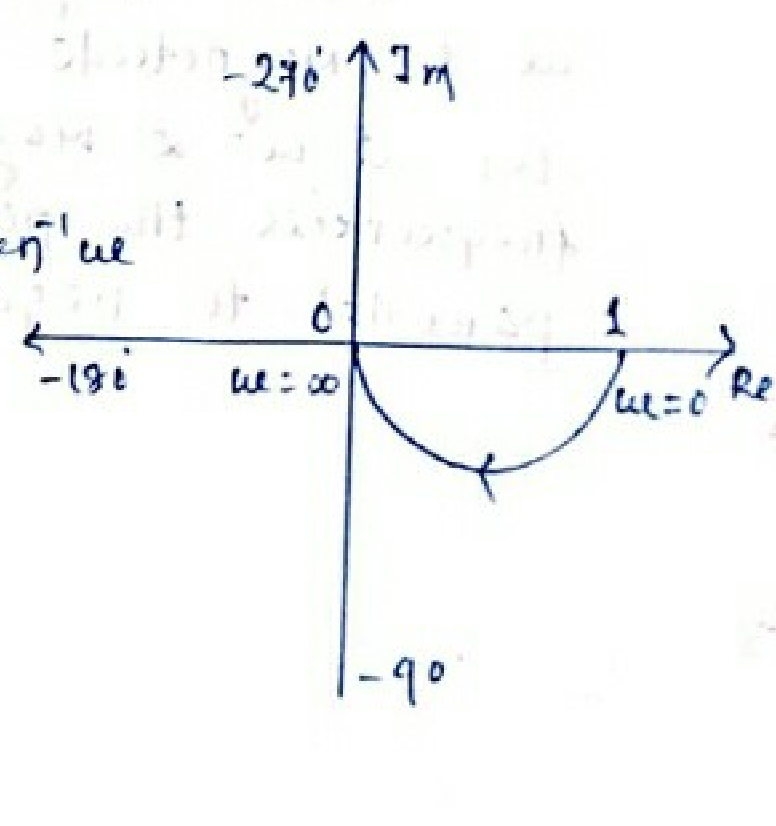
Put, $s = j\omega$

$G(j\omega) = \frac{1}{1 + j\omega}$

$= \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$

$\lim_{\omega \to 0} G(j\omega) = 1 \angle 0^\circ$

$\lim_{\omega \to \infty} G(j\omega) = 0 \angle -90^\circ$



Example - 1. Sketch the polar plot for

$$G(s) = \frac{1}{s(s+1)}$$

Sol^n:- $G(s) = \frac{1}{s(s+1)}$

Put, $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)}$$

$$G(j\omega) = \frac{1}{\omega\sqrt{1+\omega^2}} \angle -90^\circ - \tan^{-1}\omega .$$

Applying the limit for the phase angle of $G(j\omega)$

$$\lim_{\omega \to 0} |G(j\omega)| = \lim_{\omega \to 0} \frac{1}{\omega\sqrt{1+\omega^2}} = \infty$$

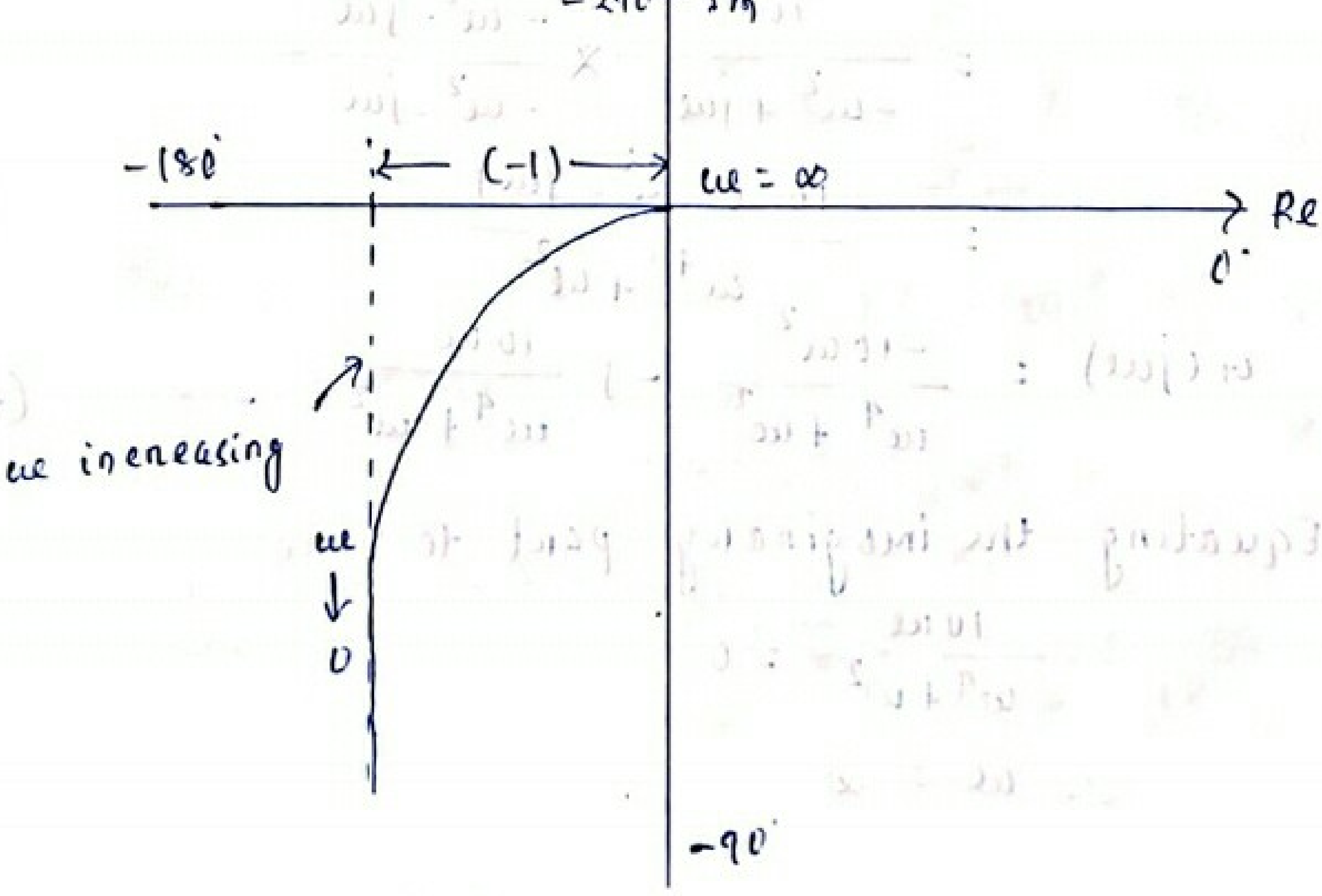
$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{1}{\omega\sqrt{1+\omega^2}} = 0$$

Taking the limit for the phase angle of $G(j\omega)$

$$\lim_{\omega \to 0} \angle G(j\omega) = \lim_{\omega \to 0} \angle -90^\circ - \tan^{-1}\omega = -90^\circ$$

$$\lim_{\omega \to \infty} \angle G(j\omega) = \lim_{\omega \to \infty} \angle -90^\circ - \tan^{-1}\omega = -180^\circ$$

The plot is asymptotic to the vertical line passing through the point $(-1, 0)$.



Example 2 Sketch the polar plot of

Sol^n :-
$$G(s) = \frac{10}{s(s+1)}$$

Put. $s = j\omega$

$$G(j\omega) = \frac{10}{j\omega(j\omega+1)} = \frac{10}{\omega\sqrt{\omega^2+1}} \angle -90^\circ - \tan^{-1}\omega$$

Taking limit for the magnitude of $G(j\omega)$.

$$\lim_{\omega \to 0} |G(j\omega)| = \lim_{\omega \to 0} \frac{10}{\omega\sqrt{\omega^2+1}} = \infty$$

$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{10}{\omega\sqrt{\omega^2+1}} = 0$$

Taking limit for the phase angle of $G(j\omega)$.

$$\lim_{\omega \to 0} \angle G(j\omega) = \lim_{\omega \to 0} \angle -90^\circ - \tan^{-1}\omega = -90^\circ$$

$$\lim_{\omega \to \infty} \angle G(j\omega) = \lim_{\omega \to \infty} \angle -90^\circ - \tan^{-1}\omega$$

$$= -90^\circ - 90^\circ = -180^\circ$$

Separate the real and imaginary part of $G(j\omega)$

$$G(j\omega) = \frac{10}{j\omega(j\omega+1)} = \frac{10}{-\omega^2 + j\omega}$$

$$= \frac{10}{-\omega^2 + j\omega} \times \frac{-\omega^2 - j\omega}{-\omega^2 - j\omega}$$

$$= \frac{10(-\omega^2 - j\omega)}{\omega^4 + \omega^2}$$

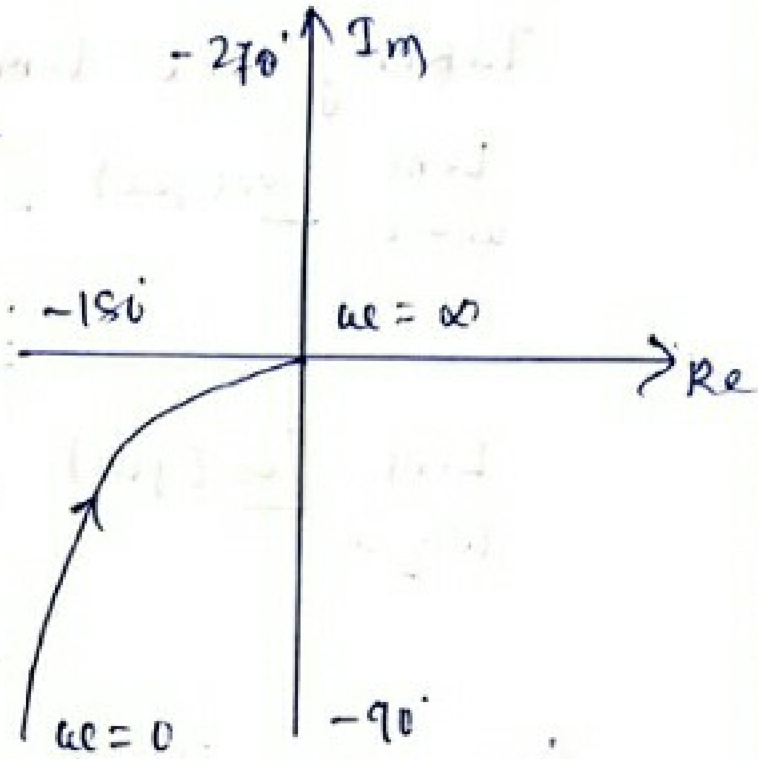
$$G(j\omega) = \frac{-10\omega^2}{\omega^4 + \omega^2} - j\frac{10\omega}{\omega^4 + \omega^2} \quad \text{----- (A)}$$

Equating the imaginary part to zero.

$$\frac{10\omega}{\omega^4 + \omega^2} = 0$$

$\therefore \omega = \infty$

Put $\omega = \infty$ in equation (A)
$G(j\omega) = 0$, i.e. plot intersect the real axis at the origin.
Equating the real part to zero.



$$\frac{-10\,\omega^2}{\omega^4 + \omega^2} = 0$$

or $$\frac{-10}{\omega^2 + 1} = 0$$

$\therefore \quad \omega = \infty$

Put $\omega = \infty$ in equation (A)
$G(j\omega) = 0$ i.e. the plot intersects the imaginary axis at the origin. The required polar plot.

Example 3

Sketch the polar plot for

$$G(s) = \frac{20}{s(s+1)(s+2)}$$

Sol^n :- Put $s = j\omega$

$$\therefore \quad G(j\omega) = \frac{20}{j\omega(1+j\omega)(j\omega+2)}$$

$$= \frac{20}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

Taking the limit for the magnitude of $G(j\omega)$

$$\lim_{\omega \to 0} |G(j\omega)| = \lim_{\omega \to 0} \frac{20}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}} = \infty$$

$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{20}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}} = 0$$

Taking the limit for the phase angle of $G(j\omega)$

$$\lim_{\omega \to 0} \angle G(j\omega) = \lim_{\omega \to 0} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}\omega/2$$

$$= -90^\circ$$

$$\lim_{\omega \to \infty} \angle G(j\omega) = \lim_{\omega \to \infty} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$= -270^\circ$$

Squaring the real and imaginary part of $G(j\omega)$

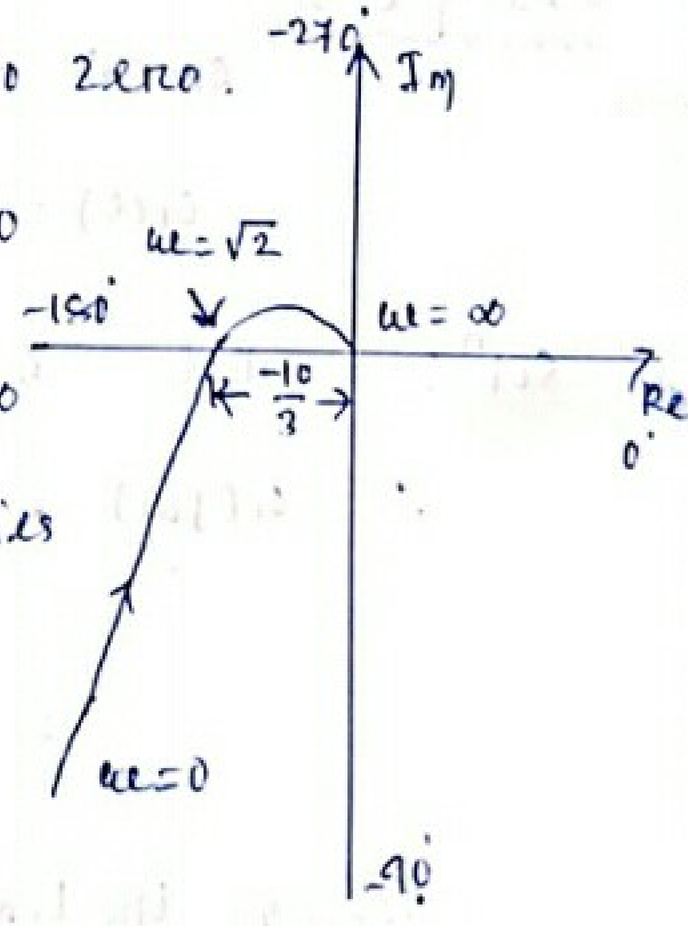
$$G(j\omega) = \frac{20}{j\omega(1+j\omega)(2+j\omega)}$$

$$G(j\omega) = \frac{-60\omega^2}{(\omega^4+\omega^2)(4+\omega^2)} + j\frac{20(\omega^3-2\omega)}{(\omega^4+\omega^2)(4+\omega^2)}$$

Equating the imaginary part to zero.

$$\frac{20(\omega^2-2\omega)}{(\omega^4+\omega^2)(4+\omega^2)} = 0$$

$$\therefore \ \omega = \pm\sqrt{2} \text{ and } \omega = \pm\infty$$



for the positive values of frequencies polar plot intersects the real axis at $\omega = \sqrt{2}$ and $\omega = \infty$ for which $G(j\omega) = \frac{-10}{3} \angle 0^\circ$ and $0 \angle 0^\circ$

Equating the real part to zero

$$\frac{-60\omega^2}{(\omega^4+\omega^2)(4+\omega^2)} = 0$$

$$\therefore \ \omega = \infty$$

The polar plot intersect the imaginary axis at $\omega = \infty$ for which $G(j\omega) = 0 \angle -270^\circ$

The required polar plot is shown in figure.

BODE PLOT

Bode plot is a graphical representation of the transfer function for determining the stability of the control system. Bode plot consists of two separate plots. One is a plot of the logarithm of the magnitude of a sinusoidal transfer function, the other is a plot of the phase angle, both plots are plotted against the frequency. The curve are drawn on semilog paper, using the log scale for frequency and linear scale for magnitude or phase angle (in degree). The magnitude is represented in decibles. Thus, Bode plot consists of

(i) $20\log_{10} |G(j\omega)|$ Vs $\log\omega$.

(ii) Phase shift Vs $\log\omega$.

The main advantage of using Bode plot is that multiplication of magnitudes can be converted into addition.

Consider open loop transfer function of a closed loop control system.

$$G(S)\cdot H(S) = \frac{K(1+ST_a)(1+ST_b)\cdots}{S^N(1+ST_1)(1+ST_2)\cdots}$$

Put, $s = j\omega$

$$G(j\omega)\cdot H(j\omega) = \frac{K(1+j\omega T_a)(1+j\omega T_b)\cdots}{(j\omega)^N(1+j\omega T_1)(1+j\omega T_2)\cdots}$$

$$\begin{aligned}20\log_{10}|G(j\omega)H(j\omega)| = &\left(20\log K + 20\log\sqrt{1+\omega^2T_a^2}\right. \\ &\left.+ 20\log\sqrt{1+\omega^2T_b^2}\right)\cdots \\ &-\left(20N\log\omega + 20\log\sqrt{1+\omega^2T_1^2}\right. \\ &\left.+ 20\log\sqrt{1+\omega^2T_2^2}\right)\cdots\end{aligned}$$

Hence, in order to get $|G(j\omega) H(j\omega)|$ we will have to obtain the individual plot and adding individual components, the resultant can be obtained, suppose, $H(s) = 1$.

Case 1, The Gain K.

$$G(s) = K$$

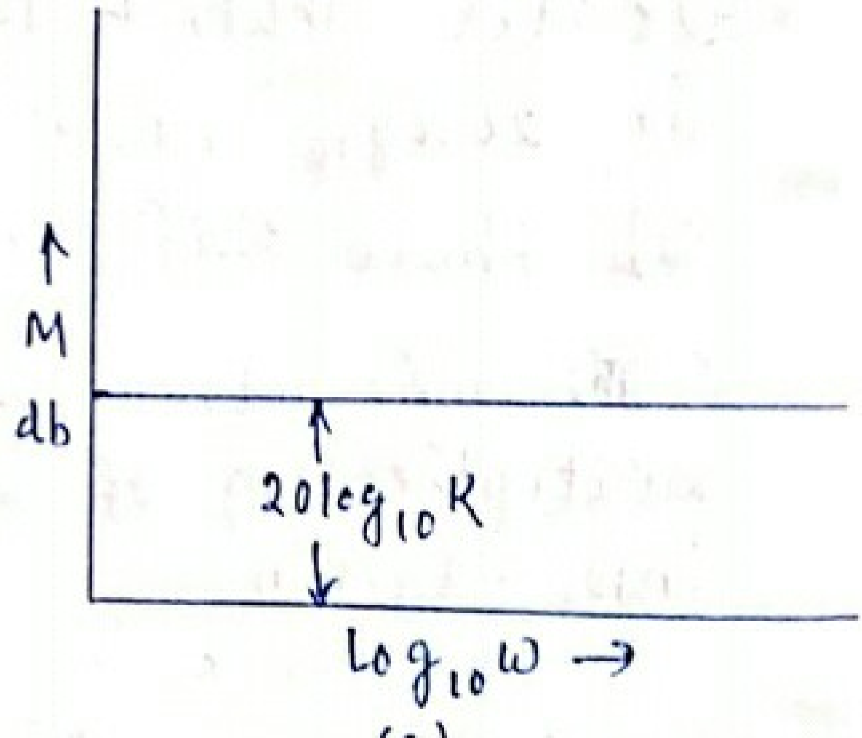
Put, $s = j\omega$

$$G(j\omega) = K$$

$$20\log_{10} |G(j\omega)| = 20\log_{10} K \quad \text{- - - - (1)}$$

$$\text{Phase angle } \phi = \angle G(j\omega) = 0^\circ \quad \text{- - - - (2)}$$

From equation (1) and (2) it is clear that the magnitude is independent of $\log_{10}\omega$ and phase angle always zero. Th plots are shown in fig.



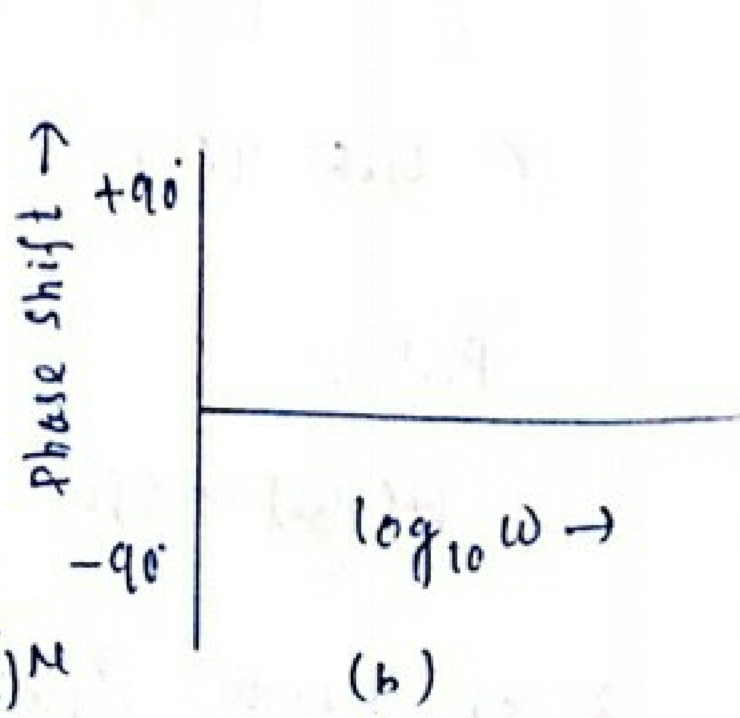
(a)

Case 2:

$$G(s) = \frac{1}{s^N}$$

Put. $s = (j\omega)$

$$G(j\omega) = \frac{1}{(j\omega)^N}$$



(b)

$$20\log_{10} |G(j\omega)| = 20\log_{10} \frac{1}{(j\omega)^N}$$

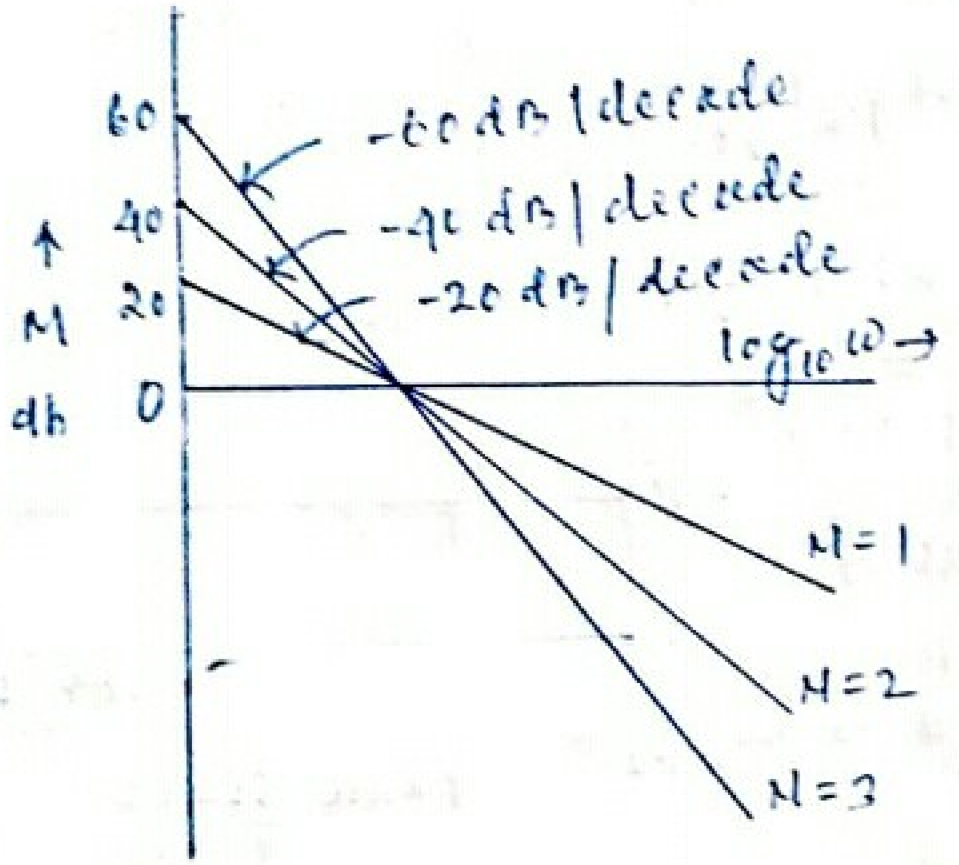
$$= 20\log_{10} (j\omega)^{-N}$$

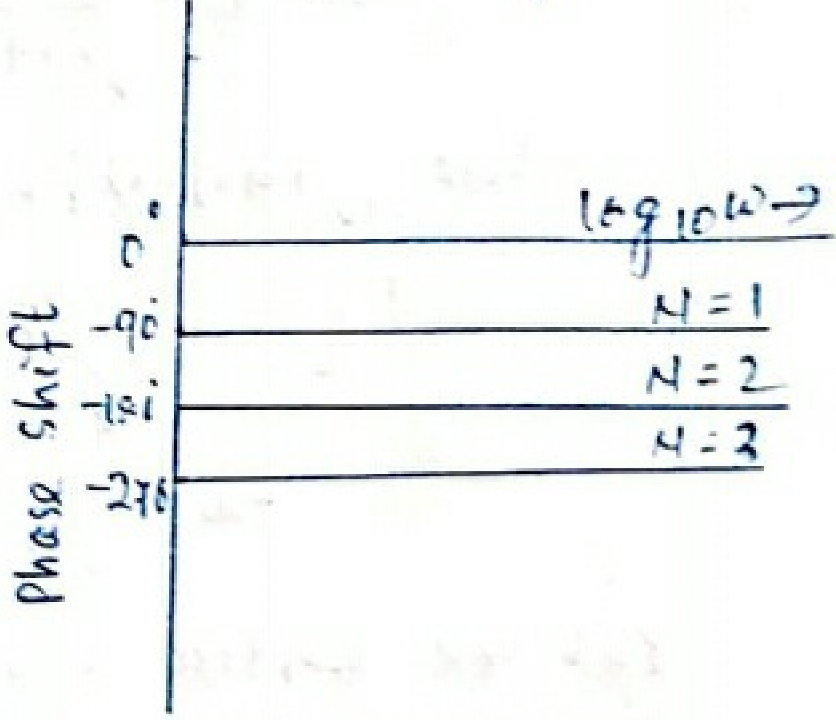
$$= -20 \, N \log_{10} (\omega)$$

$$\angle G(j\omega)^N = -90N^\circ$$

Where, $N = 1, 2, 3, \ldots$

The plot M vs $\log_{10}\omega$ is a straight line. for $N = 1$ the line has a slope of -20 db/decade and angle $-90°$. for $N = 2$, the slope of the line will be ~~-40 db~~ -40 db/decade and angle will be $-180°$ and so on.





Case 3 :

$$G(s) = s$$

Put, $s = j\omega$

$$G(j\omega) = (j\omega)$$

$$M = 20\log_{10} |G(j\omega)|$$

$$= 20\log_{10}\omega$$

$$\angle G(j\omega) = +90°$$

The plot M vs $\log_{10}\omega$ is a straight line having a slope of $+20$ dB/decade and angular phase shift of $+90°$.

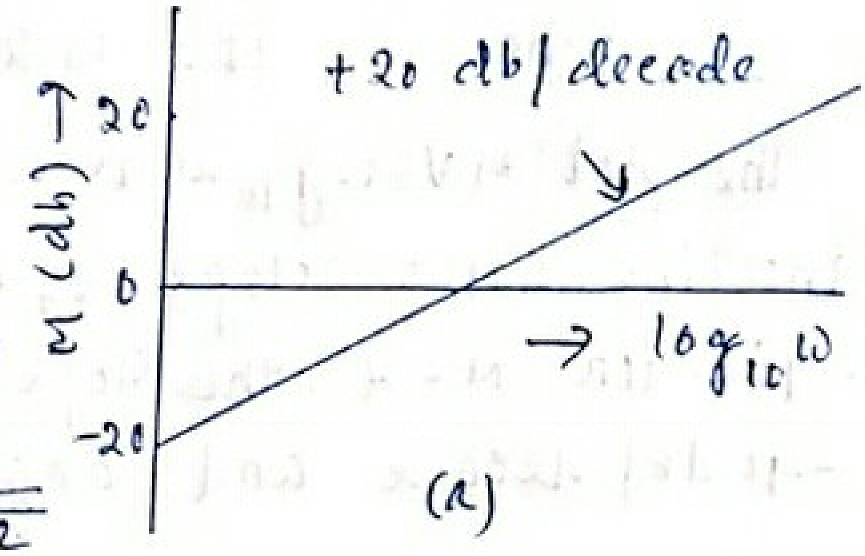
Case 4 :

$$G(s) = \frac{1}{1+sT}$$

Put, $s = j\omega$

$$\therefore \; G(j\omega) = \frac{1}{1+j\omega T}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2T^2}}$$



(a)

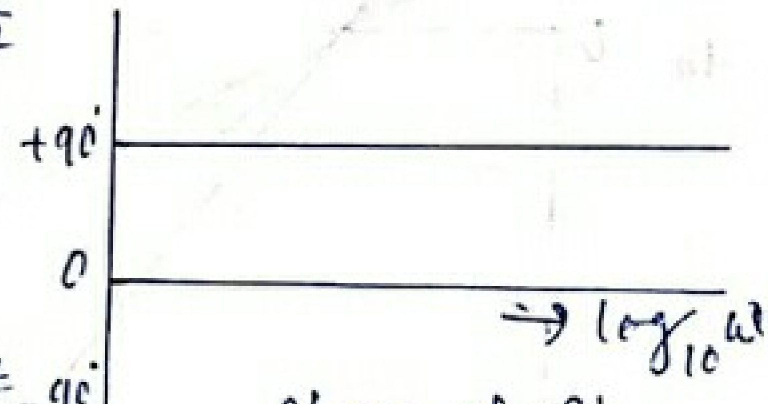
$$20\log_{10}|G(j\omega)| = 20\log_{10}\left[\frac{1}{\sqrt{1+\omega^2T^2}}\right]$$

$$= 20\log_{10} 1 - 20\log_{10}\sqrt{1+\omega^2T^2}$$

$$= -20\log_{10}\sqrt{1+\omega^2T^2}$$

Put the different value of ω, we will get $|G(j\omega)|$

Consider following two ~~cause~~ cases,



Phase shift.

(a) for $\omega T << 1$ (very low frequencies)

$$-20\log_{10}\sqrt{1+\omega^2T^2} = -20\log_{10}\sqrt{1} = 0$$

$\therefore$ $M = 0$ for $\omega T << 1$ or $\omega \le \frac{1}{T}$

(b) for $\omega T << 1$ (very high frequencies)

$$-20\log_{10}\sqrt{1+\omega^2T^2} = -20\log_{10}\sqrt{\omega^2T^2}$$

$$= -20\log_{10}\omega T$$

$\{$ for $\omega >> 1/T$ $\}$

Hence, M Vs $\log_{10}\omega$ has two parts.

(i) one part has having $M = 0$ for $\omega << \frac{1}{T}$

(ii) In other part M varies as a straight line with slope of $-20\,db/decade$ for $\omega \gg \frac{1}{T}$.

$\omega = \frac{1}{T}$ is called break frequency or corner frequency.

$$M = -20\log_{10}\omega T = -20(\log_{10}\omega + \log_{10}T)$$

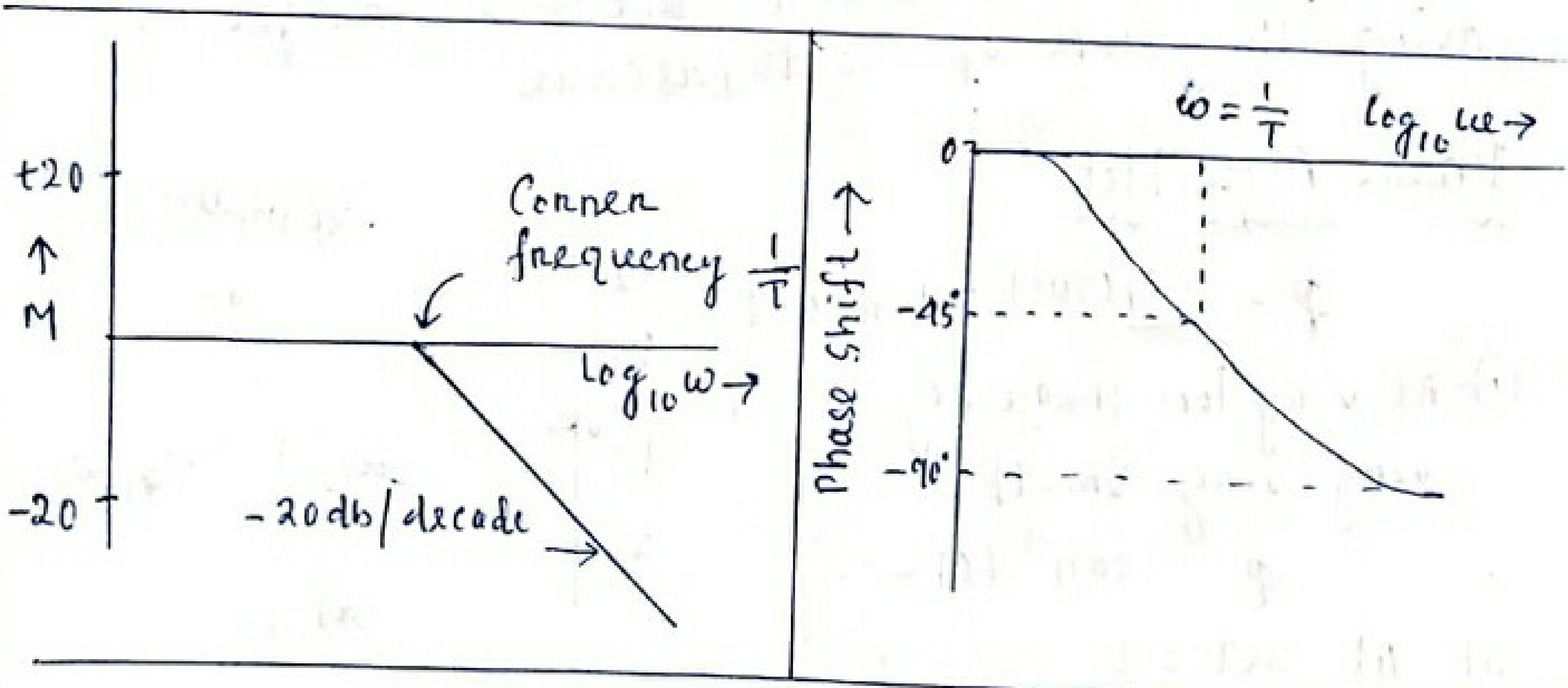
$$M = -20\log_{10}\omega - 20\log_{10}T.$$

$$= -20\log_{10}\omega + 20\log\frac{1}{T}. \quad \text{——— (1)}$$

The above two parts of the graph intersect 0 db axis is determined by equating the eq^n. (1) to zero.

$$0 = -20\log_{10}\omega + 20\log_{10}\frac{1}{T}$$

$\omega = \frac{1}{T}$ is called break frequency.



Case 5 :

$$G(s) = (1 + sT)$$

Put, $s = j\omega$

$$G(j\omega) = (1 + j\omega T)$$

$$|G(j\omega)| = \sqrt{1 + \omega^2 T^2}$$

$$20\log_{10}|G(j\omega)| = 20\log_{10}\sqrt{1 + \omega^2 T^2}$$

(i) When $\omega T << 1$

$$M = 20\log_{10}\sqrt{1} = 0 \text{ db}$$

(ii) When $\omega T >> 1$

$$M = 20\log_{10}\omega T$$

$$M = 20\log_{10}\omega T = 20\log_{10}\frac{\omega}{1/T}$$

$$= 20\log_{10}\omega - 20\log_{10}\frac{1}{T}$$

Equate the above equation to zero

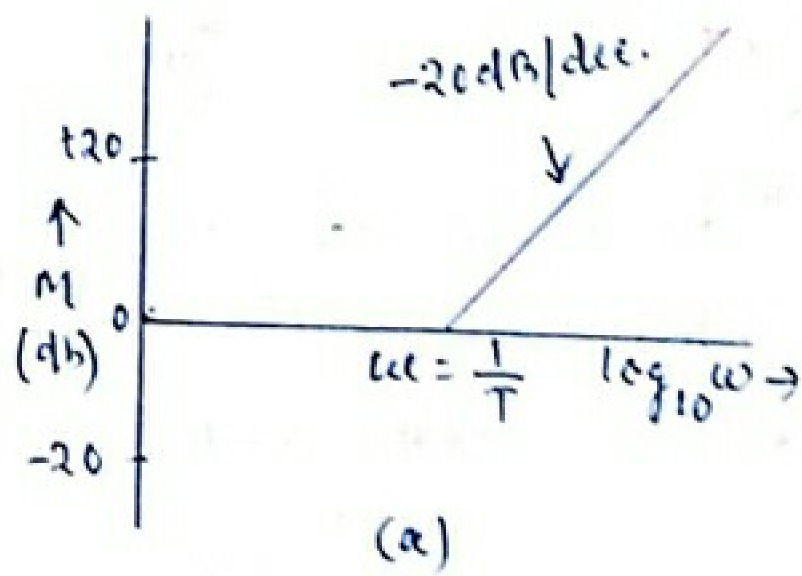
$$0 = \cancel{20\log}\ 20\log_{10}\omega - 20\log_{10}\frac{1}{T}$$

$$\omega = \frac{1}{T} \text{ Corner frequency.}$$

Thus the two part of the graph intersects the '0' db axis at $\omega = \frac{1}{T}$. The second part is a straight line having the slope of -20 db/decade

Phase Angle Plot

$$\phi = \angle G(j\omega) = \tan^{-1}\omega T$$



(a)

(i) At very low frequency ωT is very very small.

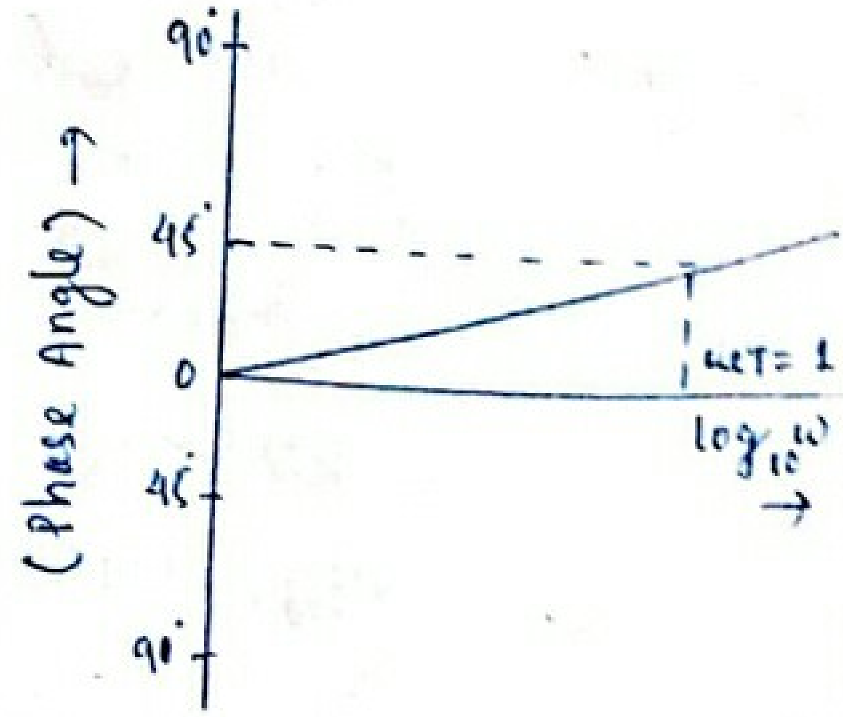
$$\phi = \tan^{-1}(0) = 0°$$

(ii) At $\omega T = 1$

$$\phi = \tan^{-1}(1) = 45°$$

(iii) At very high frequencies

$$\phi = \tan^{-1}(\infty) = 90°$$



Thus the value of ϕ gradually changes from 0° to 90° as ω increases from 0 to very high values.

Case 6 :- General second order system.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Put, $s = j\omega$

$$G(s) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n (j\omega) + \omega_n^2} = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega_n\omega + \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\xi\omega_n \cdot \omega} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}$$

$$20\log_{10}|G(j\omega)| = 20\log_{10}\left|\frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}\right|$$

$$= -20\log_{10}\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}$$

Suppose $\left(\frac{\omega}{\omega_n}\right) = u$

$$\therefore\ 20\log_{10}|G(j\omega)| = M = -20\log_{10}\sqrt{(1-u^2)^2 + 4\xi^2 u^2}$$

Consider the two cases

1. $u << 1$ i.e. $\frac{\omega}{\omega_n} << 1$

$$M = -20\log_{10}\sqrt{1} = 0\,\text{db}.$$

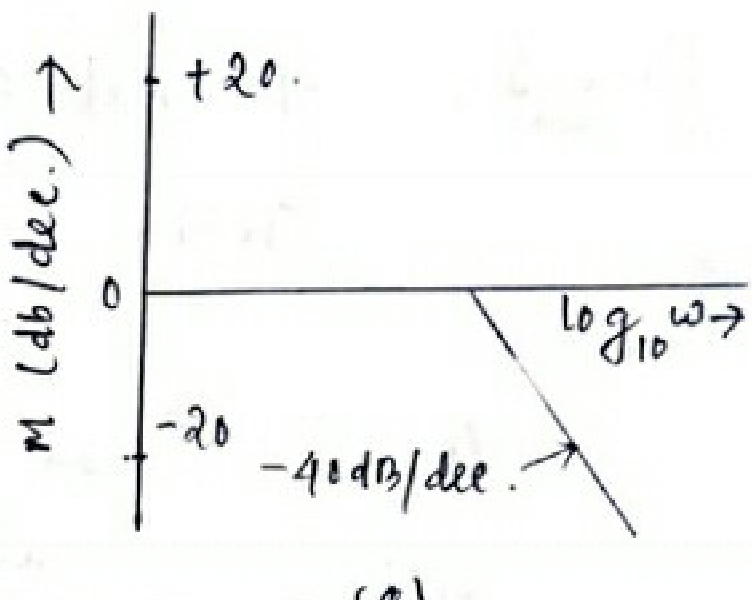
2. $u >> 1$ i.e. $\frac{\omega}{\omega_n} >> 1$

$$M = -20\log_{10}\sqrt{(u^2)^2}$$
$$= -20\log_{10} u^2$$
$$= -40\log_{10} u$$

So, it a straight line having slope of -40 db/dec and passing through the point u. Therefore, the asymptotic plot consists of.

(i) $M = 0$ $\quad u << 1$

(ii) $M = -40 \log_{10} u$ $\quad u >> 1$



(a)

Phase Angle Plot

$$\phi = \angle G(ju) = -\tan^{-1} \frac{2\xi u}{1-u^2}$$

(i) For small value of u, u^2 is small.

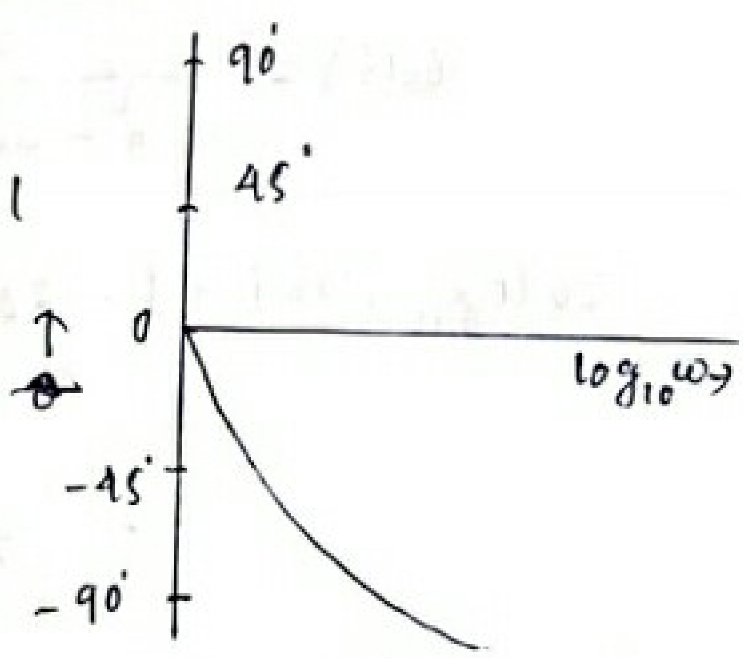
$$\therefore \quad \phi = -\tan^{-1} 2\xi u$$

(ii) For large value of u, $u^2 >> 1$

$$\therefore \quad \phi = +\tan^{-1} \frac{2\xi}{u}$$

(iii) When, $u = 1$

$$\phi = -\tan^{-1} \infty = -90^\circ$$



(b)

Initial Slope of Bode Plot

Let. $\quad G(s)\,H(s) = \dfrac{K}{s^N}$

Put, $\quad s = ju$

$$G(ju) \cdot H(ju) = \frac{K}{(ju)^N}$$

$$20\log_{10} |G(ju) \cdot H(ju)| = 20\log_{10} \left| \frac{K}{(ju)^N} \right|$$

$$= 20\log_{10} K - 20N \log_{10} \omega \quad \text{—(1)}$$

1. For, $N = 0$ $\quad$ (Type zero system)

$$20 \log_{10} |G(ju)\,H(ju)| = 20\log_{10} K.$$

This is a straight line. The graph is shown in figure.

2. For $N = 1$ (Type one system)

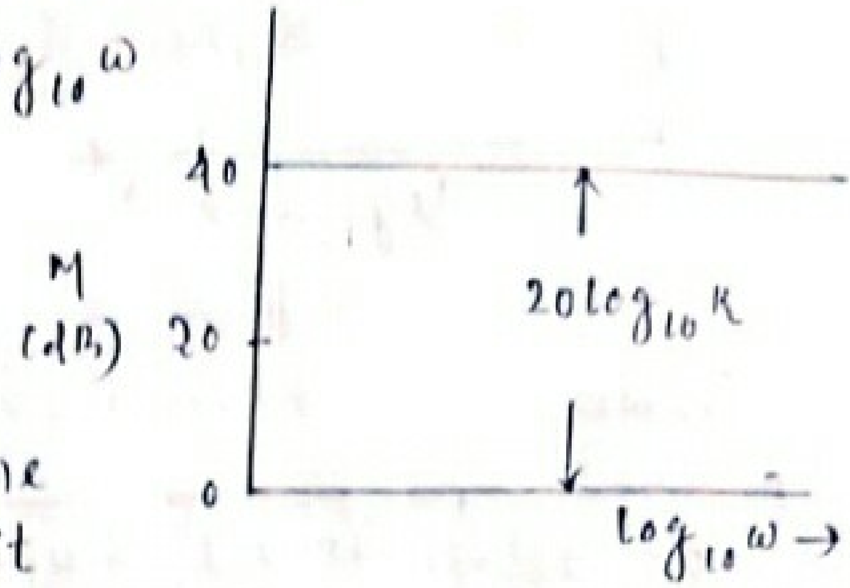
Put $N = 1$ in equation (1)

$$20\log_{10} |G(j\omega) \cdot H(j\omega)| = 20\log_{10} K - 20\log_{10} \omega$$

- Intersection with 0 db axis.

$$0 = 20\log_{10} K - 20\log_{10} \omega$$

$$\therefore \quad K = \omega.$$



Locate $\omega = K$ on 0 db axis and at this point draw a line of -20 db/decade produce it till it intersect the y-axis that will be the starting point on Bode plot.

3. For $N = 2$ (type two system)

Put $N = 2$ in equation

$$20\log_{10} |G(j\omega) H(j\omega)| = 20\log_{10} K - 20\times 2 \log_{10} \omega$$

$$= 20\log_{10} K - 40\log_{10} \omega$$

Intersection with 0 db. axis

$$0 = 20\log_{10} K - 40\log_{10} \omega$$

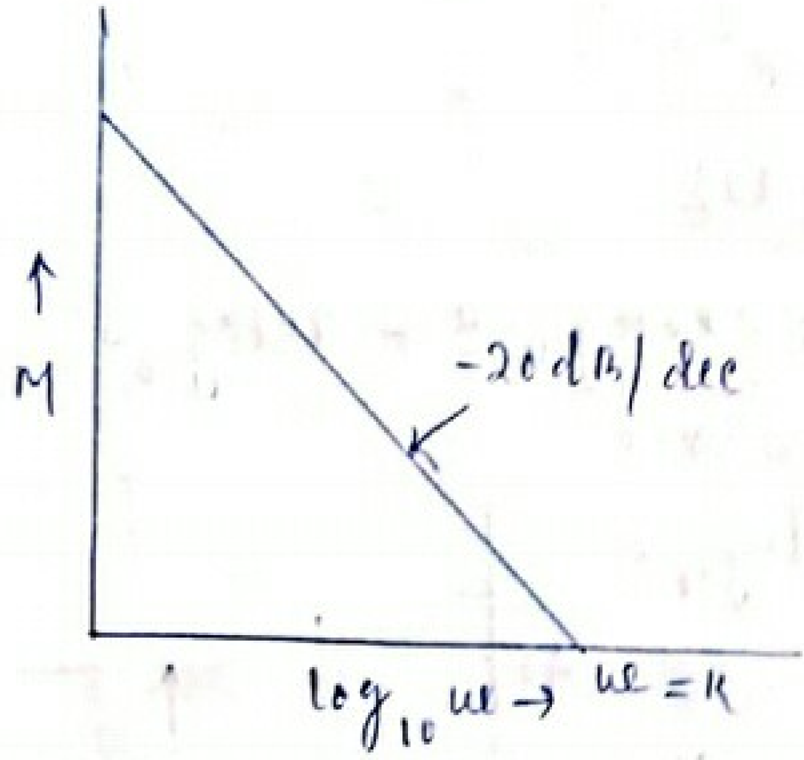
$$20\log_{10} K = 40 \log_{10} \omega$$

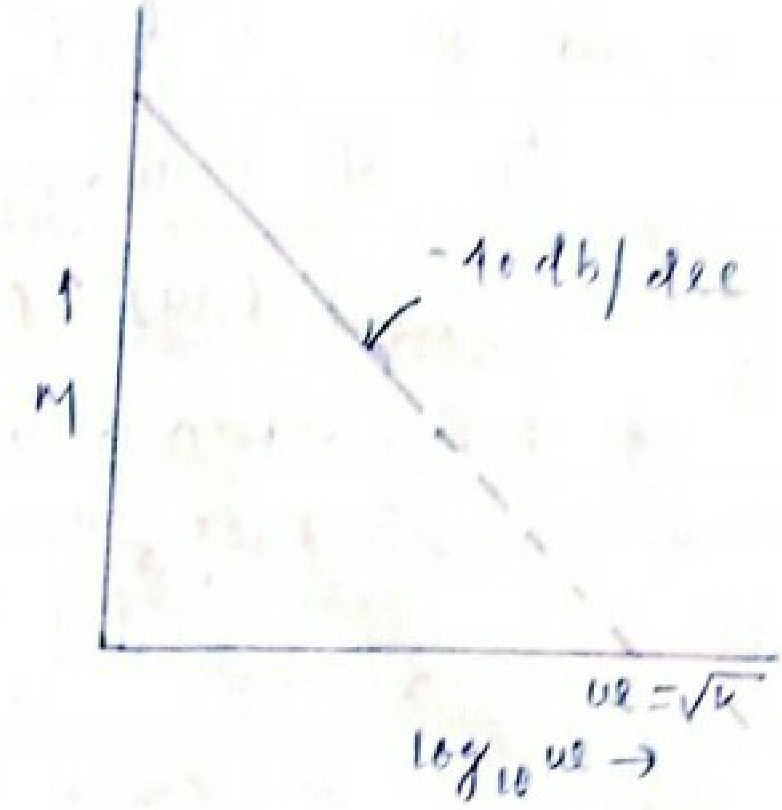
$$20\log_{10} K = 20\log_{10} \omega^2$$

$$\omega^2 = K$$

$$\omega = \sqrt{K}$$

Hence, graph intersect the 0 db axis at $\omega = \sqrt{K}$ on 0 db axis and draw a line -40 db/dec and produce it to the y-axis. Graph





Table

Type of the system N	Initial Slope 0 db Axis	Intersection with
0	0 db/decade	Parallel to 0 axis
1	-20 db/dec.	$= K$
2	-40 db/dec	$= \sqrt{K}$
3	-60 db/dec	$= K^{1/3}$
⋮	⋮	⋮
N	-20N db/dec	$K^{1/N}$

Minimum Phase systems And Non-Minimum Phase Systems.

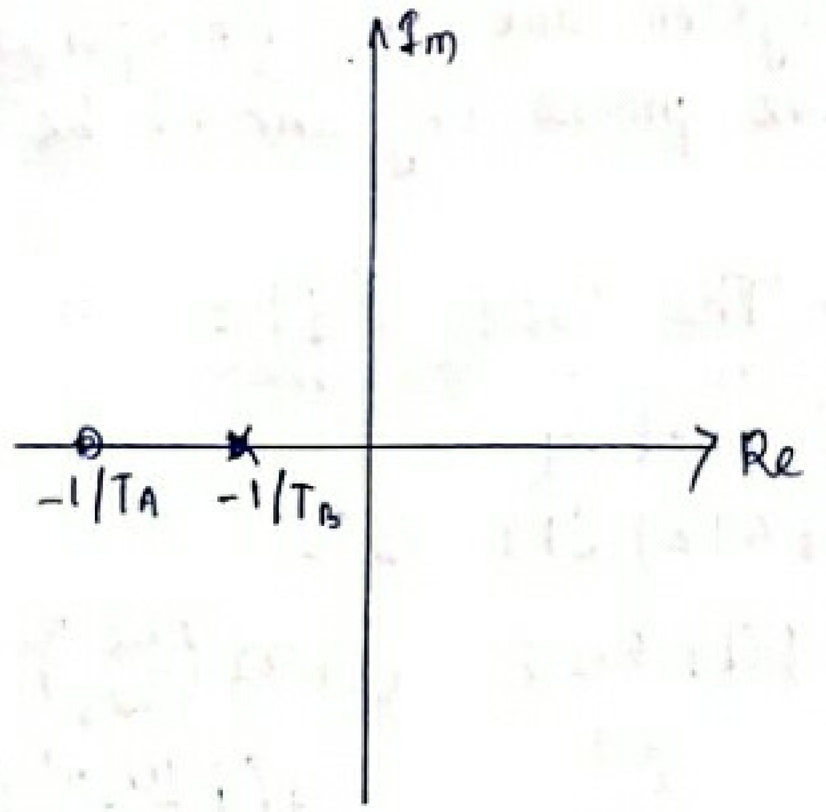
The transfer functions having no poles and zeros in the right half s-plane are called minimum phase transfer functions. System with mini Minimum phase transfer function are called minimum phase system.

The transfer functions having poles and / or zeros in the right half s-plane are called non-minimum phase transfer function. Systems with non-minimum phase transfer functions are called non-minimum phase systems.

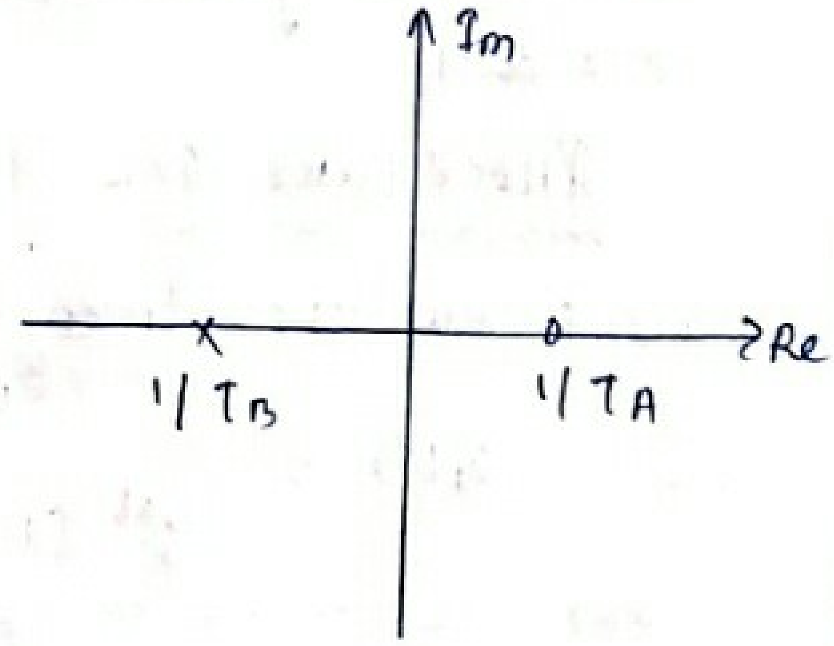
Let
$$G_1(j\omega) = \frac{1 + j\omega T_A}{1 + j\omega T_B} \quad \text{——— (1)}$$

$$G_2(j\omega) = \frac{1 - j\omega T_A}{1 + j\omega T_B} \quad \text{- - - - (2)}$$

The transfer function given by equation (1) is a minimum phase transfer function and transfer function given by equation (2) is non-minimum phase type transfer function.



(Minimum Phase)



(Non-minimum phase).

For maximum phase system, the magnitude and phase angle plots are uniquely related. It means that if the magnitude curve is specified for the frequency from zero to infinity, then the phase angle curve is uniquely related. This ~~art~~ is not applicable for non-minimum phase system.

For minimum phase system, the phase angle at $\omega = \infty$ is $-90^\circ (q-p)$ where p and q are the degrees of the numerator and denominator polynomials of transfer function. For ~~pc~~ both minimum and non-minimum phase system the slope is $(q-p)$ db/decade. But the phase angle for non-minimum system is differ from $-90^\circ (q-p)$. Thus it is possible to determine whether the system is minimum phase or non-minimum phase system. If the slope of the log-magnitude curve as ω approaches infinity is $-20(q-p)$ db/decade and the phase angle is equal to $-90^\circ (q-p)$ at $\omega = \infty$ then the system minimum phase otherwise not.

Non-minimum phase system are slow in response. In control system excessive phase lag should be avoided.

Procedure For Draw The Bode Plot

Consider the transfer function

$$G(s) = \frac{K(1+sT_a)(1+sT_b)\ldots\ldots}{s^N (1+sT_1)(1+sT_2)\ldots\left[1+2\xi\left(\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2\right]} \quad (1)$$

Where N is the number of poles at the origin

i.e. N defines the type of system.

For type zero system $K = K_p$

For type one system $K = K_v$

For type two system $K = K_a$

In above transfer function put, $s = j\omega$

$$G(j\omega) = \frac{K(1 + j\omega T_a)(1 + j\omega T_b) \dots}{(j\omega)^N (1 + j\omega T_1)(1 + j\omega T_2) \dots \left[1 + 2\xi\left(\frac{\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2\right]}$$

$$20 \log_{10} |G(j\omega)| = 20 \log K + 20 \log \sqrt{1 + \omega^2 T_a^2} + 20 \log \sqrt{1 + \omega^2 T_b^2} + \dots - 20 N \log \omega$$

$$- 20 \log \sqrt{1 + \omega^2 T_1^2} - 20 \log \sqrt{1 + \omega^2 T_2^2} \dots - 20 \log \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}$$

Phase angle

$$\angle G(j\omega) = \tan^{-1} \omega T_a + \tan^{-1} \omega T_b + \dots N(90^\circ)$$

$$- \tan^{-1} \omega T_1 - \tan^{-1} \omega T_2 \dots \cancel{\tan^{-1}(2\xi\omega\omega_n}$$

$$\dots \tan^{-1} \left[\frac{2\xi\omega\omega_n}{\omega_n^2 - \omega^2}\right] \quad \text{——— (2)}$$

Step-1 : Identify the corner frequency.

Step-2 : Draw the asymptotic magnitude plot. The slope will change at each corner frequency by +20 db/dec for zero and -20 db/dec pole. For complex conjugate pole and zero the slope will change by $\mp$ 40 db/decade.

Step-3 : (i) for type zero system draw a line upto first (lowest) corner frequency having 0 db/dec slope.

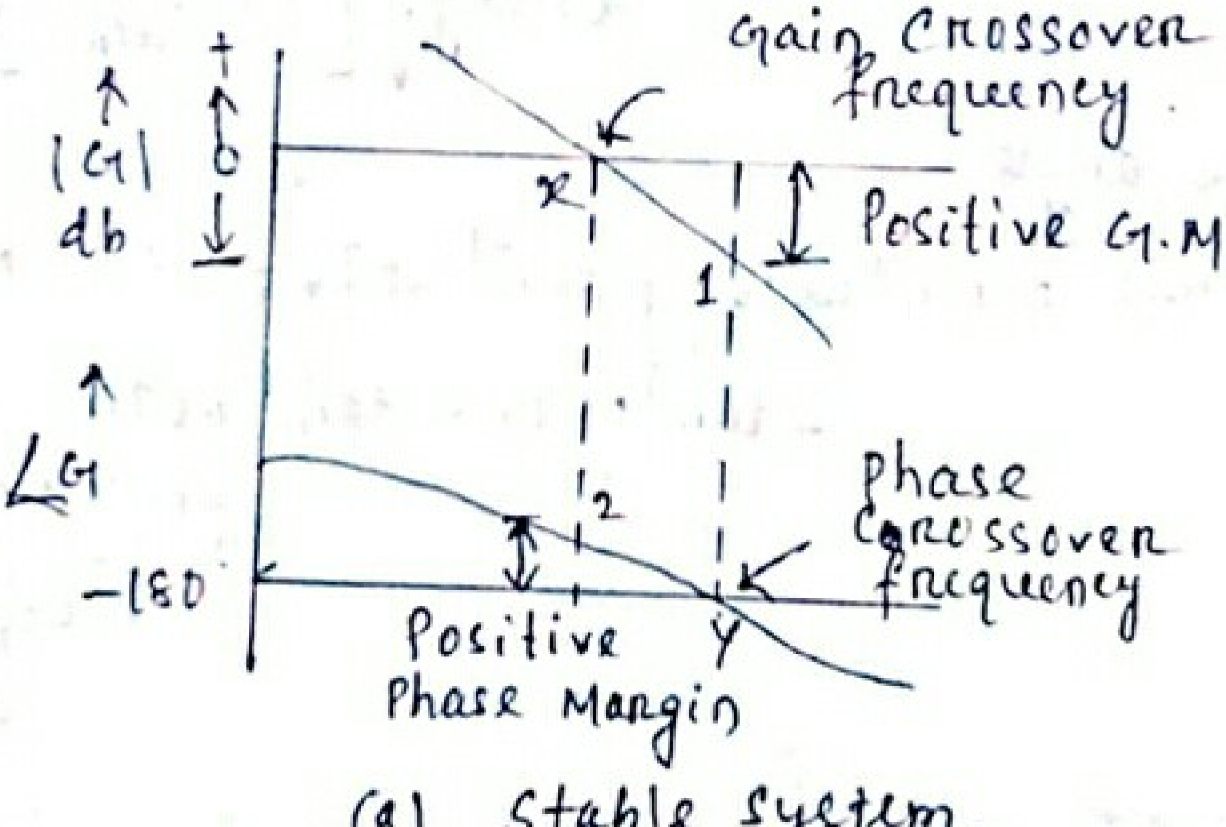
(ii) for type one system draw a line having slope -20 db/dec upto $\omega = K$. Mark first (lowest) corner frequency.

(iii) for type two system draw the line having slope -40 db/ dec. upto $\omega = \sqrt{K}$ and so on. Mark first corner frequency.

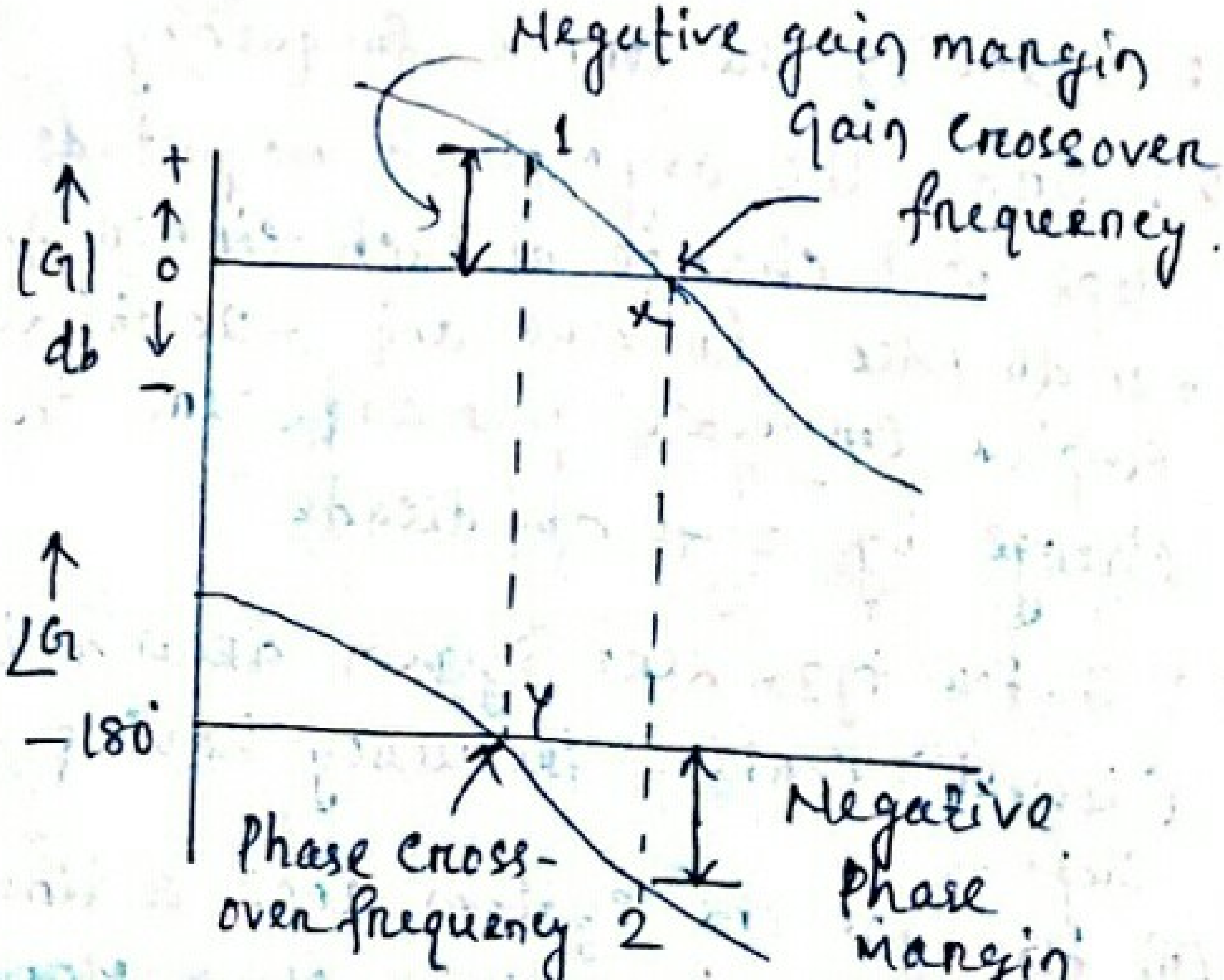
Step 4 :- Draw a line upto second corner frequency by adding the slope of next pole or zero to the previous slope and so on.

Step 5 :- Calculate phase angle for different value of ω from the equation and join all points.

Phase Margin and Gain Margin



(a) Stable system.



Phase Margin :- For gain the additional phase lag can be introduced without affecting the magnitude plot. Therefore, phase margin can be defined as the amount of additional phase lag which can be introduced in the system reaches on the verge of instability is called as phase margin (P.M.). Mathematically phase margin can be defined as.

$$P.M = \left[\left| \angle G(j\omega) H(j\omega) \right|_{\omega = \omega_{c_2}} \right] - (-180^\circ)$$

$$P.M. = 180^\circ + \left| \angle G(j\omega) H(j\omega) \right|_{\omega = \omega_{c_1}}$$

where, ω_{c_1} = Gain crossover frequency.

Nyquist Plot

Now we are going to study the Nyquist stability criterion. The Nyquist stability criterion based on the principle of argument. The principle of argument is related with the theory of mapping. So first we will study the mapping and principle of argument then Nyquist stability criterion.

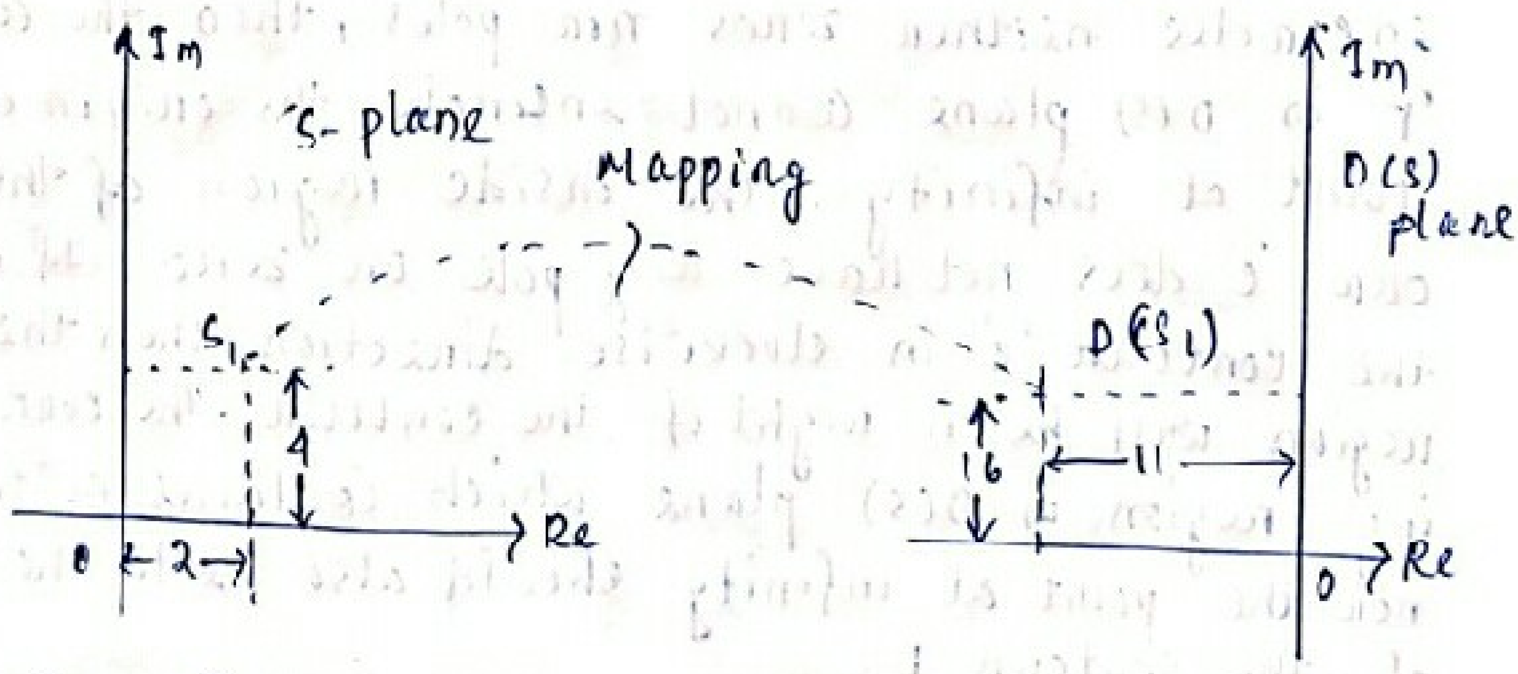
Mapping :-

Consider a function $D(s) = s^2 + 1$

Any point in the s-plane can be mapped by locating the values of u and v for the given value of s. Let, $s_1 = 2 + j4$.

$$D(s_1) = D(2 + j4) = (2 + j4)^2 + 1 = -11 + j16.$$

The ~~mapping~~ refore, we can say that every point in the s-plane maps into one and only one point in the D(s) plane. Any close contour in the s-plane maps into the closed contour in the D(s) plane. These are the properties of the mapping.



Mapping of close Contour and Principle of Argument

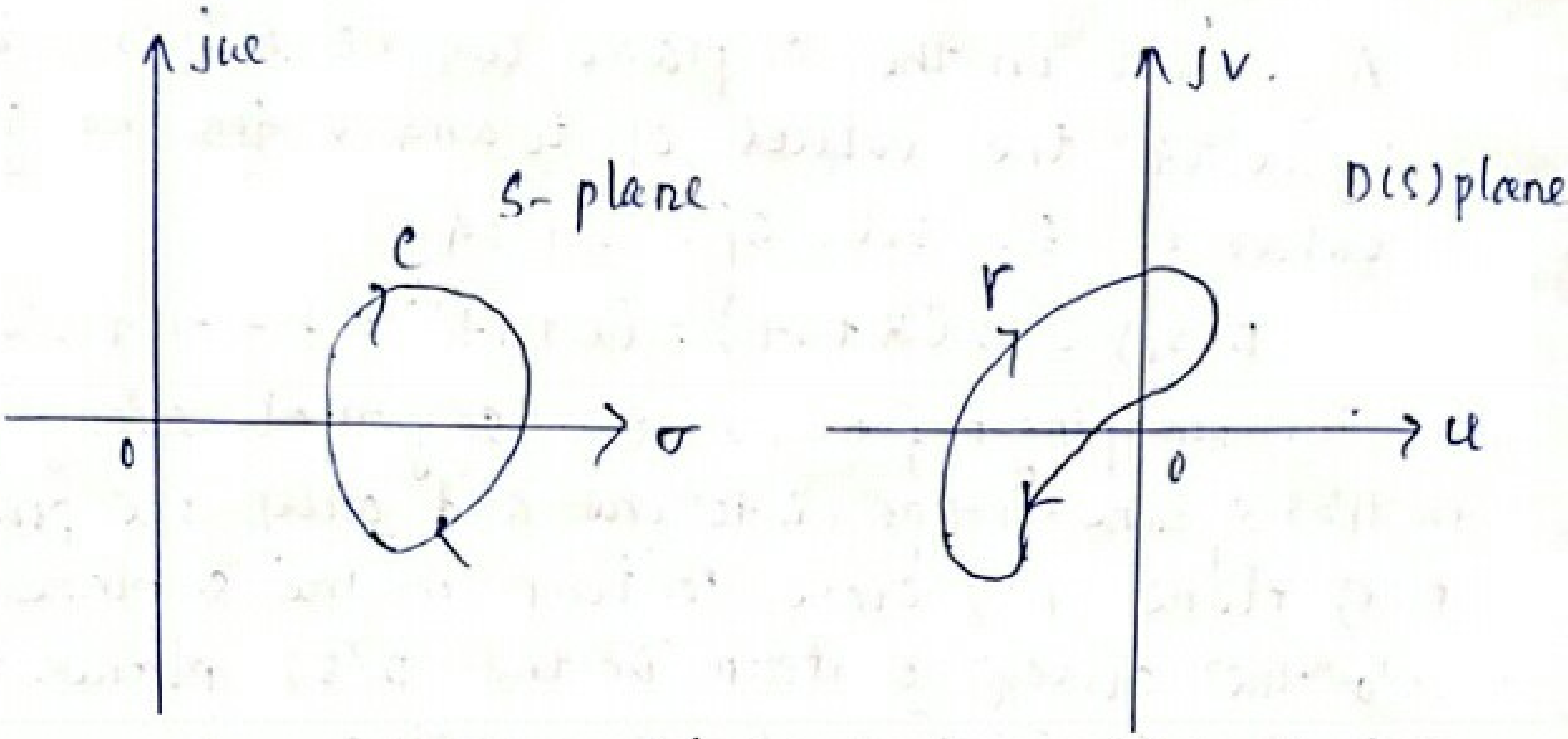
Consider the characteristic equation.

$$D(s) = 1 + G(s).H(s) = \frac{K(s-a_1)(s-a_2)\ldots}{(s-b_1)(s-b_2)\ldots}$$

Where, $a_1, a_2 \ldots$ are zero and $b_1, b_2 \ldots$ are poles.
Let s be a complex variable $s = \sigma + j\omega$, then, D(s) will also be complex

$$D(s) = u(s) + jv(s)$$

Now choose a closed path 'c' arbitararily. The contour 'c' in s-plane is mapped into the plane as contour Γ.



Now consider a contour 'c' in s-plane. This contour encircles neither zeros nor poles, then the contour 'Γ' in D(s) plane cannot encircle the origin or the point at infinity. The inside region of the contour 'c' does not have any pole or zero. If we trace the contour 'c' in clockwise direction then the inside region will be on right of the contour. The corresponding region in D(s) plane which contains neither origin nor the point at infinity should also be to the right of the contour Γ.

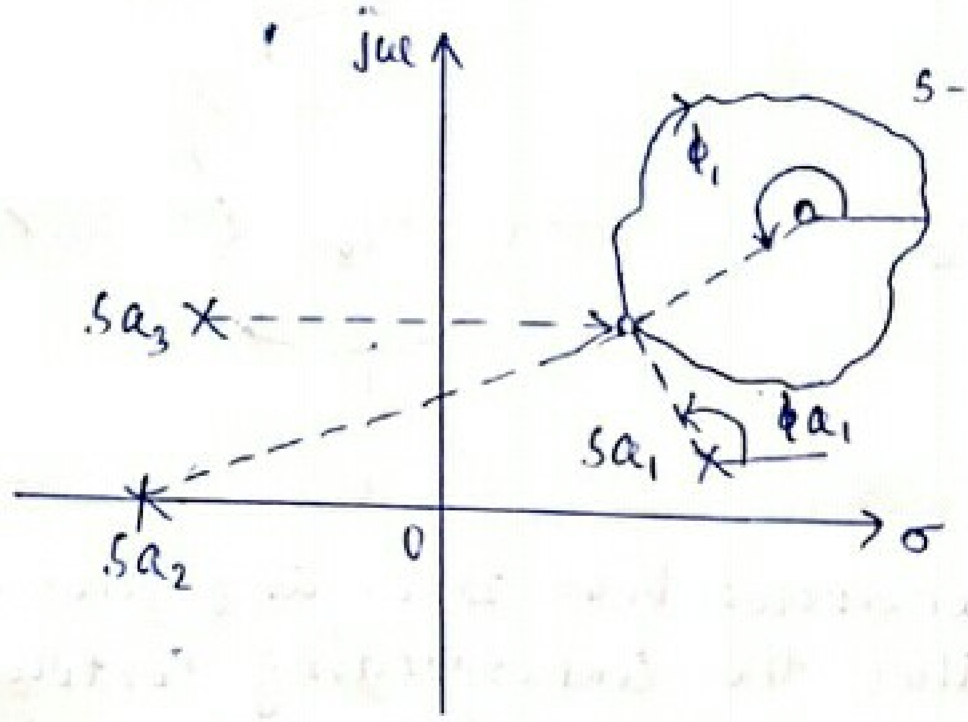
If the contour 'c' encircles a zero but not poles then the point $D_1(s)$ is given by.

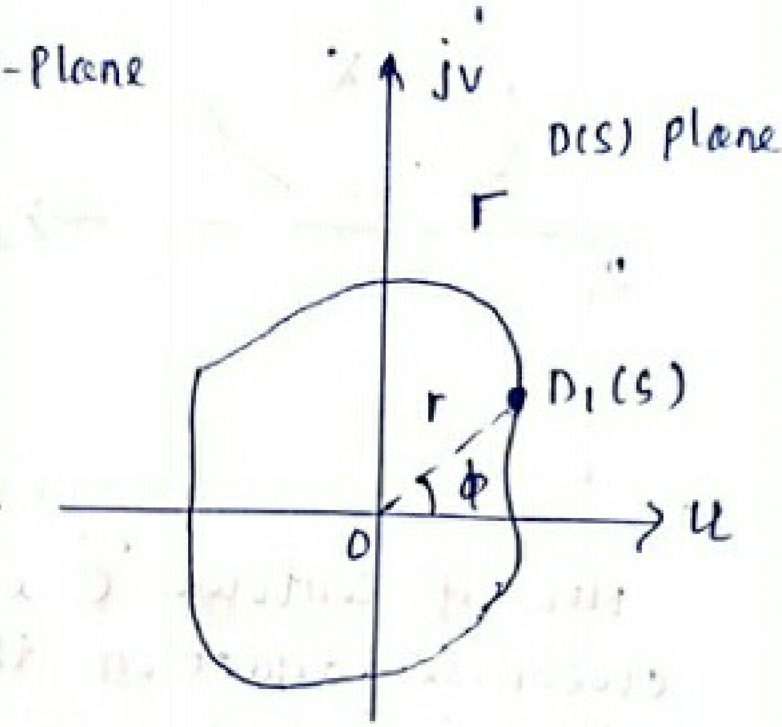
$$|D_1(s)| = \frac{|s-a_1||s-a_2|\cdots}{|s-b_1||s-b_2|\cdots}$$

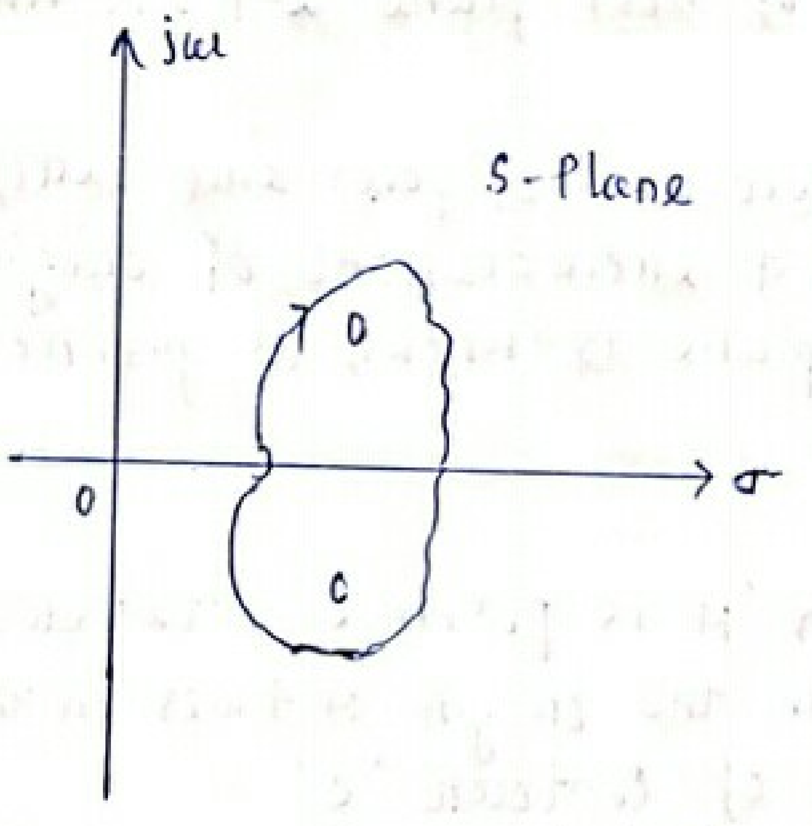
$$\angle D_1(s) = \angle s-a_1 + \angle s-a_2 \cdots - \angle s-b_1 - \angle s-b_2$$

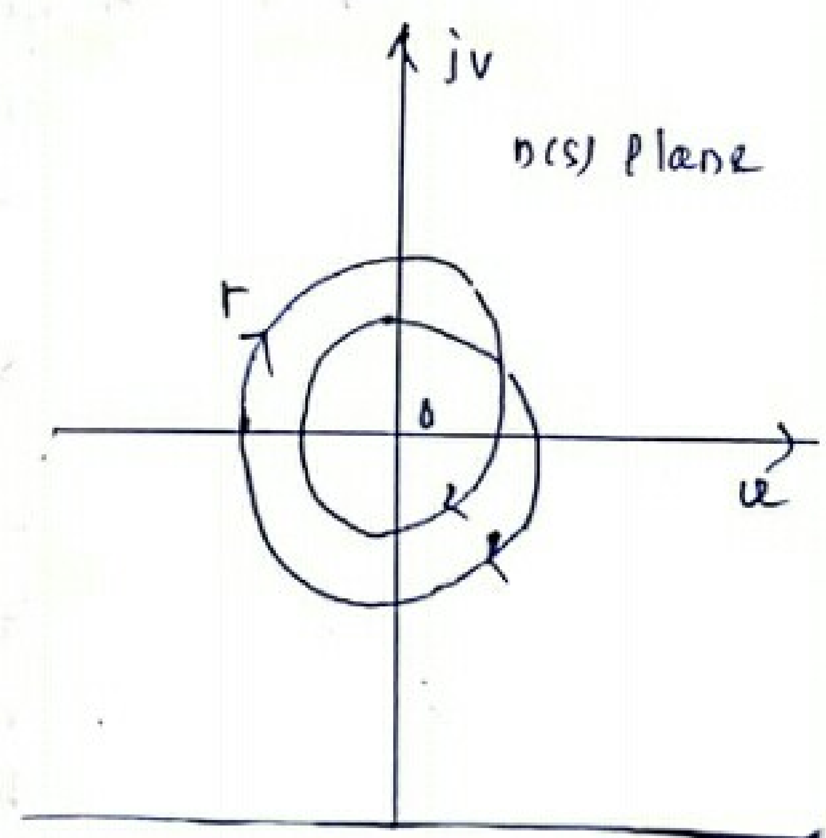
i.e. $D_1(s) = r\angle\phi$

It mean that the tip of $D_1(s)$ froms a close contour about the origin in clockwise direction. Similarly. if the contour 'c' encircles 'z' zero in clockwise direction then the contour in D(s) plane encircles the origin of D(s) plane 'z' times in clockwise direction. In the figure the contour 'c' encircles two zeros, the corresponding contour in D(s) plane.





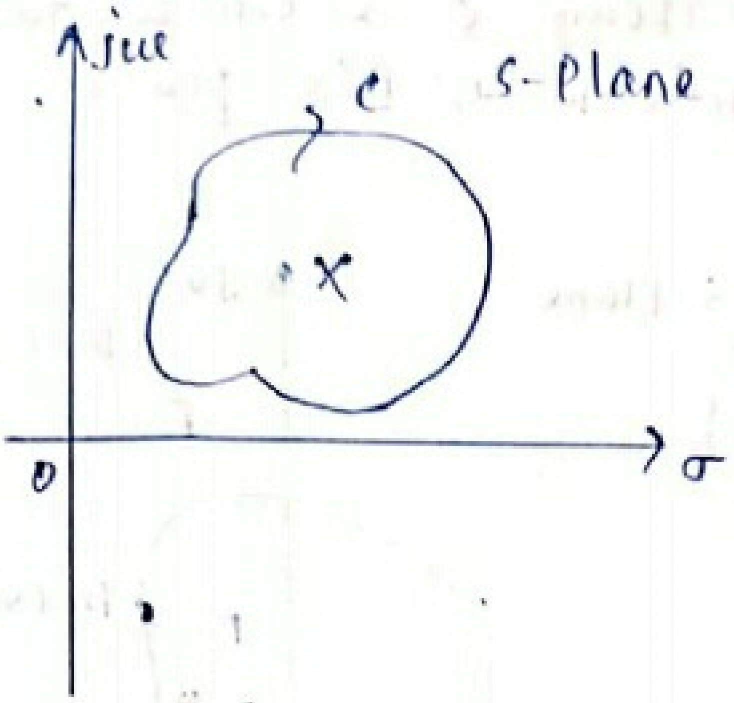


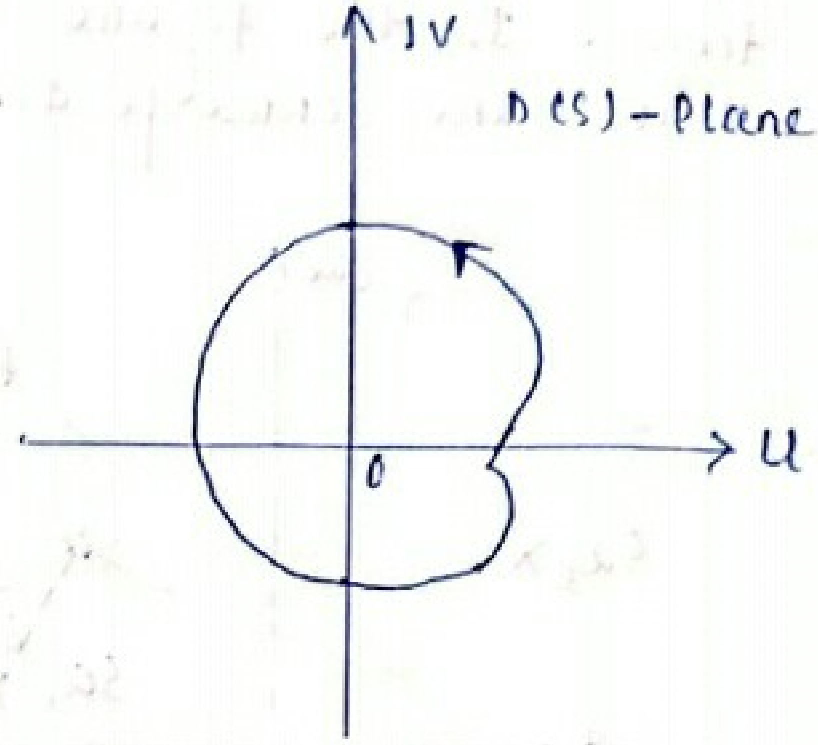


Two zeros are encircled by contour 'c'

origin encircles two times by contour Γ

Consider another case when a pole is encircled by the contour 'c' in clockwise direction then angle will be $\angle -\phi$. The phasor $s-b_1$ will also generate an angle -2π but due to in denominator net phase change will be $+2\pi$. Therefore it means that if it means that if the contour 'c' encircles 'p' number of poles in clockwise direction then the corresponding contour Γ will encircle the origin P times in contour clockwise direction.





Now, if contour 'c' encircles both zeros and poles in clockwise direction then the corresponding contour 'Γ' encircles the origin of D(s) plane $Z-P=N$ times in clockwise direction.

This relation bet^n enclosure of poles and zeros by the contour in s-plane and encirclement of origin by the contour 'Γ' in D(s) plane is known as principle of arguement.

Conclusion : $N = Z-P$.

(a) If $N>0$ i.e. $Z-P$, then 'N' is positive integer. In this case Γ will encircle the origin N times in the same direction as that of contour 'c'.

(b) If $N=0$ i.e. $Z=P$, the contour 'c' will not encircle the origin.

(c) If $N<0$ i.e. $Z<P$, then N is negative integer, in this case Γ will encircle the origin N times in opposite direction as that of 'c'.

For example: The contour 'c' encircles three poles and one zero ~~in~~ in clockwise then the number of encirclement $N = 1-3 = 2$. Then the contour 'Γ' will encircle the origin two times in counterclockwise direction.

Nyquist Criterion :-

The characteristic equation is given by

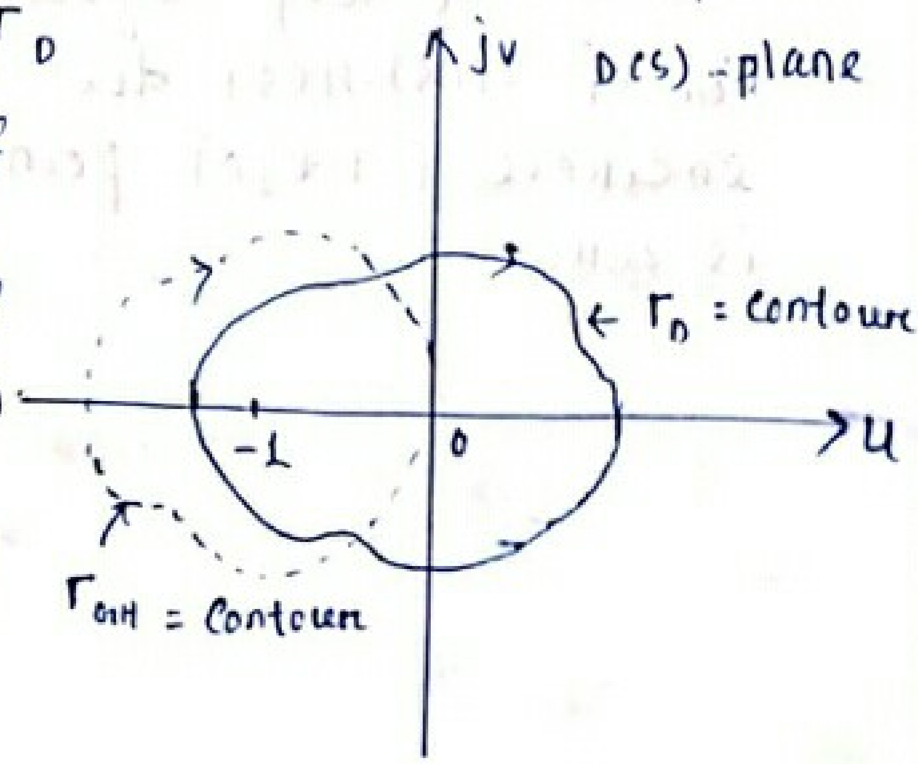
$$D(s) = 1 + G(s) \cdot H(s)$$

The zero of $D(s)$ are the roots of the characteristic equation. For a feedback system the necessary and sufficient condition is that all zero of $1+G(s).H(s)$ that is the roots of the characteristic equation must have negative real part i.e. they must lie in the left half of s-plane. In order to determine the presence of zero in right half of the s-plane we choose a contour is called nyquist contour. Let there are 'Z' zeros & 'P' poles in the right half of s-plane. If this contour is mapped in $D(s)$ plane as Γ_D then Γ_D encloses the origin N times (where $N = Z - P$) in clockwise. Hence the system is unstable because the clockwise encirclement is possible only when there are zeros of $D(s)$ in right half of s-plane.

A feedback system (close loop system) is stable if and only if there is no zeros of $D(s)$ in the right half of s-plane i.e. $Z = 0$

$$\therefore \boxed{N = -P}$$

Therefore, for a closed loop system to be stable the number of counter clockwise encirclement of the origin of $D(s)$ plane by Γ_D should equal the number of right half s-plane poles of $D(s)$ which are the poles of open loop transfer function $G(s).H(s)$.



Since, $D(s) = 1 + G(s) H(s)$

or, $G(s) H(s) = D(s) - 1$

The contour Γ_D in $D(s)$ plane can be mapped in $G(s)H(s)$ plane. Γ_{GH} by shifting horizontally to the left by one unit. Thus the encirclement of the origin by the contour Γ_D is equivalent to the encirclement of the point $(-1+j0)$ by the contour Γ_{GH}.

In most single loop feedback system $G(s).H(s)$ has no poles in the right half plane i.e. $P=0$ then closed loop system is stable if $N=P=0$.

So we can say that a close loop system with $P=0$ is stable if the net encirclement of the origin of $D(s)$ plane by Γ_D contour is zero.

Now we can ~~say that~~ state the Nyquist stability criterion as follows.

A feedback system or closed loop system is stable if the contour Γ_{GH} of the open loop transfer function $G(s)H(s)$ corresponding to the Nyquist contour in the s-plane encircles the point $(-1+j0)$ in counterclockwise direction and the number of counterclockwise encirclements about the $(-1+j0)$ equal the number of poles of $G(s).H(s)$ in the right half of s-plane i.e. with positive real parts.

In common case of open loop stable system, the closed loop system is stable if the contour Γ_{GH} of $G(s).H(s)$ does not pass through or does not encircle $(-1+j0)$ point, i.e., net encirclement is zero.

General Construction Rules of the Nyquist Path

Path ab	$s = j\omega$	$0 < \omega < \omega_0$ ... (a)
Path bc	$s = \lim_{\rho \to 0} (j\omega_0 + \rho e^{j\theta})$	$-90° \le \theta \le 90°$ ... (i)
Path cd	$s = j\omega$	$\omega_0 \le \omega \le \infty$ ... (ii)
Path def	$s = \lim_{R \to \infty} R e^{j\theta}$	$-90° - \le \theta \le 90°$ ... (iii)
Path fg	$s = j\omega$	$-\infty < \omega < -\omega_0$ ... (iv)
Path gh	$s = \lim_{\rho \to 0} (j\omega_0 + \rho e^{j\theta})$	$-90° \le \theta \le 90°$ ... (v)
Path hi	$s = j\omega$	$-\omega_0 \le \omega \le 0$ ... (vi)
Path ija	$s = \lim_{\rho \to 0} \rho e^{j\theta}$	$-90° \le \theta \le 90°$ ... (vii)

Step 1 : check $G(s)$ for poles on $j\omega$ axis and at the origin.

Step 2 : Using equation (a) to equation (ii) sketch the image of the path a-d in the $G(s)$-plane. If there are no poles on $j\omega$ axis equation (i) need not be employed.

Step 3 :- Draw the mirror image about the real axis of the sketch resulting from step 2.

Step 4 :- Using equation (v) plot the image of path def. This path at infinity usually plot into a point in the $G(s)$-plane

Step 5 :- Using equation (vii) plot the image of path ija (pole at origin)

Step 6 :- Connect all curves drawn into the previous steps.

Example - 1. Determine the closed loop stability of a control system whose open loop transfer function is

$$G(s)\cdot H(s) = \frac{K}{s(1+sT)}$$

Soln:- Given that

$$G(s)\cdot H(s) = \frac{K}{s(1+sT)}$$

Put, $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T)} \quad \text{--- (i)}$$

Rationalizing the equation (i) and separating into real and imaginary parts.

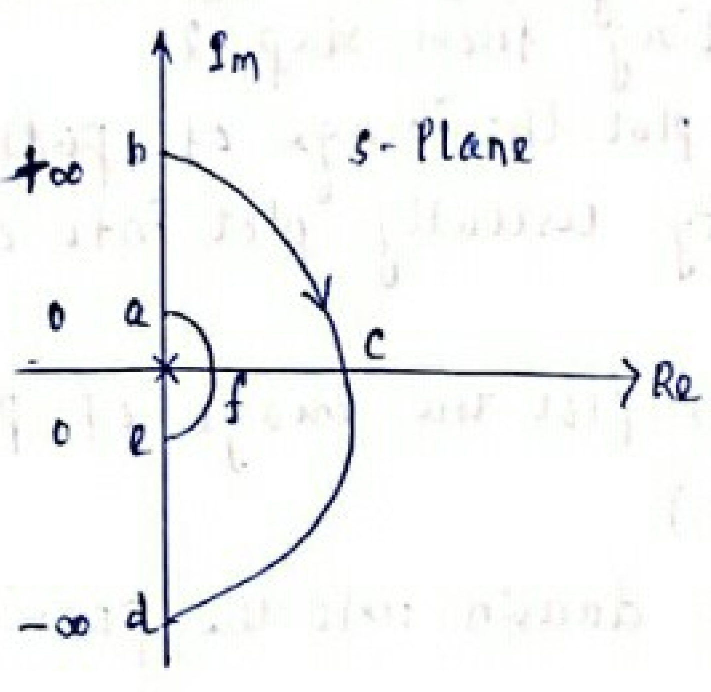
$$G(j\omega)H(j\omega) = \frac{KT}{1+\omega^2T^2} - j\frac{K}{\omega(1+\omega^2T^2)} \quad \text{--- (2)}$$

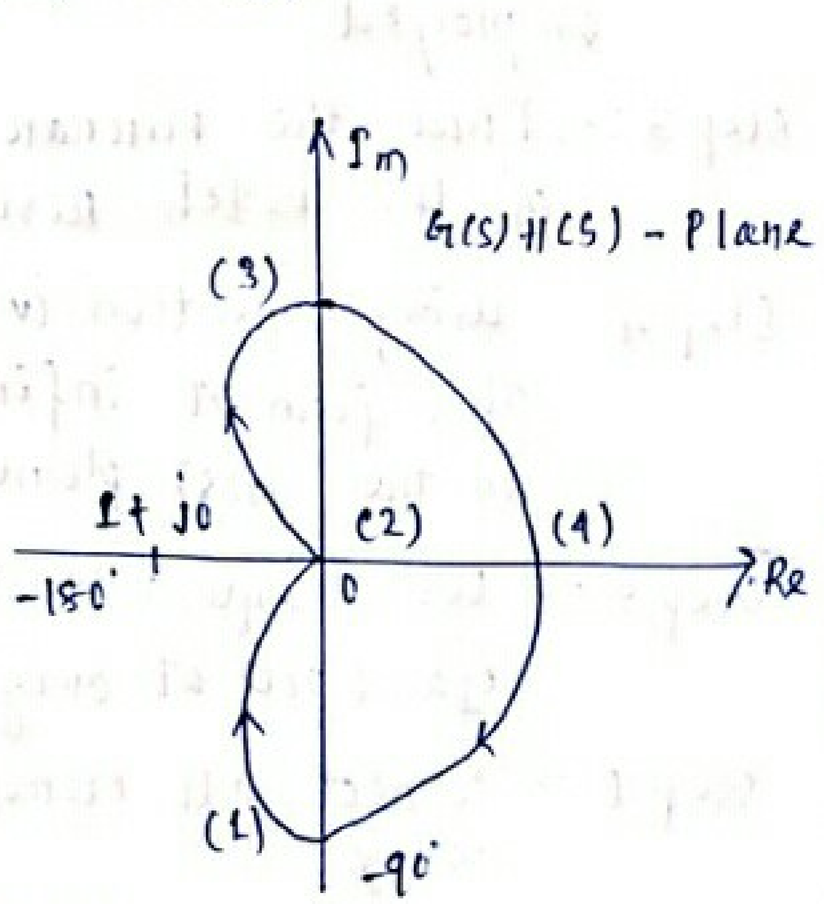
$$\lim_{\omega\to 0} |G(j\omega)H(j\omega)| = \infty$$

$$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = -90^\circ$$

$$\lim_{\omega\to \infty} |G(j\omega)H(j\omega)| = 0$$

$$\lim_{\omega\to \infty} \angle G(j\omega)H(j\omega) = -180^\circ$$





(Nyquist Plot)

The polar plot will lie in third quadrant.

The Nyquist plot is shown in figure. The part for $0 < \omega < +\infty$ is draw (1)(2) and for $-\infty < \omega < 0$ is shown by the point (2),(3) which is the mirror image of (1),(2). The semicircular detour around the origin in s-plane is mapped into a semicircular path of infinite radius representing a change of phase from $+\pi/2$ to $-\pi/2$.

As the point $(-1+j0)$ is not encircled by the plot, $N = 0$.

$N = 0$ $\qquad$ $P = 0$

$\because N = Z - P$ $\qquad$ $\therefore Z = 0$

The number of zero or roots of the characteristic equation with positive real part is nill and hence the closed loop system is stable.

Example-2 Sketch the Nyquist plot and determine the stability of a unity feedback control system.

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$ {Type zero system

Sol^n:- Given that

$$G(s)\cdot H(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$

Put, $s = j\omega$.

$$G(j\omega)H(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)} \quad \text{---(1)}$$

$$|G(j\omega)H(j\omega)| = \frac{K}{\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}} \quad \text{---(2)}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$$

$$\lim_{\omega\to 0} |G(j\omega)H(j\omega)| = K$$

$$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = 0$$

$$\lim_{\omega\to \infty} |G(j\omega)H(j\omega)| = 0$$

$$\lim_{\omega\to \infty} \angle G(j\omega)H(j\omega) = -180^\circ$$

Rationalize the equation (i) and separate the real and imaginary parts.

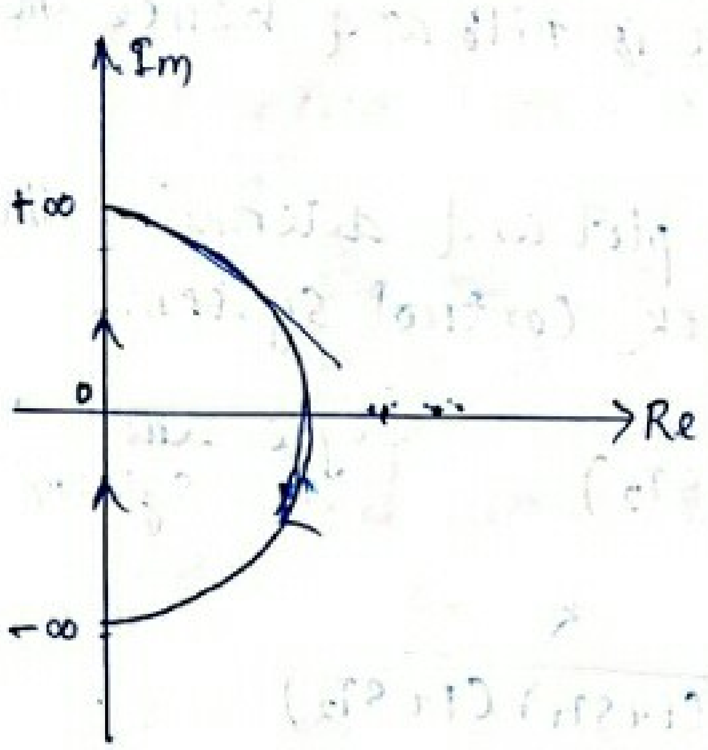
$$\frac{K}{(1+j\omega T_1)(1+j\omega T_2)} = \frac{\cancel{K(1-\omega^2 T_1 T_2)}}{}$$

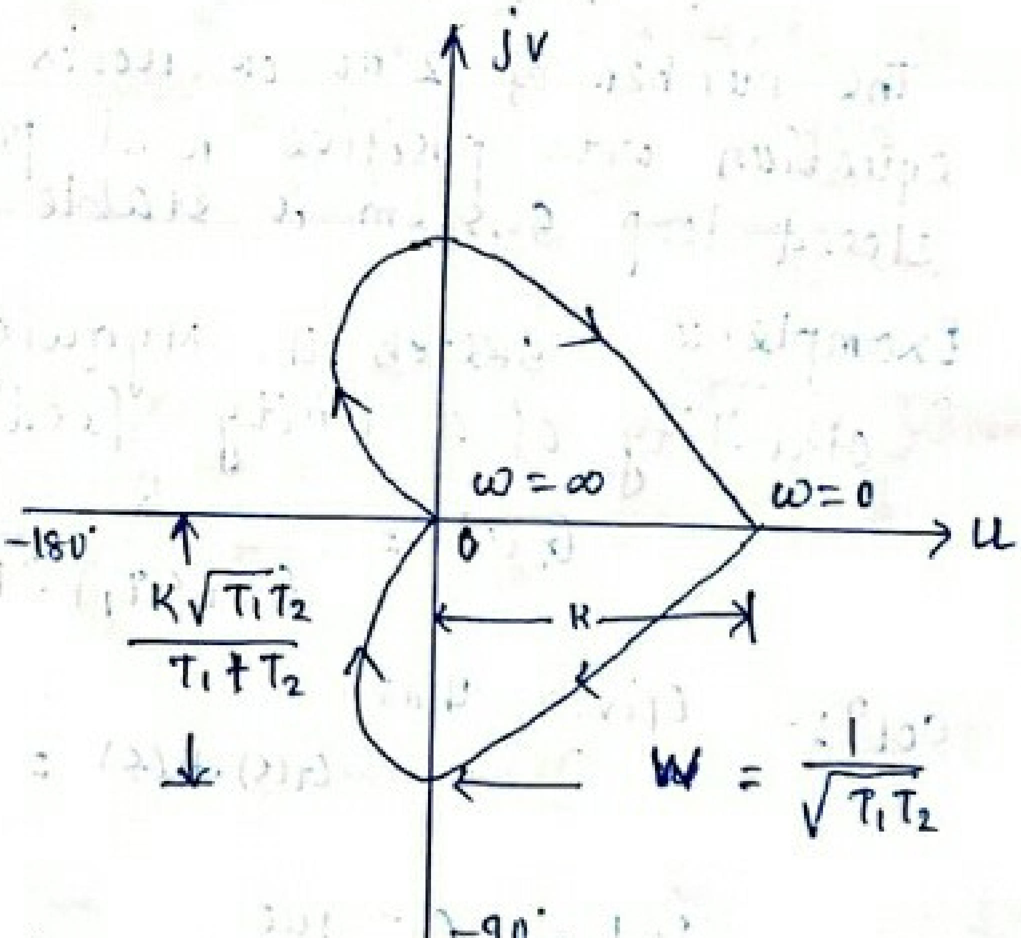
$$= \frac{K(1-\omega^2 T_1 T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)} - j\,\frac{\omega(T_1+T_2)K}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

Equate the real to zero, we get.

$$\omega = \frac{1}{\sqrt{T_1 T_2}}$$

$$|G(j\omega)H(j\omega)| = \frac{K T_1 T_2}{T_1 + T_2}$$





The plot of $G(j\omega)H(j\omega)$ is shown in above the figure. The infinite semicircular arc of the Nyquist contour maps into origin. As the point $(-1+j0)$ is not encircled by the plot.

$\therefore$ $N = 0$

$P = 0$

$\therefore$ $Z = 0$

Hence, the system is stable.

Example 3 Using Nyquist criterion, determine the stability of the feedback system which has the following open loop transfer function.

$$G(s)\cdot H(s) = \frac{K}{s^2(1+sT)}$$ {Type 2 system

Sol^n:- Given that

$$G(s)\cdot H(s) = \frac{K}{s^2(1+sT)}$$

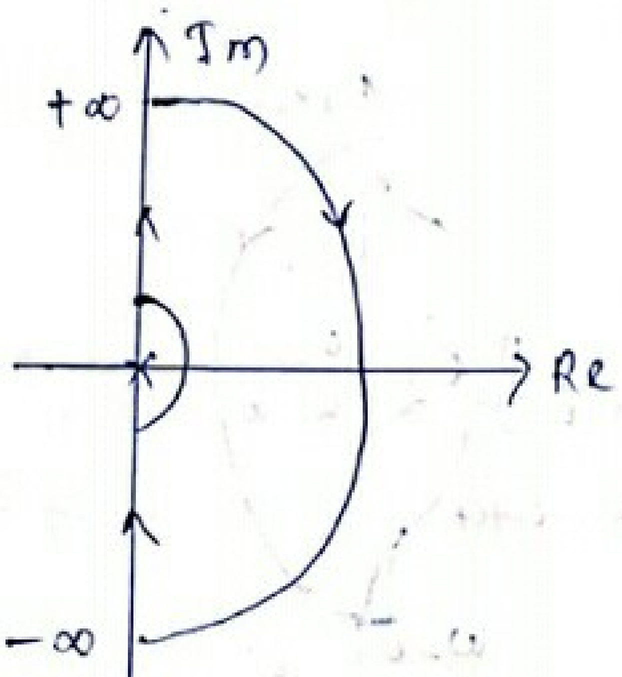
Put, $s = j\omega$

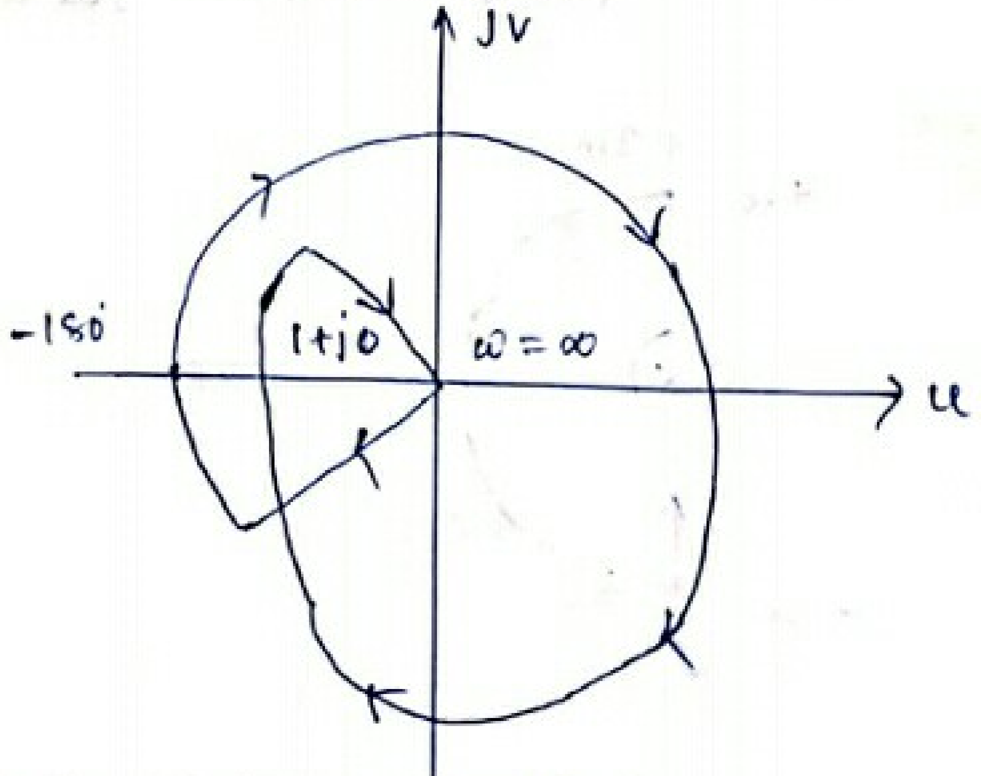
$$G(j\omega)H(j\omega) = \frac{K}{(j\omega)^2(1+j\omega T)} \quad \text{—— (1)}$$

Rationalizing the equation (1) and separating the real and imaginary part.

$$G(j\omega)H(j\omega) = \frac{K}{-\omega^2(1+\omega^2T^2)} + j\frac{K}{\omega(1+\omega^2T^2)} \quad \text{—— (2)}$$

The Nyquist diagram is shown in figure in bellow. Because of the double pole at $s=0$, a small semi-circular detour at the origin should be made.





The point $(-1+j0)$ is encircular twice,

Hence $N = 2$

$P = 0$

$\therefore Z = 2$

Hence, the system is unstable.

Example 4 :- Use Nyquist criterion, determine whether the closed loop system having the following open loop transfer function is stable or not.

$$G(s).H(s) = \frac{1}{s(1+2s)(1+s)}$$

sol^n:- Given that

$$G(s).H(s) = \frac{1}{s(1+2s)(1+s)}$$

Put, $s = j\omega$

$$G(j\omega).H(j\omega) = \frac{1}{j\omega(1+j2\omega)(1+j\omega)} \quad \text{——(1)}$$

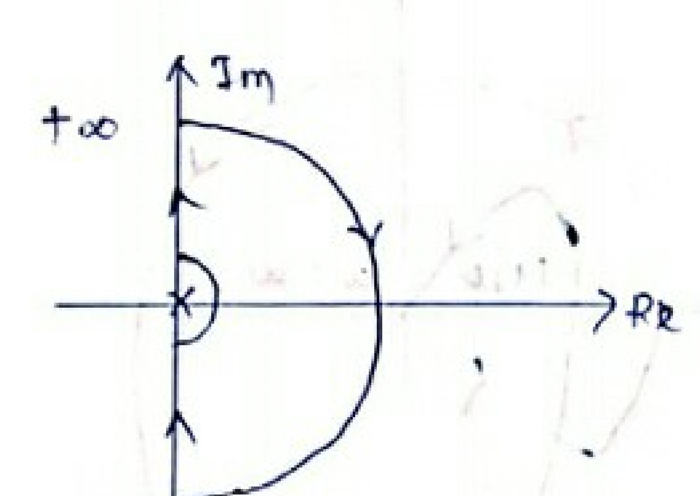
Rationalizing the equation (1) and separate the real and imaginary part.

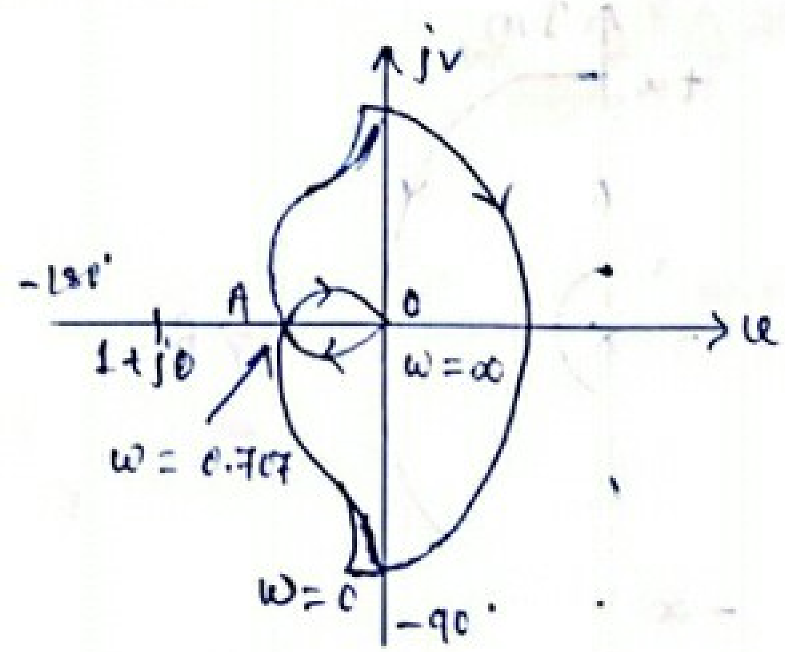
$$G(j\omega).H(j\omega) = \frac{-3}{(1+4\omega^2)(1+\omega^2)} - j\frac{1-2\omega^2}{\omega(1+4\omega^2)(1+\omega^2)} \quad \text{——(2)}$$

Equating the imaginary part to zero, we get the point of intersection on real axis

$$1-2\omega^2 = 0 \quad \therefore \omega = 0.707$$

$$|G(j\omega)H(j\omega)|_{j\omega = 0.707} = 0.66$$





Since, OA = 0.66, the point $(-1+j0)$ is not encircled

$\therefore$ $N = 0$

$P = 0$

$\therefore$ $Z = 0$

Hence, the system is stable no closed loop poles lie in the right half of s-plane.

Inverse Polar Plot

The inverse polar plot of $G(j\omega)$ is a graph of $\frac{1}{G(j\omega)}$ as a function of ω.

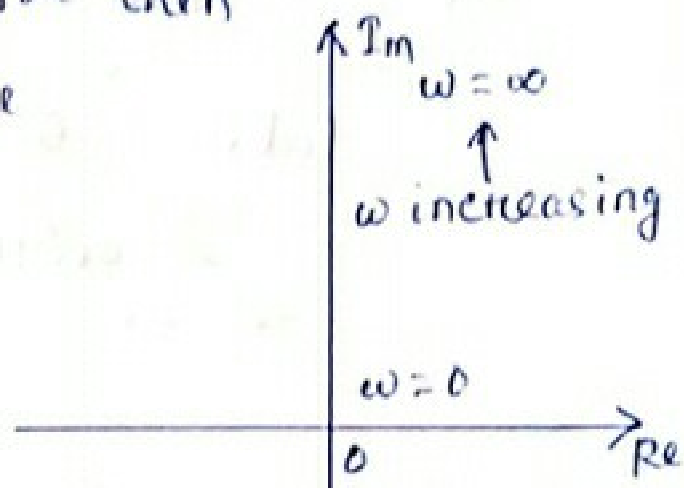
for example, if $G(j\omega) = 1/j\omega$ then

$$G(j\omega)^{-1} = j\omega$$

$$\lim_{\omega \to 0} |G(j\omega)^{-1}| = 0$$

$$\lim_{\omega \to \infty} |G(j\omega)^{-1}| = \infty$$

→ The inverse polar plot is shown figure.



(Inverse polar plot)

Example - 1 sketch the inverse polar ~~polar~~ plot of $G(j\omega) = j\omega T / 1 + j\omega T$.

sol^n:- $G(j\omega)^{-1} = \frac{1}{G(j\omega)} = \frac{1 + j\omega T}{j\omega T}$

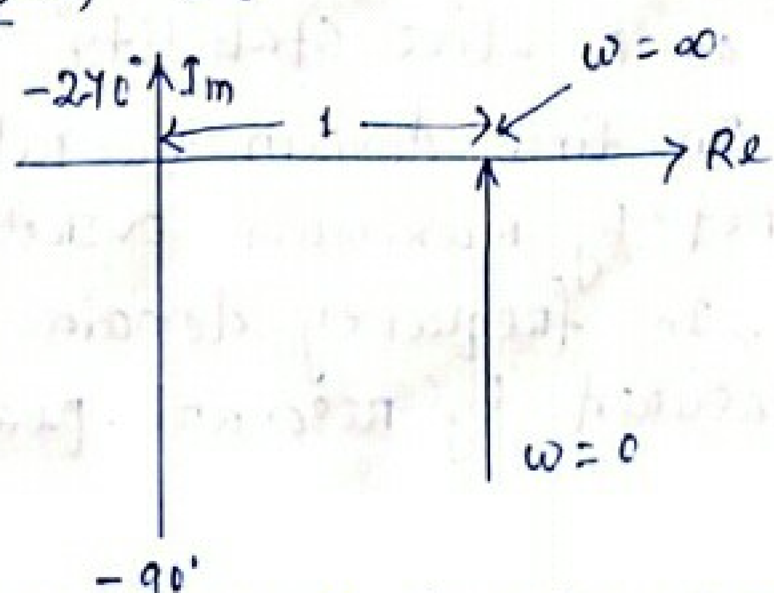
$$= \frac{1}{j\omega} + 1$$

$$\lim_{\omega \to 0} |G(j\omega)^{-1}| = \infty$$

$$\lim_{\omega \to \infty} |G(j\omega)^{-1}| = 1$$

$$\lim_{\omega \to 0} \angle G(j\omega)^{-1} = -90^\circ$$

$$\lim_{\omega \to \infty} \angle G(j\omega)^{-1} = 0^\circ$$



(Inverse polar plot of $j\omega T / 1 + j\omega T$)

Example-2 Sketch the inverse polar plot of

$$G(s) = \frac{1+sT}{sT}$$

Solⁿ:-

$$G(s)^{-1} = \frac{1}{G(s)} = \frac{sT}{1+sT}$$

Put, $s = j\omega$

$$G(j\omega)^{-1} = \frac{1}{G(j\omega)}$$

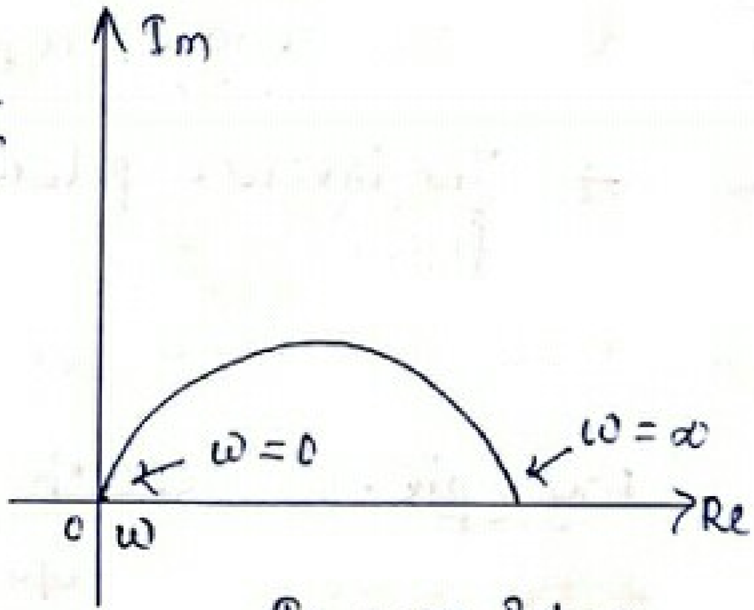
$$= \frac{j\omega T}{1+j\omega T}$$

$$\lim_{\omega \to 0} |G(j\omega)^{-1}| = 0$$

$$\lim_{\omega \to 0} \angle G(j\omega)^{-1} = 90^\circ$$

$$\lim_{\omega \to \infty} |G(j\omega)^{-1}| = 1$$

$$\lim_{\omega \to \infty} \angle G(j\omega)^{-1} = 0^\circ$$



Inverse Polar Plot of $\frac{1+sT}{sT}$

Relative And Absolute Stability

There are two types of stability namely absolute stability and relative stability. Absolute stability means whether the system is stable or not i.e. the system is stable or unstable. If the system is stable then we determine how stable it is i.e. we measure the degree of stability, it is known as relative stability.

In time domain the relative stability is measured by maximum overshoot and damping ratio. In frequency domain relative stability is measured by resonant peak (M_r).

Constant Magnitude circle (M-circle)

Let, $G(j\omega) = x + jy$

Then from equation

$$M = M(\omega) = \frac{|C(j\omega)|}{|R(j\omega)|} = \left|\frac{G(j\omega)}{1+G(j\omega)}\right|$$

$$= \frac{|x+jy|}{|1+x+iy|} = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

Squaring both side.

$$M^2 = \frac{x^2+y^2}{(1+x)^2+y^2}$$

$$M^2[(1+x)^2+y^2] = x^2+y^2$$

or, $$M^2[1+2x+x^2+y^2] = x^2+y^2$$

$$M^2+2xM^2+x^2M^2+M^2y^2-x^2-y^2=0$$

$$x^2(M^2-1)+y^2(M^2-1)+2xM^2+M^2=0$$

or, $$x^2(1-M^2)+y^2(1-M^2)-2xM^2=M^2$$

Divide both sides by $(1-M^2)$

$$x^2-y^2-2x\frac{M^2}{1-M^2} = \frac{M^2}{1-M^2}$$

$\left(\frac{M^2}{1-M^2}\right)^2$ add in both sides.

$$x^2+y^2-2x\frac{M^2}{1-M^2}+\left(\frac{M^2}{1-M^2}\right)^2 = \frac{M^2}{1-M^2}+\left(\frac{M^2}{1-M^2}\right)^2$$

$$\left(x-\frac{M^2}{1-M^2}\right)^2+(y-0)^2 = \frac{M^2}{1-M^2}+\frac{M^4}{(1-M^2)^2}$$

$$\left(x-\frac{M^2}{1-M^2}\right)^2+(y-0)^2 = \frac{M^2}{(1-M^2)^2} \quad \text{----}(1)$$

Equation (1) is the equation of the circle with centre $\left(\frac{M^2}{1-M^2}, 0\right)$ and radius $\left(\frac{M}{1-M^2}\right)$

If $M=1$. Then equation (1) becomes

$$(1+x)^2 + y^2 = x^2 + y^2$$

or, $$x = -\frac{1}{2}$$

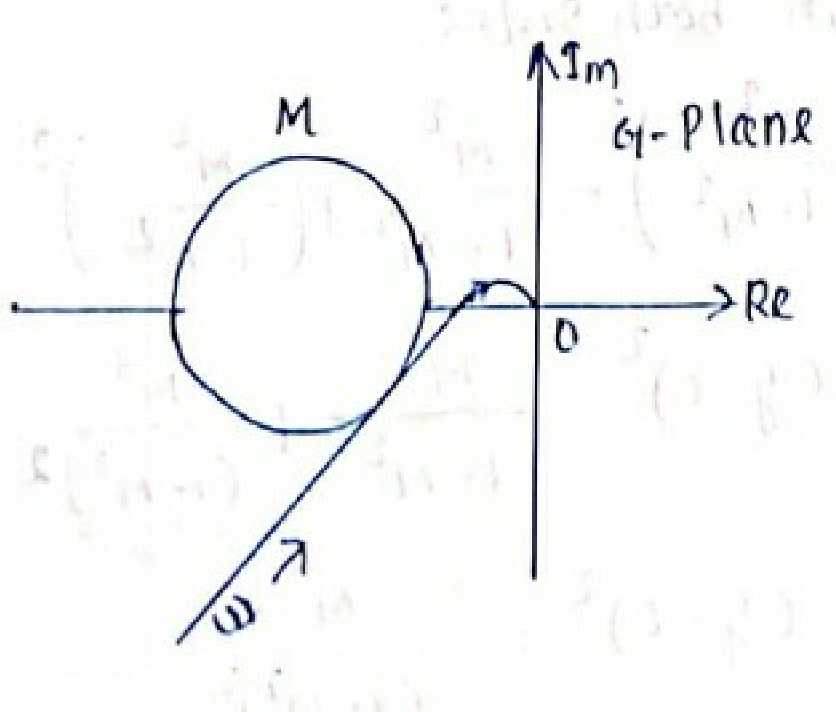
This is the equation of the straight line parallel to the y-axis and passing through $(-\frac{1}{2}, 0)$ in the $G(j\omega)$ plane.

The Constant M locii for different value of M. bellow the figure, it is clear that:

(a) The ~~locti~~ locii are symmetrical with respect to $M = 1$.

(b) The M-circle for $M>1$ are on the left side of the line $M=1$ and for $M<1$ the constant M-circles are on right side of the line $M=1$.

The intersection of $G(j\omega)$ plot (Nyquist plot) and constant M Locii gives the value of Magnitude M. The M-circle which is tangent to the $G(j\omega)$ plot will give the value of resonance peak (M_r) and resonance frequency ω_p.



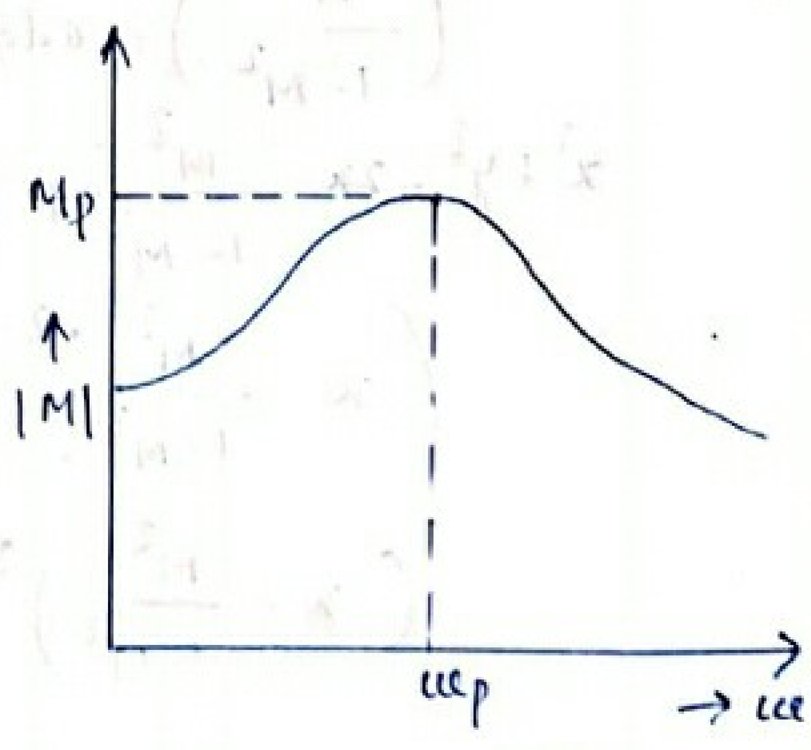
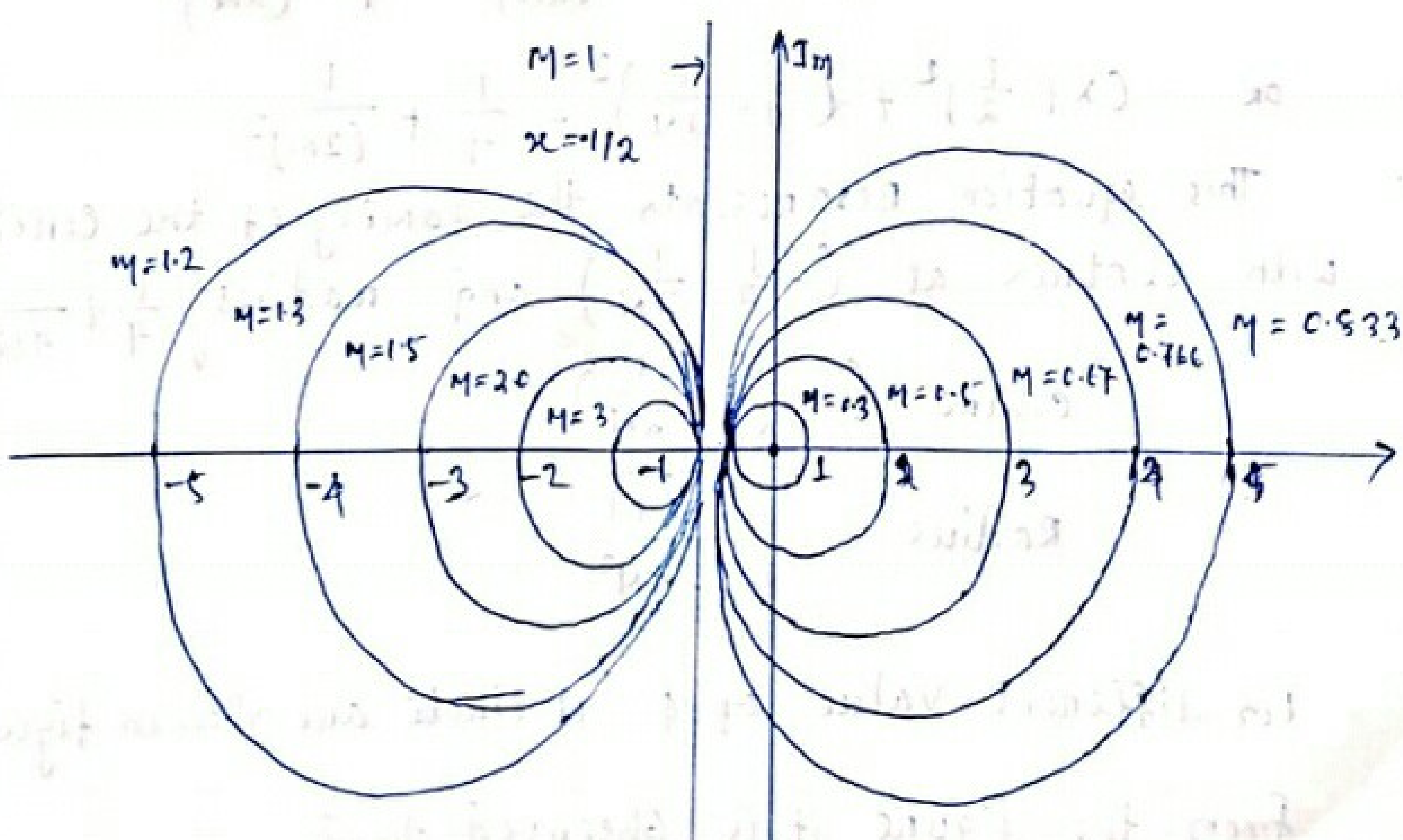


Table for Construction of M-circle

SL NO.	M	Centre $\frac{M^2}{1-M^2}, 0$	Radius $\frac{M}{1-M^2}$
1.	0.3	(0.098, 0)	0.329
2.	0.5	(0.33, 0)	0.666
3.	0.67	(0.814, 0)	1.215.
4.	0.766	(1.42, 0)	1.854
5	8.833	(2.27, 0)	2.72
6.	1.2	(-3.27, 0)	-2.72
7.	1.3	(-2.45, 0)	-1.88
8.	1.5	(-1.8, 0)	-1.2
9.	2.0	(-1.33, 0)	-0.66
10.	3.0	(-1.125, 0)	-0.375



(Constt. M- circles)

Constant N- Circles (Phase angle locii)

From equation the phase shift can be written as

$$\phi = \arg \text{ of } (x+jy) - \arg (1+x+jy)$$

$$\phi = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x} \quad \text{---} \quad (1)$$

or
$$\tan\phi = \frac{\frac{y}{x} - \frac{y}{1+x}}{1+\left(\frac{y}{x}\right)\left(\frac{y}{1+x}\right)} = \frac{y}{x^2+x+y^2} \quad \text{---} \quad (2)$$

Let.
$$\tan\phi = N$$

$$\therefore \quad N = \frac{y}{x^2+x+y} \quad \text{or} \quad N(x^2+x+y^2) = y$$

$$\Rightarrow \quad x^2+x+y^2 - \frac{y}{N} = 0$$

Add $\frac{1}{4} + \frac{1}{(2N)^2}$ to both sides, we get

$$x^2+x+y^2 - \frac{y}{N} + \frac{1}{4} + \frac{1}{(2N)^2} = \frac{1}{4} + \frac{1}{(2N)^2}$$

or
$$\left(x+\frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \frac{1}{(2N)^2}$$

This equation represents the family of the circles, with centre at $\left(-\frac{1}{2}, \frac{1}{2N}\right)$ and radius $\sqrt{\frac{1}{4}+\frac{1}{4N^2}}$.

centre $\left(-\frac{1}{2}, \frac{1}{2N}\right)$

$$\text{Radius} = \sqrt{\frac{N^2+1}{4N^2}}$$

For different value of ϕ, N circle are show in figure

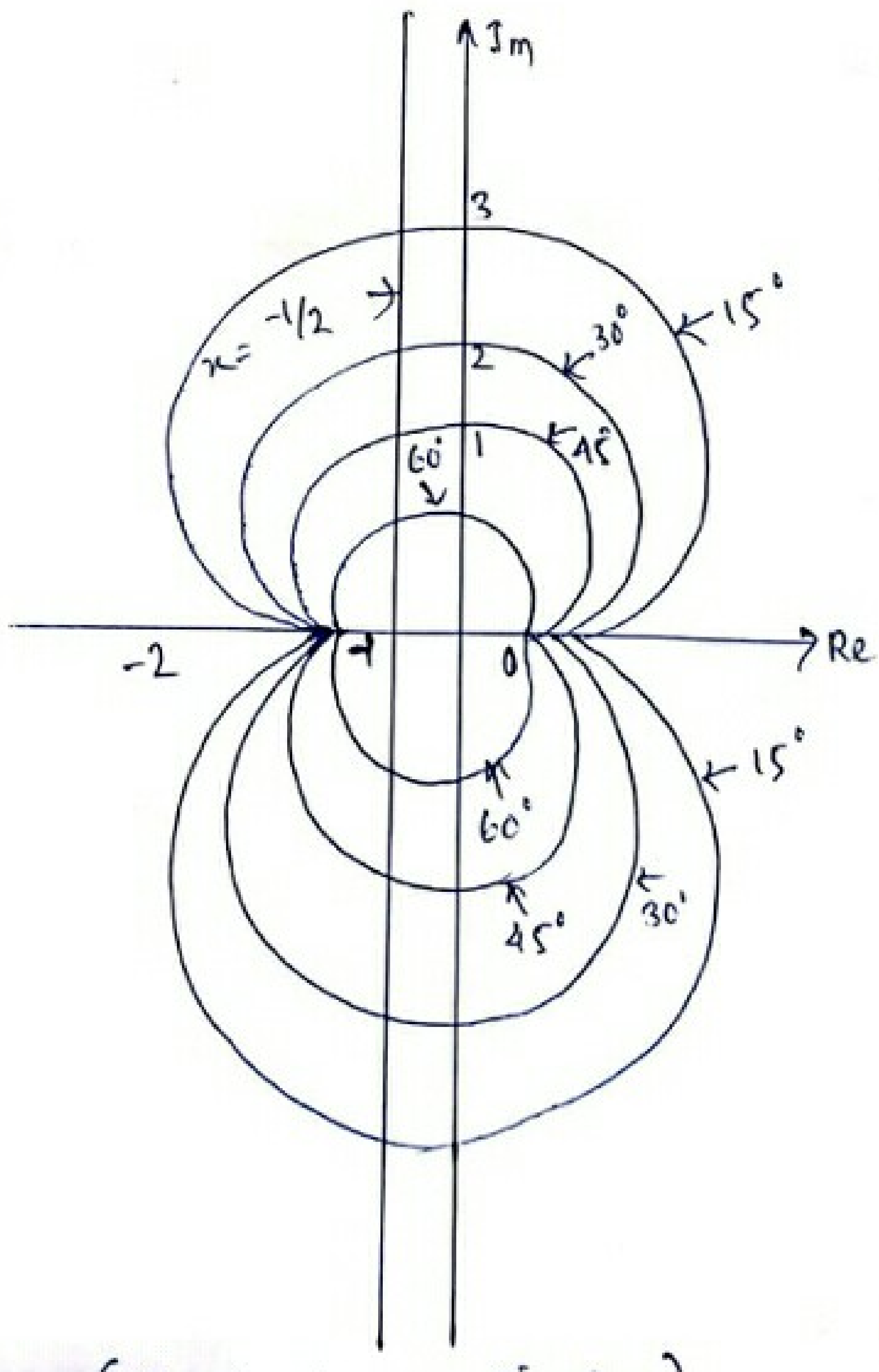
from the figure it is observed that.

(a) The centre is laying always at a distance $x = -\frac{1}{2}$ and y depends upon the phase shift.

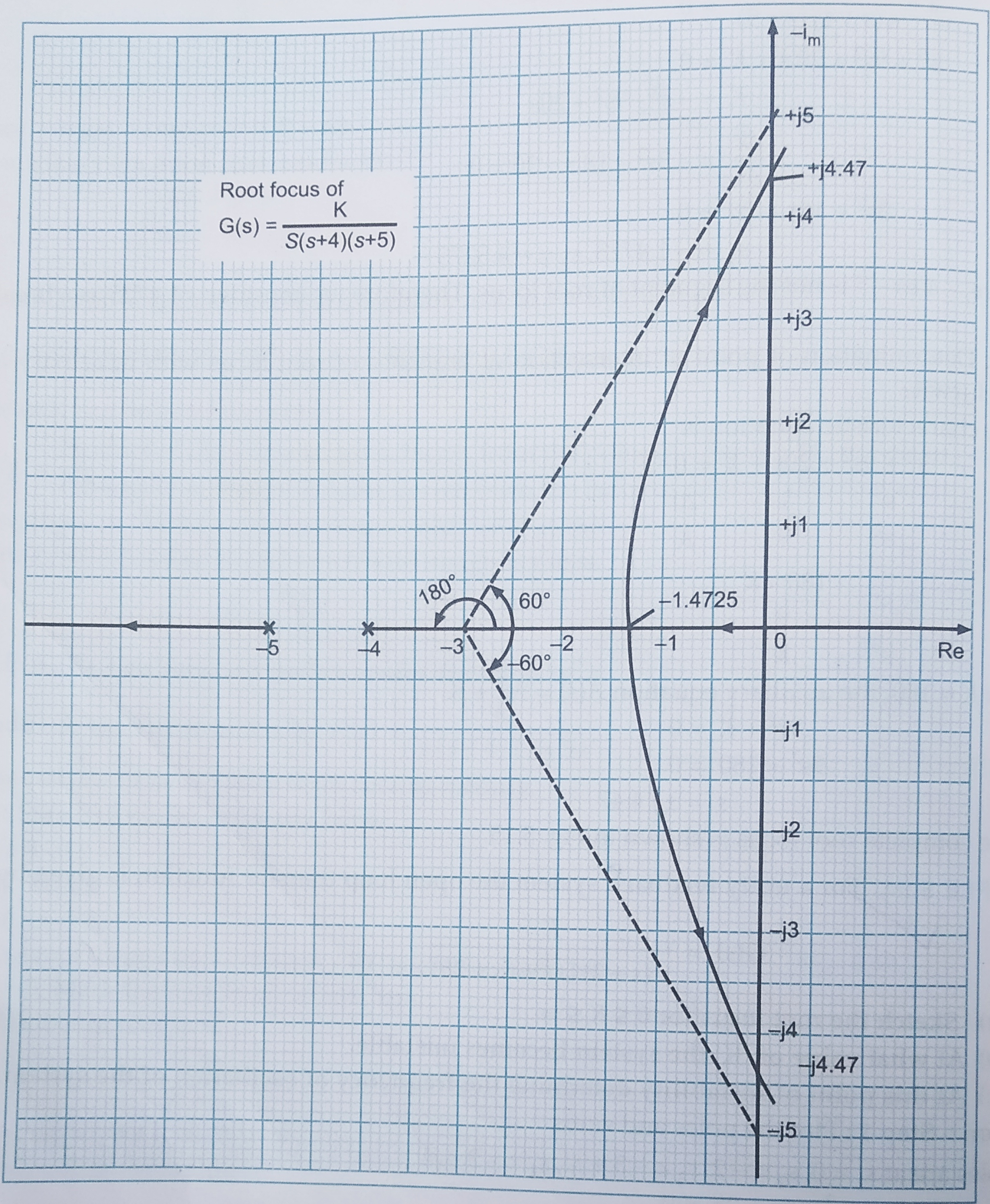
(b) All the circles passes through -1 as well as 0.

Table :-

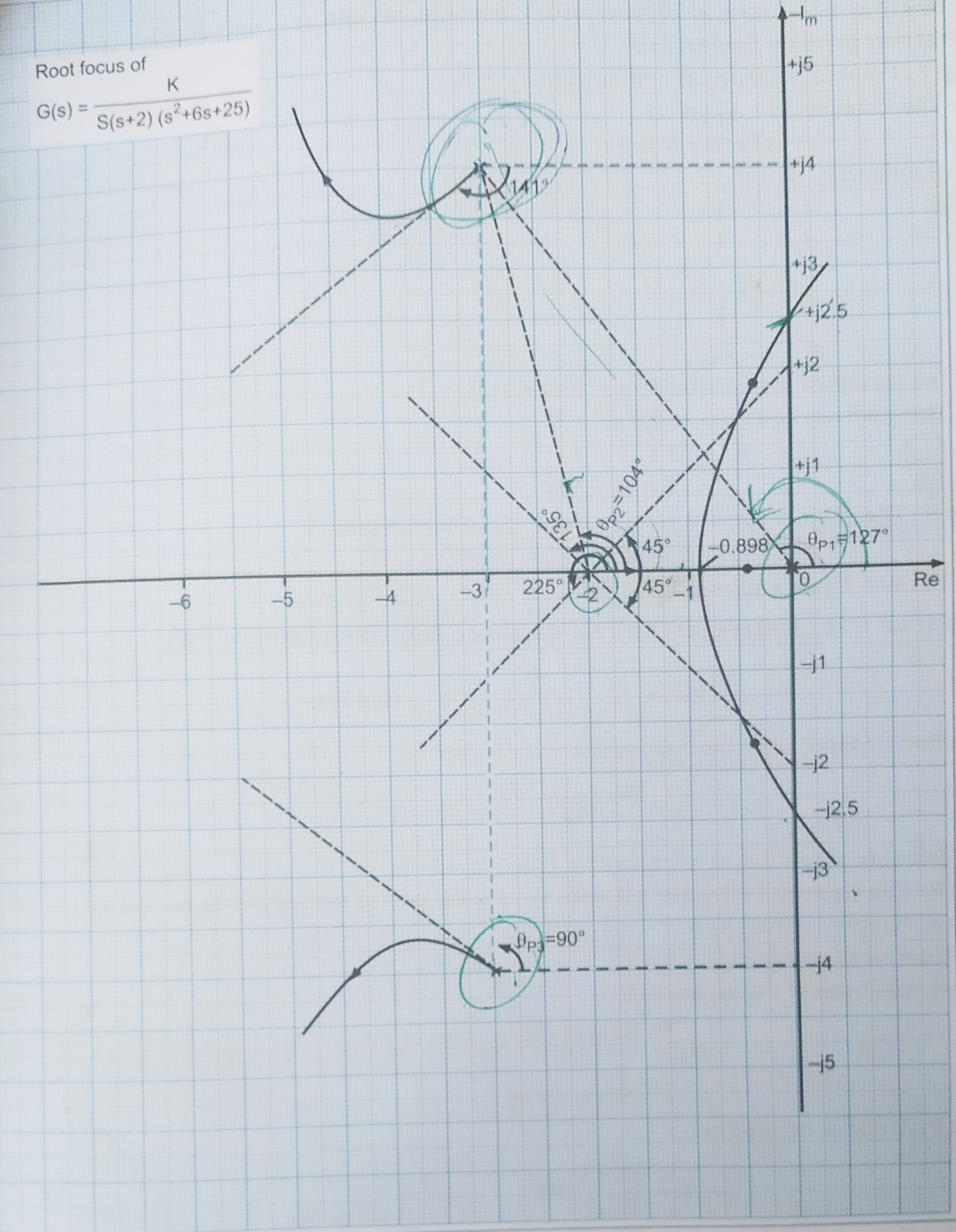
ϕ	$N = \tan\phi$	Centre	Radius
30°	0.577	(-0.5, 0.866)	1.0
45°	1.0	(-0.5, 0.5)	0.7
60°	1.732	(-0.5, 0.288) ~~(-0.5, 1.87)~~	0.577
15°	0.267	(0.5, 1.87)	1.9
-60°	-1.732	(-0.5, -0.288)	0.577
-45°	-1.0	(-0.5, -0.5)	0.7
-30°	-0.577	(-0.5, -0.866)	1.0



(Constant N- Circles)



Root focus of
$G(s) = \frac{K}{S(s+4)(s+5)}$
$-i_m$
$+j5$
$+j4.47$
$+j4$
$+j3$
$+j2$
$+j1$
-1.4725
180°
60°
$-60°$
-5
-4
-3
-2
-1
0
Re
$-j1$
$-j2$
$-j3$
$-j4$
$-j4.47$
$-j5$



Root focus of
G(s) = K / S(s+2) (s^2+6s+25)
Im
+j5
+j4
141°
+j3
+j2.5
+j2
+j1
θP2=104°
135°
45°
−0.898
θP1=127°
225°
45°
Re
−6
−5
−4
−3
−2
−1
0
−j1
−j2
−j2.5
−j3
θP3=90°
−j4
−j5

$G(s) = \frac{1000}{S(1+0.1s)(1+0.001s)}$
–60db/dec.
–40db/dec.
–20db/dec.
0db/dec.
ω_1
–20db/dec.
ω_2
–40db/dec.
$20\log_{10}K = 20\log_{10}1000$
Grain crossover frequency
PM=+26°
$|G(j\omega)|$ db
$\angle G(j\omega)$ deg.
ω (rad/sec)
60
40
20
0
60
50
40
30
20
10
0
–10
–100°
–120°
–140°
–160°
–180°
–200°
–220°
10^1
10^2
10^3
10^4

60
40
20
0
–60db/dec.
–40db/dec.
–20db/dec.
$G(s) = \dfrac{1000}{S(1+0.1s)\,(1+0.001s)}$
60
50
40
30
20
10
0
–10
|G (jω)| db
–20db/dec.
–40db/dec.
Gain crossover frequency
ω_c
–80°
–100°
–120°
–140°
–160°
–180°
–200°
–220°
∠G (jω)
–60db/dec.
Phase crossover frequency
1
10^1
10^2
10^3
10^4
ω (rad/sec)

VI- SEM ELECTRICAL/E&M/E&ETC//2019(W)
EET604(E(iii)/EMT604(E(ii))/ETT604E(iii)-CONTROL SYSTEM ENGINEERING

Full Marks: 80 Time : 3 Hours

Answer any FIVE Questions including Q No. 1 & 2
Figures in the right hand margin indicates marks

1. Answer ALL the questions. (2 X 10)
   a. Define centroid and breakaway point in root locus.
   b. What is meant by reasonant peak and bandwidth of a system?
   c. State the formula to find out angle of arrival and angle of departure.
   d. What is a Nichol's chart?
   e. Define Transfer Function.
   f. What do you mean by polar plot?
   g. What is the effect of addition of zeroes on root locus?
   h. Define rise time and peak time of a system.
   i. What is Mason's Gain formula?
   j. What is Routh's stability criterion?

2. Answer any SIX questions: (5 X 6)
   i. Determine the stability of a system whose characteristic equation is $s^4+2s^3+10s^2+8s+4 = 0$.
   ii. Explain steady state error and error constants.
   iii. Derive rise time, maximum peak overshoot and settling time for a 2^{nd} order underdamped system.
   iv. Discuss the correlation between time domain and frequency domain specifications.
   v. Draw the polar plot of given open loop system $G(s) = \frac{K}{(1+sT1)(1+sT2)}$
   vi. Find error co-efficients and steady state error of a unity feedback system whose open loop transfer function is given by $G(s) = \frac{108}{s^2(\,s+4)(s^2+3s+12)}$ when subjected to a input given by $r(t)= 2+5t+2t^2$.
   vii. Find the Y/X of the signal flow graph shown in the FIG-A below:

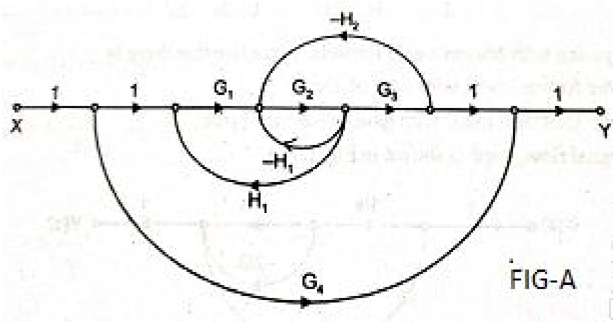


FIG-A

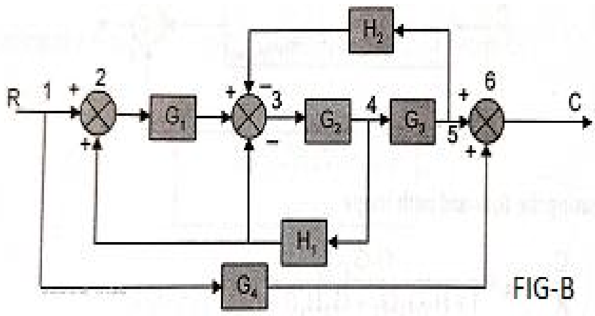


FIG-B

3. Using block diagram reduction techniques, find $\frac{C(s)}{R(s)}$ of the system given in the FIG-B. (10)

4. Sketch the root locus $G(s)*H(s) = \frac{K}{s(s+5)(s+10)}$. Also mention about the stability. (10)

5. Apply Nyquist stability criterion to the system with loop transfer function as given in the above question and mention about its stability. (10)

6. A unity feedback control system has $G(s) = \frac{80}{s(s+2)(s+20)}$. Draw the bode plot. Determine G.M., P.M., w_{gc}, w_{pc}. Also comment on its stability. (10)

7. Write short notes on any two of the following: (10)
   a) Constant M-circles and N-circles
   b) All pass and minimum phase system
   c) PID Controller